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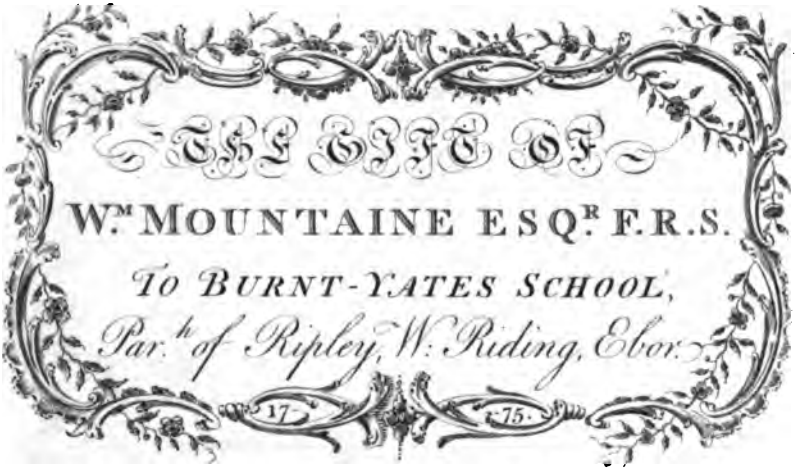

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JOHN BRISTOW,

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Allow me, Sir, before I conclude this addreſs, to return you my thanks for a long ſeries of favours; favours I am no otherwiſe able to requite than by a grateful remembrance, and which could only flow from a benevolent heart.

I am,

With the greateſt ſincerity and reſpect,

SIR,

Haughton-Park,
March 25, 1749.

Your moſt obliged,

humble Servant,

FRANCIS HOLLIDAY.

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M.DCC.XLV.

PREFACE.

THE editor of the Gentleman's Magazine having been ſollicited, by ſeveral ingenious mathematicians, to inſert in it ſo many of their pieces as would have diſguſted the far greater part of his readers, whoſe turn of mind leads them to delight in a very different kind of amuſement, he was under a neceſſity of diſregarding thoſe correſpondents. But while he complyed with the taſte of the polite and gay, the politician and the patriot, he thought himſelf indiſpenſably obliged to conſider likewiſe his friends of a more philoſophical diſpoſition, and therefore he readily joined in a ſcheme for publiſhing ſuch productions once a quarter in a ſeparate pamphlet; nothing of this kind being now on foot periodically, except the Philoſophical Tranſactions, and the annual Diaries; the former of which, however well executed, muſt be acknowledged very improper for initiating youth; and in the latter too little room is allowed for ſolutions at large to queſtions worthy of conſideration: A defect and inconvenience, which it is the intention of this work to provide againſt.

The ſtudy of the mathematics has in all ages and countries where learning hath prevailed, been regarded as one of the moſt valuable parts of ſcience; and, as it is now riſing into great eſteem in this nation, we hope this undertaking will meet with a favourable reception, and in ſome meaſure anſwer our deſign, which is to encourage the labours of thoſe who are eminently ſkilled in this branch of uſeful knowledge, a knowledge that accuſtoms the mind to reflection, and gives it a habit of cloſe and demonſtrative reaſoning.

Many other parts of ſcience are founded only on probable conjectures, in the midſt of which the mind not knowing where to fix, and wanting ſufficient principles to direct its ſearches, wanders in the dark, and at length gives them over as uſeleſs and unſatisfactory. The mathematics, on the contrary, being founded on demonſtration, and the unalterable nature of things, enlighten the mind, give it a thirſt for truth, quicken and ſharpen the invention, ſtrengthen and extend the imagination, improve and refine the reaſoning faculty, and are of infinite uſe both in the neceſſary and luxurious improvements of mechanical arts. This ſtudy is, particularly, the baſis of thoſe ſciences which give us a clear and extenſive knowledge of the ſyſtem of the world, and, as they lead us to the moſt profound reverence of the ALMIGHTY, and wiſe Creator, ſo they free us from the mean and narrow thoughts that ſpring from prejudice, ſuperſtition and bigotry. In ſhort, the ſtudy of the mathematics is not only an agreeable and beneficial amuſement in a retired and country life, but is the beſt expedient for forming the minds of youth, by taking them off from the fruitleſs and airy exerciſes of the fancy, and rendering them ſerious, diligent and inquiſitive in the ſearch of knowledge and truth.

These arguments might be illustrated by many fine observations drawn from the antient Greek *philosophers, as* Plato, Aristotle, Hipparchus, Apollonius Pergaeus, Proclus, Diophantus, Theon, Archimedes, *&c. and also from the fathers of the christian church, as* Origen, Tertullian, Gregory Nazianzen, Lactantius, *&c. But we shall content ourselves with mentioning the sentiments of our own countryman, the celebrated and judicious Mr* Locke. Would you have a man reason well, *says he*, you must exercise his mind in observing the connexion of ideas, and following them in train. Nothing does this better than *mathematics*, which therefore, I think, should be taught all those who have time and opportunity; not so much to make them mathematicians, as to make them *reasonable creatures*. I have mentioned mathematics as a way to settle in the mind a habit of reasoning closely, and in train: not that I think it necessary all men should be deep mathematicians; but that having got the way of reasoning, which that study necessarily brings the mind to, they might be able to transfer it to other parts of knowledge, as they shall have occasion: for in all sorts of reasoning, every single argument should be managed as a mathematical demonstration.—I have mentioned *mathematics*, wherein *algebra* gives new helps and views to the understanding; and I think the study of them is of infinite use, even to grown persons. First, By experimentally convincing them that to make any one reason well, it is not enough that he have parts wherewith he is satisfy'd, and that serve him well enough in his ordinary converse; a man in these studies will see that, however good he may think his understanding, yet in many things, and those very visible, it may fail him. This would take off that presumption, that most men have of themselves in this part; and they would not be so apt to think their minds wanted no helps to enlarge them, that there could be nothing added to the acuteness and penetration of their understandings. Secondly, The study of the mathematics would shew them the necessity there is, in reasoning, to separate all the distinct ideas, see the habitudes that all those concerned in the present enquiry have to one another, and to lay by those which relate not to the proposition in hand, and wholly to leave them out of the reckoning. This is that which, in other subjects besides quantity, is what is absolutely requisite to just reasoning, tho' in them it's not so easily observed, nor so carefully practised. In those parts of knowledge where 'tis thought *demonstration* has nothing to do, men reason, as it were, in the lump; and if, upon a summary and confused view, or upon a partial consideration, they can raise the appearance of *probability*, they usually rest content, especially if it be in a dispute where every little straw is laid hold on, and every thing, that can but be drawn in any way to give colour to the argument, is advanced with ostentation. But that mind is not in a posture to find the truth, that does not distinctly take all the parts asunder, and, omitting what is not at all to the point, draw a conclusion from the result of all the particulars which any way influence it.

The CONDUCTOR.

Miſcellanea Curioſa, &c.

THE eaſe, perſpicuity, and general uſe of the following extract, with the great character of its author, entitle it to ſerve for an INTRODUCTION preferable to any compoſition of our own.

Of the proportion of velocity and forces in bodies in motion. Philoſophical Tranſactions, N° 401. *By Dr* S. Clarke.

EVERY effect muſt neceſſarily be proportionate to the cauſe of that effect, that is, to the action of the cauſe, or the power exerted at the time when the effect is produced. To ſuppoſe any effect proportional to the ſquare or cube of its cauſe, is to ſuppoſe that an effect ariſes partly from its cauſe, and partly from nothing.

In a body in motion, the force ariſing from the quantity of the matter, as its cauſe, muſt neceſſarily be proportional to the quantity of the matter: and the force ariſing from the velocity of the motion, as its cauſe, muſt neceſſarily be proportional to the velocity of the motion. The whole force, therefore, ariſing from theſe two cauſes, muſt neceſſarily be proportional to theſe two cauſes taken together. And, therefore, in bodies of equal bigneſs and denſity, or in one and the ſame body, the quantity of matter continuing always the ſame, the force muſt neceſſarily be always proportional to the velocity of the motion. If the force were as the ſquare of the velocity, all that part of the force, which was above the proportion of the velocity, would ariſe either out of nothing, or (according to Mr *Leibnitz*'s philoſophy) out of ſome living ſoul eſſentially belonging to every particle of matter.

Whenever any effect is in a duplicate proportion, or as the ſquare of any cauſe, it is always either becauſe there are two cauſes acting at the ſame time, or that one and the ſame cauſe continues to act for a double quantity of time.

The reſiſtance made to a body, moving in any fluid medium, is

in a duplicate proportion to the velocity of its motion; becauſe, in proportion to its velocity, it is reſiſted by a greater number of particles in the ſame time; and again, in proportion to its velocity, it is reſiſted by the ſame particles ſingly with a greater force, as being to be moved out of their places with greater velocity.

Light decreaſes in a duplicate proportion of its diſtance from the ſun, becauſe the rays divaricate according to two dimenſions; according to the dimenſion upwards or downwards, and according to the dimenſion ſideways. But, according to the third dimenſion forwards from the ſun, a ray of light undergoes no alteration, becauſe the particles of which it conſiſts, being all emitted with an equal velocity, continue every where at equal diſtance from each other.

One and the ſame cauſe, acting in a double quantity of time, produces the ſame effect as two equal cauſes acting in a ſingle quantity of time. One and the ſame force, in two parts of time, will cauſe a body in motion to deſcribe the ſame ſpace, as double the force would do in one part of time. The ſpace deſcribed therefore by a body in motion is not as the force, but as the force and time taken together. A body, with any the leaſt aſſignable force, will move through infinite ſpace, if it meets with no reſiſtance in an infinite time. And in ſpaces, where there is an uniform reſiſtance to motion, the ſpace deſcribed before the motion ceaſes muſt needs be as the force and as the time together; becauſe a double force will carry a body twice as far in the ſame time, and will alſo cauſe the motion to be twice as long time in deſtroying by an uniform reſiſtance. The ſpace deſcribed, therefore, before the motion ceaſes, is in this caſe demonſtrably as the ſquare of the force. A body thrown upwards with double force, will be carried four times as high, before its motion be ſtopped by the uniform reſiſtance of gravity, becauſe the double force will carry it twice as high in the ſame time, and moreover require twice the time for the uniform reſiſtance to deſtroy the motion. The caſe is the ſame in accelerated motion, in bodies accelerated by a ſucceſſion of elaſtic impreſſions, or falling with a motion accelerated by the uniform power of gravity, or by any other uniform power whatſoever. The ſpace deſcribed muſt needs be as the force, and as the time wherein the force operates.

What I have thus demonſtrated concerning any force, conſidered as the cauſe producing an effect, and concerning the time, during which the force operates, is on all hands acknowledged to be true concerning velocity: and, therefore, velocity and force, in this

caſe,

caſe, are one and the ſame thing. So that to affirm force to be as the ſquare of the velocity, is to affirm that the force is equal to the ſquare of itſelf.

Now from hence appears very clearly the ground of the error theſe Gentlemen have fallen into, and of their miſapplication of the experiments they build upon.

The effect of a force impreſs'd on a moveable body is the motion of that body from one place to another. Now foraſmuch as the effect cannot but be proportional to its cauſe, hence Mr *Leibnitz* (whom the other Gentlemen have followed) contends that the ſpace deſcribed by a body, in falling, is proportional to the force by which it is impelled during its fall; which ſpace being agreed to be as the ſquare of the velocity (as being proportional to the velocity and to the time taken together) hence they infer that the force likewiſe is as the ſquare of the velocity.

But, from what has been ſaid, it is plain that the ſpace deſcribed, in theſe and all other the like caſes, is not as the force only, but as the force, and as the time wherein the force acts; that is to ſay, as the ſquare of the force. For the cauſe of the quantity of the ſpace deſcribed is not barely the quantity of the force, but alſo the continuance of the time wherein the force acts. The force, therefore, and the time taken together, being neceſſarily as the ſpace deſcribed, (as the velocity and the time taken together are on all hands acknowledged to be,) it follows that the velocity and force are equal, and not the force as the ſquare of the velocity.

When two unequal bodies, faſtened at the ends of the arms of a balance of unequal length, counterpoiſe each other, and vibrate in equal times, as they muſt neceſſarily do, being faſtened to the arms of the ſame balance (which is an obſervation Mr *Leibnitz* lays great ſtreſs upon) in that caſe indeed the forces will be as the ſpaces deſcribed, but not therefore as the ſquare of the velocities. For, in that caſe, the velocities themſelves are as the ſpaces deſcribed, becauſe the times are equal.

When a body projected with a double velocity enters deeper into ſnow or ſoft clay, or into a heap of ſpringy or elaſtic parts, than in proportion to its velocity, it is not becauſe the force is more than proportional to the velocity, but becauſe the depth it penetrates, into a ſoft medium, ariſes partly from the degree of the force or velocity, and partly from the time wherein the force operates before it be ſpent.

In the colliſion of hard bodies, it is, I think, agreed on all hands, that

that it is demonſtrated by reaſon, and confirmed by experience, that when a perfectly hard ball, moved with whatever degrees of velocity, ſtrikes full upon another hard ball, equal in bigneſs and weight, and without any motion in it; if the balls be unelaſtic, they will both go on together the ſame way, dividing the motion equally between them, with half the velocity the firſt ball had originally: but if they be perfectly elaſtic, the moving ball will communicate its whole motion and velocity to the quieſcent ball, and itſelf lye ſtill in the other's place. Were it true, now, that the force of the moving ball was as the ſquare of its velocity, theſe experiments would then ſhew (which is infinitely abſurd) that the force, or *vis inertiae*, in the quieſcent ball, or the dead force was always proportional to the ſquare of the velocity (which theſe Gentlemen affect to call the living force) of the moving ball, whatever its velocity were: or the force in both might juſt as reaſonably be ſuppoſed to be as the cube, or the quadrato-quadrate, or any other power of the velocity of the moving ball; which is turning the nature of things into redicule.

A TABLE *of the times when the paths of the* Satellites *of* Jupiter *and* Saturn *become convex toward the Sun. And a method propoſed for demonſtrating the Earth's motion, and inveſtigating the parallaxes of the Sun and Planets. By* X. Y.

THIS table exhibits the times in which the paths of the *Satellites* of *Jupiter* and *Saturn* are *concave* and *convex* toward the Sun; computed from their known *periods* and diſtances, from Sir *Iſaac Newton's Principia*, or Dr *Smith's Optics*, B. III. by the rule given by me at page 279. No. III. of the *Miſcellaneous Correſpondence*, printed for *E. Cave*, which may be acceptable to ſuch as love to contemplate the *geometry* of *nature*, even abſtracted from practical uſe.

Satell.	Concave before, or after oppoſition.				♃	Convex before, or after conjunction.			
	d.	h.	′	″		d.	h.	′	″
I		17	37	52½			3	36	25½
II	1	16	32	7½			2	6	49
III	2	19	25	58½			18	33	53½
IV	5	20	16	49		2	12	45	43½
					♄				
I		19	48	54			2	50	33½
II	1	5	45	44			3	5	27
III	1	19	20	45			10	53	13
IV	5	10	13	25		2	13	24	14½
V	23	13	48	43		16	9	9	29½

But

But the caſe of the innermoſt *Satellites*, furniſhing likewiſe matter of obſervation, deſerves to be conſidered a-part.

1. The *epicycloids* which they deſcribe, being *contracted* as ſoon as they become convex, the *heliocentric* motion of the Satellites becomes retrograde. Thus the I. ſatellite of *Jupiter* is retrograde for $3^h\ 36'\ 25''\frac{1}{2}$, before its conjunction, and for the like time after it; and that of *Saturn* for $2^h\ 50'\ 33''\frac{1}{2}$.

2. I find likewiſe that the 1ſt *Satellite* of *Jupiter*, $7^h\ 34'\ 5''$ before its Conjunction, paſſes through the *double-point* of the curve; and by the like time, after the conjunction, will have returned to the ſame *points*: and that the 1ſt *Satellite* of *Saturn*, $4^h\ 57'\ 5''$ before its conjunction, paſſes through a *double-point*, to which it is to return after $9^h\ 54'\ 10''$.

3. Whence it follows, that either of theſe points may, on certain occaſions, do the office of a *fixt ſtar*, that ſhould be placed as that point.

For example, *Jupiter*, being not far from his *oppoſition*, if his 1ſt *Satellite* can be obſerved as it paſſes through the *double-point*, and is nearly in conjunction with a *fixt ſtar*; and again at the interval of $15^h\ 8'\ 10''$ when it returns to it, the *difference* in longitude of its diſtances from the ſtar will be the *parallax* of the *Earth*'s orbit ſeen from *Jupiter*, which anſwers to $15^h\ 8'\ 10''$. By this means the *diſtances* of *Jupiter* and the *Earth* from the *Sun* may be compared, and the *Earth*'s motion *demonſtrated*.

4. Or if *theſe* are aſſumed as known from other principles, and the computed difference of the places of the *double-point*, which reſults from the annual parallax only, ſubtracted from the obſerved difference, what remains, if reduced to the *equinoctial*, will give the *diurnal* parallax of the correſponding *chord* of the obſerver's *parallel* of *latitude*.

5. This *parallax* will be greater, the nearer to the *equator* the obſervation is made; and may be ſomewhat increaſed by making the ſecond obſervation, as near as may be, at the diſtance of 12^h, inſtead of $15^h\ 8'\ 10''$, and allowing for the difference; although ſcarcely in any caſe can it riſe to *half* the *Sun*'s horizontal parallax.

Yet, in the perfection to which inſtruments are daily advancing, even *this angle* may poſſibly ſhew itſelf on a micrometer, ſo as to be of ſome uſe in determining the parallaxes; eſpecially as it may be eaſily purged of every inequality, and its ſmall quantity be made at leaſt to ſtand clear by itſelf, provided only the *micrometer*

is well contrived and adjuſted, and the teleſcope of a due length and aperture.

6. The interval 15^h 8′ 10″ being computed upon the *mean* motion of *Jupiter*, may be corrected upon his *actual* motion at the time of obſerving, either by a new calculation, or by a table prepared on purpoſe.

7. And, in determining the *double-point*, regard muſt be had to the time in which *light* is propagated to us from *Jupiter*, which muſt be added to the time between the *double-point* and *conjunction*, and the ſum taken from the *apparent* time when the *Satellite* was in the middle of *Jupiter*'s ſhadow. In this, however, a difference of ſome minutes in time is of no conſequence; becauſe the ſmall errors, which it produces, will lie on the ſame ſide of the *point*, and will be nearly equal.

Some other uſes of this *double-point* will readily occur, as when *Mars* happens to be in conjunction with *Jupiter*, &c.

Feb. 23, 174¼. X. Y.

The following is a demonſtration of four Theorems *publiſhed in the* Gent. Mag. *for* October 1737, (*to which we refer*) *for determining accurately the* Sun's parallax; *by the celebrated mathematician Mr* N. Facio, F.R.S.

THEOREM I.

THAT in thoſe ſtereographic maps, where a terreſtrial or celeſtial hemiſphere is projected upon a plane parallel to a meridian, the eye being ſuppoſed in the ſurface of the ſphere, and the line drawn from the eye thro' the center of the ſphere being perpendicular to the plane of the projection, all the angles formed upon the ſphere (where any circles great or ſmall, or their tangents interſect each other) are equal to the angles repreſenting them in the projection.

DEMONSTRATION.

Fig. 1. 1. In the right line, SPTA, let SP to PT be as SA to AT; and upon the diameter, PA, deſcribe the *ſtereographic* ſphere, ALPA, having its poles in S and T. From any point, L, in that ſphere, draw the lines LS, LP, LT. And, I ſay, that SL is always to LT in the determined proportion of SP to PT.

2. For to the angle, TLP, make the angle, PLE, equal; and

and prolong the line, LE, till it cuts the line, AP: And, becauſe of the equality of thoſe angles, the interſection will fall upon the very point, S. For thus, and thus only, will LS to LT be as SP to PT.

3. I have called the ſphere, ALPA, *ſtereographic*, becauſe, in the common ſtereographic projections of the ſphere of the world, ſuppoſing S and T to be the poles of the projection, and the circle ALPA any one of the circles parallel to the equator, the nature of the projection itſelf requires, that SP be to PT, as SA to AT.

THEOREM II.

Definition. IF ſuch a ſtereographic projection, as I have juſt now deſcribed, be extended on all ſides *in infinitum*, ſo that it may contain a repreſentation of the whole ſphere; and if about each pole all the parallels be drawn in it by intire circles from minute to minute, or from ſecond to ſecond; and the whole be turned about the infinite axis paſſing thro' the poles: I call any of the ſpherical ſurfaces thus formed by an intire revolution, a *ſtereographic ſphere*.

The THEOREM.

In any propoſed ſtereographic ſphere, OLP, *having its centre* C *upon the prolonged axis* ST, *any two lines* LS, LT, *drawn from any point* L *of that ſphere to the poles* S *and* T *of the projection, are to one another in one and the ſame proportion. And, by conſequence, if the centres of the Sun and of the Earth be placed in the poles,* S *and* T; *and if the centre of the Moon deſcribe any orbit, either circular, or more compoſed, while it moves upon the ſurface of the ſtereographic ſphere* OLP; *the lines drawn from the centre of the Moon to the centres of the Sun and of the Earth, will be to one another in one and the ſame proportion.* FIG. 2.

DEMONSTRATION.

4. Suppoſing S and T, placed upon the line SPTA, to be the poles of a common ſtereographic projection of the ſphere TLOS; and AP to be the diameter of a circle ALP parallel to the equator; I ſay, that SP is to PT, as SA to AT. FIG. 3.

5. Let O repreſent the eye placed in the equator; and draw the line S*p*O*a*; and upon it draw the perpendicular lines P*p*, TO and

TO and A *a*. And if S *p* be to *p* O, as S *a* to *a* O; then, by conſequence, SP will be to PT, as SA to AT. But I ſay, that S *p* is indeed to *p* O, as S *a* to *a* O.

6. For let the lines OP, and OA be drawn: and, by the nature of the projection, the angles POT and TOA will be equal: and to them alſo will the angles *p* PO, and *a* AO be equal. Now taking P *p*, or its equal S *p*, for radius; then *p* O is tangent of a certain determined angle: and taking A *a*, or its equal S *a*, for radius, then is *a* O tangent of the ſame determined angle. From whence it follows that really S *p* is to *p* O, as S *a* to *a* O.

7. And here it is evident reciprocally, that if A and P be made the poles of another common ſtereographic projection ALP, then will the circle TLOS be a parallel to the equator thereof, and repreſent what I have called a ſtereographic ſphere; for SP will be to PT, as SA to AT.

THEOREM III.

Fig. 2. *IF, in a ſtereographic ſphere* OLP, *the gravitations of the Moon towards the Sun and towards the Earth be directly as* s *the maſs of the Sun, and* t *the maſs of the Earth; and reciprocally as* $\overline{SL}^2$ *and* $\overline{LT}^2$, that is, *if thoſe gravitations be as* $\frac{s}{SL\text{ quad.}}$ *and* $\frac{t}{TL\text{ quad.}}$, *which is the caſe in the ſolar ſyſtem; then the direction of the two united gravitations of the Moon will tend to one and the ſame focus* F, *or* f, *placed ſome where upon the line or axis* ST, *or rather upon the line* PT. *And, by conſequence, equal areas will be deſcribed about that focus in equal times.*

DEMONSTRATION.

Fig. 4. 8. Let S be the centre of the Sun, T the centre of the Earth, or rather the common centre of gravity of the Earth and of the Moon. Let ALP be a ſtereographic ſphere, upon which the centre of the Moon deſcribes at certain times her orbit. From any point L of that orbit, draw the line LT; and make TY parallel to SL; and let LT be to TY, as the gravitation of the Moon toward the Earth, to the gravitation of the Moon toward the Sun; and draw the line LY, ſhewing the direction of thoſe two united gravitations. I ſay, that LY will cut the line ST in a determined

determined focus F, from which the line drawn to the centre of the Moon will deſcribe areas proportional to the times of their deſcription. But, I ſuppoſe, with Sir *Iſaac Newton*, that the centre S, and the centre T, or rather the common centre of gravity of the Earth and of the Moon, do remain at reſt.

9. For S L is to L T in a determined proportion of S P to P T, becauſe A L P is a ſtereographic orbit. And L T is to T Y in the determined proportion of the gravitation of the Moon toward the Earth, to the gravitation of the Moon toward the Sun. And therefore S L will be to T Y, or S F to F T, in a determined proportion; and, by conſequence, the ſituation of the focus F will be determined upon S T.

THEOREM IV.

AND, in general, *if the gravitations be as* s *and* t *directly,* Fig. 2. *and as* SL^n *and* LT^n *directly, taking n for any index whatſoever, affirmative or negative*; that is, *if thoſe gravitations be as* $s \times SL^n$ *and* $t \times LT^n$; *the direction of thoſe two united gravitations will tend towards*, or in oppoſition *to, one and the ſame focus* F, *or f, placed ſomewhere upon the axis* S T, *or rather upon the line* T P.

DEMONSTRATION.

10. In any common ſtereographic projection of the ſphere Fig. 5. O A C, deſcribed upon a plane C E I tangent of the ſphere; the eye O being ſuppoſed in the ſurface; and the line O C, drawn from the eye thro' the centre of the ſphere, being perpendicular to the plane of the projection: I ſay, that all the angles formed upon the ſphere (where any circles, great or ſmall, or their tangents, interſect each other) are equal to the angles repreſenting them in the projection.

11. For upon the plane that touches the ſphere in A, and whoſe ſection with the plane of the projection is perpendicular to A E and to C E, tangents of the circle O A C, take the angle B A D formed by A B and A D, two tangents of the ſphere, piercing the plane of the projection in the determined points B and D. And I ſay, firſt, that the point A, being projected in I by the right line O A I, the tangent A E will be projected by the line E I equal to A E. For let the tangent O T be drawn parallel to C E I; and the angle E A I will be meaſured by half the

arc A O. Likewiſe the angle A I E, or its equal A O T, will be meaſured by half the arc A O; and ſo the triangle A E I will be equicrural. But the points B and D, being in the very plane of the projection, they do project themſelves. And ſo the tangent A B is projected by its equal B I, and the tangent A D is projected by its equal D I, and the baſis B D remaining common, the triangle B A D is projected by its equal triangle B I D; and the angle B A D is projected by its equal B I D: which was to be demonſtrated. [*To be continued.*]

Maddersfield Oct. 6, 1744. *Nicolas Facio.*

A univerſal ſpherico-catoptric theorem, tranſlated from the Latin *in* N° 295 *of the* Phil. Tranſ. *By Mr* HOLLIDAY.

FIG. 6. LET D E F be a portion of the *concave ſpherical ſpeculum*, whoſe centre is B, ſemidiameter B E, or B D. Likewiſe let A be the *radiating* point placed in the axis, from which let the *radiating* line A D flow, which let be reflected to the point D in D C. Now the diſtance of the focus C, from F the vertex of the *ſpeculum*, is to be inveſtigated.

Whence it is to be obſerved, that we ſuppoſe the point D near to E, for the rays, being more remote, will glide from the eye (which we have conſtituted in the axis A E) nor do they tend any thing to the viſion of the image. Moreover, becauſe the arc D E is indefinitely ſmall, the angles D A B, A D B (as alſo their ſum D B C) are very ſmall; and therefore they will have the ſame ratio among themſelves, as their oppoſite ſides.

Theſe being premiſed, let A B $=b$, B D $=$ B E $=r$, B C $=x$ C E $(=r-x$ or for brevity's ſake) $=y$. The quantities b and r are known (for the ſemidiameter of the *ſpeculum*, and the diſtance of the lucid point from the vertex are given) but x and y are ſought and unknown. Now in the triangle D A B it will be $\angle$ D A B : $\angle$ A D B :: r : b. Alſo in the triangle D B C, $\angle$ B D C $=\angle$ A D B, by the nature of reflexion; and $\angle$ D B C $=\angle$ D A B $+\angle$ A D B, from *Euclid*'s elements. Therefore when $\angle$ D B C is as $r+b$, and $\angle$ B D C as b; likewiſe it will be $\angle$ D B C : B D C :: $r+b$: b; and D C : B C :: $r+b$: b. But becauſe the point D is very near to E, we muſt eſteem D C $=$ C E. Therefore C E : B C :: $r+b$: b. That is y : x :: $r+b$: b, and (by com-

compounding) $y+x : y :: r+2b : r+b$; but $y+x=r$, whence Fig. 6.
$r : y :: r+2b : r+b$. Therefore $y=\frac{rr\times rb}{r\times rb}$. Q. E. I.

If $r+b$ (=AE) be put $=d$, the theorem will be reduced into a more contracted form, as thus, $y=\frac{rd}{2d-r}$.

COROLLARY I.

Hence, xd will be $=dy-ry$, or $AE\times BC=AB\times CE$, or, which is the ſame, the line AE is divided harmonically in the points A, B, C, E; for the above ſaid equality of the rectangles is proper to a line cut according to harmonical proportion. This truth is manifeſt, for $y=\frac{dr}{2d-r}$, and $x=r-y=r-\frac{dr}{2d-r}$; wherefore, by ſubſtituting theſe values, the equation will become evident. Hence in every *ſpheric ſpeculum*, the lines DA, DB, DC, DE are harmonical; and the *radiating* point, centre, focus, are points making the diviſion harmonical.

COROLLARY II.

1. By putting $d>r$, y will be had from the calculus, or $\frac{rd}{2d-r}>\frac{r}{2}$ always. That is, if the diſtance of the *radiating* point be greater than the ſemidiameter of the *ſpeculum*, the diſtance of the focus will always be greater than a fourth part of the diameter. Likewiſe will $\frac{rd}{2d-r}<r$ always. That is, the diſtance of the focus will be always leſs than the ſemidiameter of the *ſpeculum*.

2. Put $d=r$, it will be $\frac{rd}{2d-r}$, or $y=r$; that is, if the *radiating* point be ſet in the centre of the *ſpeculum*, its image there will be united with it.

3. If d be put $=r$, then the expreſſion of y will either be poſitive or negative, or infinite, as $2d$ is greater or leſs, or equal to r.

4. If $2d<r$, that is, if $d>\frac{r}{2}$, then the *radiating* point and focus lie on the ſame parts of the *ſpeculum*.

5. If $2d<r$, or $d>\frac{r}{2}$, then the image is placed in the axis produced beyond the vertex of the *ſpeculum*.

6. If

Fig. 6.

6. If $2d=r$, or $d=\frac{r}{2}$, the image is infinitely distant, or the reflecting ray is parallel to the axis.

COROLLARY III.

By the help of this calculus, it may expeditiously be determined how the motion of the image answers to the motion of the *radiating* object in respect of the *speculum*. Let, as before, the distance of the image from the *speculum* be $=\frac{dr}{2d-r}$, when the distance of the object is d. Now suppose the distance of the object any how changed, and from d, become nd, n denoting either a whole number or a fraction, and so instead of the former equation $y=\frac{dr}{2d-r}$, we shall have for a new focus, another equation, $F=\frac{ndr}{2nd-r}$. And if n be supposed to express a whole number, this second distance of the object will be greater than the first; but if it be a fraction, then it will be less than the first.

These being put, if $d>r$, and n be an integer, it will be $F<y$, that is, it will be $\frac{ndr}{2nd-r}<\frac{dr}{2d-r}$ or $2nddr-ndrr<2nddr-drr$, which is manifest. That is, if in the *concave speculum* the distance of the object be greater than the semidiameter, then the object receding from the *speculum*, the image will accede to it. Again, let n denote a fraction, and then will be found $2nddr-ndrr>2nddr-drr$, or $F>y$; that is, the object acceding to the *speculum*, the image will recede.

Now suppose $d<\frac{r}{2}$; as also let there be any other distance nd of the object, it may be supposed to be always less than $\frac{r}{2}$. Then will $2nddr-ndrr$, and $2nddr-drr$ be negative quantities; or $ndrr-2nddr$, and $drr-2nddr$ positive quantities. And if n be equal to a whole number, it will be $ndrr-2nddr>drr-2nddr$, or $F>y$; but if n be a fraction, then will $ndrr-2nddr<drr-2nddr$, or $F<y$; that is, if n the distance of the object in the *concave speculum* be less than $\frac{1}{4}$ of the *speculum*'s diameter, then the object receding from the *speculum*, the image will recede too; or the object acceding towards the *speculum*, the image will accede also.

COROL-

COROLLARY IV.

In the equation $y=\frac{dr}{2d-r}$, if d be put infinite, then will $y=\frac{r}{2}$; which is a rule for parallel rays, or for the *radiating* object removed to an infinite distance. The same will follow by putting b infinite, in the equation $y=\frac{rr+rb}{r+2b}$.

COROLLARY V.

In the equation $\frac{dr}{2d-r}$, by changing the negative sign of the quantity r into the positive, will $y=\frac{dr}{2d+r}$; or in the equation $y=\frac{rb+br}{r+2b}$, by changing the positive sign into the negative, will $y=\frac{rb-rr}{2b-r}$; which exhibits a rule in the *convex speculum*, towards the *radiating* object. This changing of the sign is manifest, for if in the *concave speculum* $d=r+b$, so in the *convex* $d=b-r$.

COROLLARY VI.

In the *convex speculum* (things standing as we have observed at *Corol.* 3. concerning the *concave*) it will appear that (if n be a whole number) $2rndd+ndrr>2rndd+drr$; and (n being a fraction) that $2rndd+ndrr<2rndd+drr$. That is, that the object receding from the *speculum*, or acceding towards the same, the image will recede or accede also.

Moreover it is evident in a *convex speculum*, that the object receding to an immense distance, its image will nevertheless not pass above $\frac{1}{4}$ part of the diameter from the vertex, but there stand in a point, in the middle between the centre and the vertex. For d or b being put infinite, then will $y=\frac{dr}{2d}$, or $\frac{br}{2b}$, that is $\frac{r}{2}$.

We come now to illustrate what has been said, by the resolution of a catoptric problem.

Fig. 6.

PROBLEM.

TO find ſuch a poſition of a radiating object in reſpect of a given ſpeculum, *that the radiating object to its image made from the* ſpeculum *may have a given ratio.*

SOLUTION.

Let the given ratio be as r to q, and let O denote the object, y the diſtance of the image of the *ſpeculum*, I the image, and d the diſtance of the object. Now it will be O : I :: d : y (that is, the object and image are direct proportionals to their diſtances from the vertex of the *ſpeculum*); and becauſe it is required that O : I :: r : q, ſo likewiſe d : y :: r : q; or (by writing the expreſſion of y) $d : \frac{dr}{2d-r} :: r : q$; whence $2ddq-rdq=rdr$, $2dq=rr+qr$, and $d=\frac{rr+rq}{2q}$. Wherefore ſeeing $dr=\frac{rrr+rrq}{2q}$ — $: \frac{rr}{q}=\frac{qr^3+q^2r^2}{2qr^2}$ $=\frac{r+q}{2}$, which is the diſtance of y, or the image, from the *ſpeculum* congruous to this diſtance of the object: Therefore, if the object be put to the diſtance $\frac{rr+rq}{2q}$, its image made to the diſtance $\frac{r+q}{2}$ compared with it, will have the ſame ratio, as q to r, or will be O : I :: r : q. For O : I :: d : y :: $\frac{rr+rq}{2q} : \frac{r+q}{2}$ :: r : q.

Q. E. I.

Reduction of algebraic fractions into more ſimple ones, and recurring ſeries; by Mr Ab. De Moivre, *F. R. S.* N° 373. Phil. Tranſ. *Tranſlated by Mr* Holliday.

PROPOSITION I.

LET there be any fraction, as $\frac{1}{1-ex+fxx-gx^3}$, &c. *whoſe numerator is any given quantity, and denominator a multinome any how compoſed of the given quantities* 1, e, f, g, &c. *and the indeterminate* x; *I ſay that the fraction above ſaid will be reducible to more ſimple fractions.*

Caſe

Caſe 1.] Let the propoſed fraction be $\frac{1}{1-ex+fxx}$; ſuppoſe the denominator $1-ex+fxx=0$, and let $\frac{1}{m}$, $\frac{1}{p}$ be the roots of this equation; or making $xx-ex+f=0$, let m, p, be the roots of the new equation; put $A=\frac{m}{m-p}$, and $B=\frac{p}{p-m}$, and the propoſed fraction will be equal to the ſum $\frac{A}{1-mx}+\frac{B}{1-px}$.

Caſe 2.] Let the propoſed fraction be $\frac{1}{1-ex+fxx-gx^3}$; ſuppoſe $x^3-ex^2+fx-g=0$, and let m, p, q be the roots of this equation; put $A=\frac{mm}{m-p\times m-q}$, $B=\frac{pp}{p-m\times p-q}$, $C=\frac{qq}{q-m\times q-p}$; and the propoſed fraction will be equal to the ſum $\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}$.

Caſe 3.] Suppoſe this fraction $\frac{1}{1-ex+fxx-gx^3+hx^4}$; now ſuppoſe $x^4-ex^3+fxx-gx+h=0$, and let m, p, q, s be the roots of this equation; put $A=\frac{m^3}{m-p\times m-q\times m-s}$, $B=\frac{p^3}{p-m\times p-q\times p-s}$, $C=\frac{q^3}{p-m\times p-q\times p-s}$, $D=\frac{s^3}{s-m\times s-p\times s-q}$, and the propoſed fraction will be equal to the ſum $\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}+\frac{D}{1-sx}$.

Caſe 4.] Let this fraction be propoſed, $\frac{1}{1-ex+fxx-gx^3+hx^4-kx^5}$; ſuppoſe $x^5-ex^4+fx^3-gx^2+hx-k=0$, and let m, p, q, s, t, be the roots of this equation; put $A=\frac{m^4}{m-p\times m-q\times m-s\times m-t}$, $B=\frac{p^4}{p-m\times p-q\times p-s\times p-t}$, $C=\frac{q^4}{q-m\times q-p\times q-s\times q-t}$, $D=\frac{s^4}{s-m\times s-p\times s-q\times s-t}$, $E=\frac{t^4}{t-m\times t-p\times t-q\times t-s}$, and the propoſed fraction will be equal to

the

the Sum $\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}+\frac{D}{1-sx}+\frac{E}{1-tx}$.

The law of this reduction, and the continuation thereof are evident, and it would be needleſs to enter into an explanation.

COROLLARY I.

If all the roots are equal, the propoſed fraction cannot be reduced to more ſimple ones.

COROLLARY II.

If ſome of the roots are equal, and others unequal, the propoſed fraction may be reduced to more ſimple ones. For inſtance, ſuppoſe the propoſed fraction be $\frac{1}{1-ex+fxx-gx^3}$, and making $x^3-exx+fx-g=0$, according to the preſcript. Let the roots of this equation be m, p, q, whereof m and p are equal; they will be ſimple fractions into which the propoſed one is reſolved $\frac{mm}{m-p\times m-q\times 1-mx}+\frac{pp}{p-m\times p-q\times 1-px}+\frac{qq}{q-m\times q-p\times 1-qx}$, adding the two former into one ſum, it will be (by dividing the numerator and denominator by $m-p$) $\frac{mp-q\times m+p+mpqx}{m-q\times p-q+1-mx\times 1-px}$, or $\frac{mm-2qm+m^2qx}{m-q|^2\times 1-mx|^2}$, or $\frac{m}{m-q\times 1-mx|^2}-\frac{qm}{m-q|^2\times 1-mx|^2}$; conſequently therefore the fractions reduced will be $\frac{m}{m-q\times 1-mx|^2}-\frac{qm}{m-q|^2\times 1-mx|^2}+\frac{qq}{m-q|^2\times 1-qx}$.

COROLLARY III.

If the ſimple fractions, into which the propoſed fraction is reſolved, involve imaginary quantitities, then whatſoever is imaginary will always be deſtroyed by the addition of two or more fractions taken in an equal number.

COROLLARY IV.

Of the combination of ſimple fractions, and the apt limitation of the roots, will ariſe many uſeful theorems; for inſtance, ſuppoſe

poſe the propoſed fraction be $\frac{1}{1-ex+fxx-gx^3+hx^4}$, and making as before $x^4-ex^3+fx^2-gx+h=0$. Let the roots of the equation be m, p, q, s, and the fractions into which the propoſed is reſolved, be $\frac{A}{1-mx}+\frac{B}{1-px}+\frac{C}{1-qx}+\frac{D}{1-sx}$. Now if we put $q=-m$, and $s=-p$, adding the two former, and alſo the two latter, the propoſed fraction will be reduced to $\frac{m+p-mpx}{2\times m+p\times 1-mx\times -px}+\frac{m+p+mpx}{2\times m+p\times 1+mx\times 1+px}$. But if we make $p=-m$, and $s=-q$, and adding the two former, and likewiſe the two latter, the propoſed fraction will be reduced to $\frac{mm}{mm-qq\times 1-mmxx}+\frac{qq}{qq-mm\times 1-qqxx}$.

PROPOSITION II.

IF there be any fraction whoſe numerator is a given quantity, and the denominator a trinomial *or* quadrinomial, *or a* quinquenomial, *&c. not affected with radicality, and howſoever compoſed of given quantities,* 1, e, f, g, h, *&c. and the indeterminate* x, *and the numerator be divided by the denominator, that we may have an infinite ſeries; I ſay, it will be, if any terms of this ſeries be taken in equal ſpaces diſtant from one another, the infinite ſeries reſulting from thence will be ſummable.*

EXAMPLE I.

Let the propoſed fraction be $\frac{1}{1-x-xx}$; let it be reduced to an infinite ſeries, namely to $1+x+2xx+3x^3+5x^4+8x^5+13x^6+21x^7+34x^8$, *&c.* Let all the alternate terms be taken beginning from the firſt, and likewiſe all the alternate terms beginning from the ſecond, and we ſhall have theſe two ſeries, namely

$$1+2xx+5x^4+13x^6+34x^8, \text{ \&c.}$$
$$x+3x^3+8x^5+21x^7+55x^9, \text{ \&c.}$$

Suppoſe the denominator of the propoſed fraction, $1-x-xx=0$; now when the indices of the powers of x, in the new ſeries, exceed one another by the common difference 2, put $xx=z$, and by the aſſiſtance of theſe two equations, $1-x-xx=0$, and $xx=z$, x may be exterminated by the common rules of algebra, and it will be-

come $1-3z-zz=0$; now reſtoring x it will be $1-3xx+x^4=0$; this equation divided by the firſt, the quotient will be $1+x-xx$; let the terms of the quotient be taken alternately, becauſe the terms are taken alternately in the propoſed ſeries, and hence will ariſe two ſums, $1-xx$, and x; let theſe ſums conſtitute the numerators of two fractions, whoſe common denominator let be $1-3xx+x^4$, then $\frac{1-xx}{1-3xx+x^4}$, and $\frac{x}{1-3xx+x^4}$, will be the reſpective ſums of the new ſeries.

EXAMPLE II.

But if the ſum of the terms diſtant two ſpaces from one another be deſired, let $1-x-xx$ be made $=0$, as before. Now ſeeing the indices of the powers in the new ſeries exceed one another by the common difference 3; put $x^3=z$, and $1-4z-zz$ will be $=0$; wherefore reſtoring x, $1-4x^3-x^6$ will be made equal to 0; let $1-4x^3-x^6$ be divided by $1-x-xx$, the quotient will be $1+x+2xx-x^3+x^4$, whoſe terms taken *ordinatim* to two ſpaces will make three ſums, *viz.* $1-x^3$, $x+x^4$, $2xx$, which taken one by one, will be the numerators of three fractions, to which if the common denominator $1-4x^3-x^6$ be appointed, the three fractions will be $\frac{1-x^3}{1-4x^3-x^6}$, $\frac{x+x^4}{1-4x^3-x^6}$, and $\frac{2xx}{1-4x^3-x^6}$, the three ſums of all the terms diſtant from one another by two intervals or ſpaces, beginning reſpectively from the firſt, ſecond and third term. And by the ſame method might we proceed to thoſe that are three, four or five ſpaces diſtant from one another, whether the denominator be a quadrinomial, or a multinomial, howſoever compoſed of finite terms.

PROPOSITION III.

IF unity be divided by a trinomial, *howſoever compoſed of the given quantities*, 1, e, f, g, &c. *and the indeterminate quantity* x; *I ſay, that every term of this ſeries reſulting from this diviſion will be aſſignable.*

Let the trinomial be $1-ex+fxx$; ſuppoſe $x^2-ex+f=0$, and let m and p be the roots of the equation, and let $n+1$ be the place of the deſired term; that is, let n expreſs the interval between the firſt term and the term ſought. Make $A=\frac{m}{m-p}$, $B=\frac{p}{p-m}$, and

the

the desired term will be, *viz.* $\overline{Am^n+Bp^n}\times x^n$. After the same manner, if unity be divided by the quadrinomial $1-ex+fx^2\times-gx^3$: put $x^3-exx+fx-g=0$, and let m, p, q, be the roots of the equation; and make $A=\frac{mm}{m-p\times m-q}$, $B=\frac{pp}{p-m\times p-q}$, $C=\frac{qq}{q-m\times q-p}$, and the desired term will be $\overline{Am^n+Bp^n-Cq^n}\times x^n$; and the same law will hold good in any multinomial whatsoever.

Methinks it will not be improper to add (from the Latin*) to the above,* The section of an angle, *which the acute Mr* De Moivre *hit upon. See* N° 374. Phil. Trans.

This great man says, that in the year 1707 he hit upon a method, whereby the equation being given of this form,

$$ny+\frac{nn-1}{2.3}Ay^3+\frac{nn-9}{4.5}By^5+\frac{nn-25}{6.7}Cy^7, \&c. =a;$$

Or of this form,

$$ny+\frac{1-nn}{2.3}Ay^3+\frac{9-nn}{4.5}By^5+\frac{25-nn}{6.7}Cy^7, \&c. =a,$$

where the quantities A B C, &c. represent the co-efficients of the preceding terms, he determined the roots after this manner:

By putting $a+\sqrt{aa+1}=v$, in the first case,

$a+\sqrt{aa-1}=v$, in the second case,

then will $y=\frac{1}{2}\sqrt[n]{v}-\frac{1}{\sqrt[n]{v}}$ in the first case,

$y=\frac{1}{2}\sqrt[n]{v}+\frac{1}{\sqrt[n]{v}}$ in the second case.

Now by what artifice these *formulas* were found, a recourse to the demonstration of the following *Theorem*, no doubt, will make it evident.

Let $x=$ versed sine of any arc,

$t=$ versed sine of another arc,

$1=$ radius.

Let the arc of the former be to the latter as 1 to n, then having assumed the two equations, which we may call known,

$$1-2z^n+z^{2n}=-2z^nt,$$

$$1-2z+zz=-2zx,$$

by expunging z, an equation will arise, whereby the relation between x and t is determined.

COROL-

COROLLARY I.

If the latter arc be the ſemicircumference, the equations will be

$$1+z^n=0$$
$$1-2z+zz=-2zx;$$

from which, if z be expunged, an equation will ariſe, whereby the verſed ſines of the arcs are determined, which let be to the ſemicircumference, taken 1, 3, 5, &c. times, as 1 to n.

COROLLARY II.

If the latter arc be the circumference, the equations will be

$$1-z^n=0,$$
$$1-2z+zz=-2zx;$$

from which, if z be expunged, an equation will ariſe, whereby the verſed ſines of the arcs are determined, which let be to the circumference, taken 1, 2, 3, 4, &c. times, as 1 to n.

COROLLARY III.

If the latter arc be 60°, the equations will be,

$$1-z^n+2z^{\frac{n}{2}}=0$$
$$1-2z+zz=-2zx;$$

from which, if z be expunged, an equation will ariſe, whereby the verſed ſines of the arcs are determined, which let be to the arc of 60°, multiplied

by $\left\{\begin{matrix}1,\ 7,\ 13,\ 19,\ 25\\ 5,\ 11,\ 17,\ 23,\ 29\end{matrix}\right\}$ &c. as 1 to n.

If the latter arc be 120°, the equations will be

$$1+z^n+z^{2n}=0,$$
$$1-2z+zz=-2zx;$$

from which, if z be expunged, an equation will ariſe, whereby the verſed ſines of the arcs are determined, which let be to an arc of 120° multiplied

by $\left\{\begin{matrix}1,\ 4,\ 7,\ 10,\ 13\\ 2,\ 5,\ 8,\ 11,\ 14\end{matrix}\right\}$ &c. as 1 to n.

N. B. *In our next we propoſe to give an univerſal method of finding the ſums of the powers of an arithmetical progreſſion, of which queſtion* 258 *in the* Ladies Diary *is one of its eaſieſt caſes, which was communicated to us by our ingenious friend* Spicilegus.

A

A Theorem on increments, with its demonſtration. Tranſlated from the Latin *of Dr* TAYLOR's Methodus Incrementorum, *by Mr* HOLLIDAY.

The THEOREM.

SUPPOSE z and x be two variable quantities, whereof z is uniformly augmented by given increments z, and let nz=v, v−z=v̇, v̇−z=v̈, &c. I ſay, then, at what time z (by augmenting) becomes z+v, x likewiſe by increaſing will become x+ẋ $\frac{v}{1z}+x\frac{v\dot{v}}{..1.2z^2}+x\frac{v\dot{v}\ddot{v}}{...1.2.3z^3}+$ &c.

DEMONSTRATION.

x	x	x	x	x	&c.
$x+\dot{x}$	$\dot{x}+\dot{x}$	$\dot{x}+\dot{x}$	$\dot{x}+\dot{x}$	$\dot{x}$ &c.	
$x+2\dot{x}+\ddot{x}$	$\dot{x}+2\dot{x}+\ddot{x}$	$\ddot{x}+2\dot{x}+\ddot{x}$	&c.		
$x+3\dot{x}+3\ddot{x}+\dddot{x}$	$\ddot{x}+3\dot{x}+3\ddot{x}+\dddot{x}$	&c.			
$x+4\dot{x}+6\ddot{x}+4\dddot{x}+\ddddot{x}$	&c.				
&c.					

The ſucceſſive values of x are collected by a continual addition, x, $x+\dot{x}$, $\dot{x}+2\dot{x}+\ddot{x}$, $\ddot{x}+3\dot{x}+3\ddot{x}+\dddot{x}$, &c. as is manifeſt by the operation expreſſed in the annex'd table. But in theſe values of x the numeral coefficients of the terms x, $\dot{x}$, $\ddot{x}$, &c. are formed in the ſame manner as the coefficients of the correſponding terms in the power of the binomial. And (by Sir *Iſaac Newton*'s theorem) if the index of the power be n, the coefficients will be 1, $\frac{n}{1}$, $\frac{n}{1}\times\frac{n-1}{2}$, $\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}$, &c. Therefore in what time z by augmenting becomes $z+nz$, that is, $z+v$, x will be made equal to this ſeries $x+\frac{n}{1}\dot{x}+\frac{n}{1}\times\frac{n-1}{2}\ddot{x}+\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}\dddot{x}+$ &c. But $\frac{n}{1}=\left(\frac{nz}{z}=\right)\frac{v}{z}$, $\frac{n-1}{2}=\left(\frac{nz-z}{2z}=\right)\frac{\dot{v}}{2z}$, $\frac{n-2}{3}=\left(\frac{nz-2z}{3z}=\right)\frac{\ddot{v}}{z}$, &c. wherefore in what time z by increaſing becomes $z+v$, in the ſame

ſame time x by increaſing will become $x+x\frac{\dot{v}}{1z}+x\frac{\dot{v}^2}{1.2z^2}+x\frac{\dot{v}^3}{1.2.3z^3}+$ &c.

COROLLARY I.

z, x, $\dot{x}$, $\ddot{x}$, &c. remaining as above, by changing the ſign of v, in what time z by decreaſing becomes $z-v$, in the ſame time x by decreaſing will become $x-x\frac{\dot{v}}{1z}+x\frac{\dot{v}^2}{1.2z^2}-x\frac{\dot{v}^3}{1.2.3z^3}$, &c. or, according to our notation, $x-x\frac{\dot{v}}{1z}+x\frac{\dot{v}_1^2}{1.2z^2}-x\frac{\dot{v}_2^3}{1.2.3z^3}$, &c. $\dot{v}$, $\dot{v}_1$, $\dot{v}_2$, &c. being converted into $-\dot{v}$, $-\dot{v}_1$, &c.

COROLLARY II.

If, for the evaneſcent increments, be wrote the fluxions proportional to them, and all the $\dot{v}$, $\dot{v}$, v, $\dot{v}_1$, $\dot{v}_2$, &c. be made equal, at what time z by flowing uniformly becomes $z+v$, x will become $x+\dot{x}\frac{v}{1z}+\ddot{x}\frac{v^2}{1.2z^2}+\dddot{x}\frac{v^3}{1.2.3z^3}$, &c. or, by changing the ſign of v, at what time z by decreaſing becomes $z-v$, x by decreaſing will become $x-\dot{x}\frac{v}{1z}+\ddot{x}\frac{v^2}{1.2z^2}-\dddot{x}\frac{v^3}{1.2.3z^3}$, + &c.

The following tables are founded on the bills of mortality for the city of *London*. As the ingenious author of them exhibited an explanation thereof at page 76, *Gent. Magazine* for the year 1738, we ſhall here deſiſt carrying it on any farther, and only refer the reader thereto. He infers that the column of 6 *per Cent.* is ſomewhat higher than Dr *Halley's*; yet as the doctor's tables are calculated for *Breſlau*, he preſumes his may be more ſuitable for this kingdom. The third column (he ſays) is founded upon this conſideration, that of any number of perſons, who ſhall be left alive at ſuch an age, as, for inſtance, ſuppoſe 25, which is 426; if at 49, there ſhall be 213 left; then it is an even wager, that perſon will live 24 years; the three next columns exhibit the preſent value of an annuity at the rate of 4, 5, or 6 *per Cent.*

TABLES

TABLES of ANNUITIES,

Founded on the Bills of Mortality for LONDON.

By Mr B. DARTING.

Age.	Of 1000, left alive at *London*.		Value of one pound Annuity for life at 4 per Cent.	Ditto at 5 per Cent.	Ditto at 6 per Cent.	Age.	Of 1000, left alive at *London*.		Value of one Pound Annuity for life at 4 per Cent.	Ditto at 5.	Ditto at 6.
1	1000	8	6.7327	6 4632	6.2097	43	267	16¼	12.0356	11.1440	10 3852
2	613	36½	19.0825	16.6813	14.7307	44	258	16½	11.9223	11.0577	10.2460
3	574	38¼	19.4228	16.9079	14.8710	45	249	16	11.6523	10.8377	10.1590
4	545	38¼	19.4408	16.9280	14.8762	46	240	15¾	11.5203	10.7256	10.0230
5	527	38¼	19.4408	16.9280	14.8762	47	231	15½	11.3819	10.6056	9 9062
6	518	38	19.3678	16.8679	14.8460	48	222	15⅛	11.2084	10.4505	9.7622
7	508	37½	19.2525	16.7713	14.7917	49	213	15	11.1184	10.3796	9.7122
8	500	37	19.1425	16.7113	14.7367	50	205	14½	10.9631	10.2438	9.5051
9	494	36¼	18.9682	16.5868	14.6460	51	197	14¼	10.7031	10.0175	9.4023
10	490	35½	18.7846	16.4542	14.5582	52	189	14	10.5631	9.8986	9.2945
11	487	34¾	18.5812	16.3130	14.4571	53	181	13½	10.2556	9.6486	9.0736
12	484	33¼	18.3412	16.1500	14.3221	54	174	13⅛	10.0856	9.4835	8.9126
13	482	33	18.1476	16.0025	14.2302	55	167	12¼	9.8356	9 2636	8.7326
14	480	32	17.8735	15.8027	14.0840	56	160	12⅓	9.5850	9.0332	8.5438
15	478	31	17.5885	15.5928	13.9291	57	153	12	9 3850	8.8632	8.3838
16	475	30¼	17.3621	15.4250	13.8048	58	146	11½	9.0704	8.5864	8.1368
17	472	29½	17.1437	15.2760	13.7257	59	139	11	8.7605	8.3064	7 8868
18	468	28¾	16.8830	15.0612	13.5361	60	132	10¾	8.5404	8.1164	7.7
19	464	28	16.6630	14.8981	13.4061	61	125	10¼	8.1708	7.8657	7.49
20	459	27¼	16.4015	14.7051	13.2605	62	118	9⅝	8.....	7 6217	7.27
21	453	26½	16.1527	14.5102	13.1070	63	112	9½	7.7953	7.4178	7.08
22	447	25¾	15.8927	14.2852	12.9483	64	106	9	7.4353	7.1078	6.8017
23	440	25¼	15.7121	14.1840	12.8333	65	100	8⅔	7.0527	6.8478	6.5697
24	433	24¾	15.5270	14.....	12.7000	66	94	8¼	6.9127	6.6232	6.4097
25	426	24	15.2470	13.7986	12.5500	67	88	8	6.7327	6.4632	6.2097
26	418	23½	15.0517	13.6435	12.4283	68	82	7¾	6.5534	6.3932	6.0496
27	410	23	14.8568	13.4886	12.3034	69	76	7⅔	6.5520	6.2463	6.
28	402	22½	14.6544	13.3235	12.1730	70	70	7⅔	6.5520	6.2463	6.
29	394	22	14.4511	13.1630	12.0415	71	64	8	6.7327	6.4632	6.2097
30	386	21½	14.2501	13.....	11.9007	72	58	8⅓	6.9627	7.3178	6.4097
31	378	21	14.0291	12.8212	11.7640	73	53	8⅔	6.8230	6.4432	6.2800
32	370	20½	13.8901	12.6422	11.6155	74	48	7¾	6.5534	6.3932	6.0496
33	361	20	13.5903	12.4622	11.4700	75	44	7¼	6.1820	5.9563	5.7423
34	352	19⅔	13.4611	12.3315	11.3581	76	40	6¼	5.8123	5.6063	5.5123
35	343	19⅓	13.2931	12.2121	11.2581	77	37	6	5.2421	5.0757	4.9173
36	334	19	13.1339	12.0853	11.1581	78	34	5¼	4.8218	5.4700	4.5623
37	325	18⅔	12.9728	11.9553	11.0266	79	32	4¼	4.2310	4.1304	4.0254
38	315	18⅓	12.8214	11.8245	10.9165	80	30	4	3.63	3.5460	3.4651
39	305	18	12.6592	11.6896	10.8276	81	27				
40	295	17¾	12.4352	11.5896	10.7184	82	23				
41	285	17½	12.4093	11.4856	10.6522	83	19				
42	276	17¼	12.2356	11.3340	10.5272	84	15				

Of the Maxima *and* Minima *occurring in the celeſtial motions. By the ſagacious mathematician, Mr* Ab. De Moivre, *F.R.S. Done into* Engliſh *from* N° 360 Philoſ. Tranſ. *by Mr* Holliday.

Fig. 7. LET ABP be an elliptic orbit of a planet, AP the *axis tranſverſus*, CB the ſemi-conjugate axis, S the Sun, Q the other focus of the ellipſis; thro' S draw SM parallel to CB, and the point M will be a *maximum*, in which the diſtance from the Sun increaſes or decreaſes, with the greateſt velocity, and $SM = AC - \frac{\overline{SC}^2}{CA}$. If L be taken a mean proportional between the ſemi-axes AC, CB, L will be the point in which the equation of the centre is a *maximum*, or when the angular motion is made equal to the mean motion. But if the eccentricity is not greater than in moſt planets, $BL = \frac{1}{2}BM$ *quam proxime*, and $SL = \sqrt{\sqrt{AC^4 - AC^2SC^2}}$.

If the point N is to be inveſtigated, in which the mutation of the velocity of the real motion in the curve is a *maximum*, the problem is a ſolid. For $2NS = 4AC - 2NQ$, to $3NQ - AC$, as $\overline{AC}^2 - \overline{CS}^2 = \overline{CB}^2$ to $\overline{NQ}^2$. Wherefore if we put $AC = a$, $CB = c$, and $NQ = y$, we ſhall have this equation, $y^3 - 2ay^2 + \frac{1}{4}c^2y - \frac{1}{4}ac^2 = 0$, which ſolv'd, y, or NQ, will be the diſtance of N, the point ſought, from either focus of the ellipſis. But in orbs little eccentric, ſuch of the planets, if CD be made $= SQ$, and AK be put equal to AD joined, the other part of the axis, KP, will be equal to NS, the diſtance of the point N from the ſun *quam proxime*. But if it be a parabolic orb, SN will be to SP, as 5 to 4, and the angle NSP will be 53° 8′ *fere*, whoſe ſine is $\frac{4}{5}$ of the radius.

But the point O, in which the apparent motion, or angular acceleration of a planet deſcending, or the retardation of a planet aſcending is a *maximum*, will be obtained thus: on AC take $CG = \frac{1}{6}AC$, and make the angle CSF 30°, and put CE equal to SF, and let GH be equal to GE; I ſay, if the diſtance SO be made equal to PH, the mutation of the angular motion of a planet deſcribing the elliptic orbit ABOP will be a *maximum* in the point O; namely in that place of the orbit, the ſecond differences of the equations of the planet's centre will be found a *maximum*. But $SO = \frac{7}{6}AC - \sqrt{\frac{1}{36}AC^2 + \frac{1}{3}SQ^2}$. But if it be a parabolic orb, as in comets, SO will be to SP, as 8 to 7; and the angle OSP will be 41° 24′ $\frac{1}{2}$, or whoſe ſine is to radius, as $\frac{1}{4}\sqrt{7}$ to 1.

Laſtly

Laſtly the tangent's orbit is a *minimum*, that is, changing with the leaſt velocity, in the point R, if S R be made equal to $\frac{2}{3}$ of the greater axis A B. But if the eccentricity S C be leſs than $\frac{1}{3}$ of P C, there will be no *minimum*, but this velocity always decreaſeth with which the tangent is revolved, to the *Aphelion* itſelf, as it is in the motions of all planets, neither will it be a parabolic orbit, becauſe its axis is extended *in infinitum*.

An Essay *to find the* Sun's *horizontal parallax by the tranſit of* Venus *over the* Sun's *diſk, to be obſerved in the year* 1761, *the* 26*th day of* May, *at* 10^h 51′ *in the forenoon in* 15° *of* ♊.

By Mr Alex. Findlay, *teacher of navigation at* Aberdeen.

THE ingreſs of the tranſit may be ſeen from the Earth's ſuperficies at K; the planet in B appears to enter the Sun's diſk at D, but if it were ſeen from the Earth's centre at C, then the planet appears in E, to enter the Sun's diſk at D, and to go off at R. Fig. 8.

In this ſcheme D R is the Sun's diameter 31′ 20″=1880‴, the duration of the tranſit is 8^h 16′ 7″=29767″.

	I	
As ♀ diſtance from the ☉ W C	837.33	2.922893
Is to the time of the tranſit	297.67	2.473755
So is the ſun's diameter D R	1880	3.274158
To an angle as the ☉ appears at ♀ H L	668.4	2.825020

	II	
As ♀ diſtance from the ☉ W C	837.33	2.922893
Is to ♀ diſtance from the Sun W ☉	308.67	2.489437
So is the < at the ☉, as it appears at ♀	668.4	2.825020
To ♀ greateſt horizontal parallax K E C	246.4	2.391564

equal to 4′ 6″ 21‴.

	III inverſly	
As ♀ diſtance from the ☉ W C	837.33	2.922893
Is to her greateſt horizontal parallax K E C	246.4	2.391564
So is the Sun's leaſt diſtance from ☉ ☉ C	1146	3.059153
To the Sun's greateſt horizontal parallax	180=3′	2.255304

Note, *The diſtances are taken in Earth's ſemidiameters.*

A new method to find the length of the Earth's ſhadow beyond the Moon, and alſo to find the Sun's diſtance from the Earth, in Earth's ſemidiameters.

As the diſtance of the Earth's diſk and ſhadow 13′ 4″ is to the Moon's mean diſtance from the Earth 59 26, ſo is the Earth's ſhadow 49′ 0″ to the length of the Earth's ſhadow beyond the Moon, to which add the Moon's diſtance from the Earth, the ſum is the whole length of the Earth's ſhadow 274 25.

As the mean difference of the Earth's disk and ſhadow : is to the whole length of the Earth's ſhadow : : ſo is the mean difference of the Sun's ſemidiameter and Earth's disk : to a fourth number ; to which add the whole length of the Earth's ſhadow, the ſum will be the Sun's mean diſtance in Earth's ſemidiameters.

A new Epact, with its uſe.

This epact is grounded upon the exceſs of 13 lunations in days, which is near 19 days more than the ſolar year. To find this epact yearly, multiply the golden number of the year propoſed by 19, and divide the product by 30, the remainder is the epact.

EXAMPLE.

Suppoſe I would know when the Moon changes in the month of *April* 1745; the number for the month of *April* is 2, and the golden number for the year 1745 is 17; then 17 × 19 = 323, which divided by 30, remains 23 for this new epact. Now for the new moon, take the number belonging to the month from the epact, the remainder is the day of the new moon; borrow 30 if need be.

To find the moon's age, add the day of the month, and number of the month, from their ſum take the epact, remains the moon's age; borrow 30, when need is.

To find the day of the month by the moon's age, add the moon's age to the epact, from the ſum taken the number of the month, remains the day of the month; borrow 30 when ſubtraction cannot be had.

A. FINDLEY.

New mathematical QUESTIONS *to be anſwered.*

QUESTION I. *By Mr* HOLLIDAY.

SUPPOSE in any plain triangle, two lines drawn from the angles at the baſe = 40, and 90 chains, meeting exactly in the midſt

midſt of the perpendicular let fall from the angle oppoſite the baſe, which angle is 100 degrees. Required the ſides and area, with its analytical inveſtigation?

QUESTION II. *By Mr* J. POWLE.

IN a right-angled triangle ABC, right-angled at B, let two lines AG, CF, be drawn from the two acute angles to the oppoſite ſides, cutting off equal ſegments, that is, making BF= BG; and the point of interſection be in a perpendicular let fall from the right angle: now there's given AG=43.5182; CF=34.5522; to find the area of the triangle?

QUESTION III. *By Mr* PETER KAY.

THERE are 4 numbers in geometrical proportion, whoſe ſum is given $=b$, the ſum of their ſquares $=c$, and the ſum of their cubes $=d$; what are the numbers?

QUESTION IV. *By Mr* ROB. FEARNSIDE.

REquired the value of x when $\overline{x}|^{x}$ is a *maximum*?

QUESTION. V. *By Mr* HOLLIDAY.

GIVEN $\overline{x}|^{x}+x^{x}-x^{-x}+x^{\frac{1}{x}}=200$, quere x?

QUESTION VI. *By Mr* J. POWLE.

GIVEN $x^{x}=a^{\frac{1}{x}}$, an equation to an exponential curve; to find the area, when $x=10$, and $a=100$?

QUESTION VII. *By* HURLOTHRUMBO.

GIVEN by obſervation the difference of azimuths of three known ſtars at the ſame time; 'tis required from thence to inveſtigate the altitudes of thoſe ſtars, as well as the latitude of the place.

This queſtion was firſt propoſed in the Gent. Mag. *for* June 1738, *but was wrong printed, the word* difference *being omitted.*

QUESTION VIII. *By* ΦΙΛΟΣ.

IN a fruſtum of a pyramid, DEFC, whoſe dimenſions and weight are given, and alſo the weights of two bodies hanging to the fruſtum, one at each end: required the diſtance from either end,

end, where the fulcrum muſt be placed, that this compound body may be in equilibrium?

QUESTION IX. *By Mr* Rob. Robinson.

TWO ſhips depart from two ports, under the ſame meridian; the one runs between north and weſt 3059.7 miles into the latitude 54 north, and finds her enlarged diſtance, according to *Wright*'s projection, 4256.5 miles; the other ſails S 44° 42′ W, and falls in with the former. I demand the latitude each ſhip departed from, with their difference of longitude, and departure, or meridian diſtance made good?

QUESTION X. *By Mr* Holliday.

IF a perſon throw up 16 half-pence, what is the odds that there does not come 9 heads or 9 tails, exactly in two trials?

QUESTION XI. *By Mr* J. Powle.

Fig. 9. LET $x^3-y^4=ax^2y$ expreſs the relation of A C to B C. Now there is given the curvilineal area A B C$=545\frac{11}{70}$, together with the invariable quantity $a=3$, to find the *abſciſſa* A C, and ordinate C B?

QUESTION XII. *By Mr* John Moore.

SUPPOSE there be an open ſpheroidical veſſel, whoſe content is given ($=c$), and the ratio of its tranſverſe to its conjugate diameter, as b to a; and ſuppoſing this veſſel to have a leſſer internal ſuperficies than any other of the ſame kind and magnitude: 'tis required to find the dimenſions thereof, and the tranſverſe diameter of the *ellipſis*, from whence it is generated?

QUESTION XIII. *By Mr* J. Bulman.

SUPPOSE the diſtance between St *Paul*'s church at *London* and St *Paul*'s at *Deptford* to be 5 miles, and that being at *Greenwich* I fix upon two ſtations C and D, at each of which I can ſee both churches and the other ſtation; ſuppoſe alſo a line to be drawn from one church to the other, and from one ſtation to the other, and from each ſtation to each church, which churches let be repreſented by A and B; laſtly, ſuppoſe I obſerve the angle ACB$=38°$ 40′; ACD$=118°$ 33′; CDA$=34°$ 20′, and CDB $=48°$ 27′; required the number of acres conrained within the boundary lines, by a geometrical conſtruction, and alſo the analytical inveſtigation thereof? QUESTION

QUESTION XIV. *By* GAMSTON RETFORD.

FROM a given point A, in the plane of a rectangular billiard table BCDE, 'tis required to ſtrike a ball of a given diameter, ſo that after three reflections, as at F, G, H, it ſhall paſs thro' the ſame point A? FIG. 10.

QUESTION XV. *By Mr* J. ASH.

TO find the greateſt cylinder, that can poſſibly lye, in a ſpace deſcribed by the rotation of the figure A *m* P about an axis BC? its *abſciſſa* AP being 30, its correſpondent *ordinate* P *m* (rightly applied) 20, and the equation expreſſing the nature of the curve, $ax=y^4$. FIG. 11.

QUESTION XIV. *By Mr* THOMAS DOD.

SUPPOSING a heavy body was to deſcend from the height of the Earth's atmoſphere =47.12 miles directly towards the top of the tower of *London*, whoſe latitude is 51° 32', with a velocity of 16 feet in the firſt ſecond of time, &c. 'Tis required to know how far the body will fall from the foot of the tower (ſuppoſing the Earth to have an annual and diurnal motion) and alſo the time it will be in motion?

QUESTION. XVII. *By Mr* J. TURNER *of* York.

SUPPOSE the mean diſtance of the Earth from the Sun to be (according to *Street's Caroline* tables) 100000 equal parts, of which the eccentricity contains 1732; the longitude of the Earth's aphelion from the ſtar of *Aries* to be 8' 8° 20'; the mean diſtance of *Mercury* from the Sun 38710 ſuch parts, and his eccentricity 7970; and the longitude of his aphelion from the firſt ſtar of *aries* 7' 13° 48' (and for the preſent we will ſuppoſe the plane of the Earth's orbit and that of *Mercury* to co-incide, as they nearly do, making only an angle of 6° 54') it's required from hence to find *Mercury's* greateſt elongation?

QUESTION XVIII. *By M* SAMUEL FARRER.

IF the relation between the *abſciſſa* and *ordinate* of a curve be expreſſed by $ay^nx^n=y^{2n+1}+x^{2n+1}$, the area will be $\frac{yx}{2}+\frac{ax^{n+1}}{4n+2xy^n}$. Quere the inveſtigation?

QUESTION XIX. *By Mr* SAMUEL BAMFIELD.

SUPPOSE there is a regular plate of ſilver (or any other metal equal in ponderoſity and thickneſs, *&c.*) in the form of a quadrangle whoſe ſides are $\left\{\begin{matrix} AB=5 . BC=11 \\ CD=9 . DA=13 \end{matrix}\right\}$, and the diagonal line from A to C= 12. 'Tis required to find that point, where it may (on the top of a needle or ſuch like,) be ſuſpended horizontally?

QUESTION XX. *By* HURLOTHRUMBO.

WHAT form muſt a maſs of homogeneous matter take, that the attraction thereof on a corpuſcle, ſomewhere in or without its ſurface, *&c.* may be the greateſt poſſible? And what is the ratio of this attraction to the greateſt whereby the corpuſcle can be affected, when the ſame matter is in form of a ſphere?

SOLUTIONS *to the following queſtions in the ſupplement to the* Gentleman's Diary, *communicated by Mr* FARRER *of* London.

QUESTION I. *By Mr* THO. HARBIDGE; *anſwered by Mr* S. FARRER.

GIVEN the ſum of the ſides of a right-angled plain triangle =90, the diameter of its inſcribed circle $=\frac{4}{5}$; its required to find each ſide ſeparately?

SOLUTION.

Let ACB be the propoſed triangle; from C, as a centre, with the radius CB, let a circle be deſcribed; and continue AC both ways to meet the periphery in D and E; joyn D, B, and draw AF parallel and BE perpendicular thereto; then it's plain that AB+AE is equal to the ſum of the three ſides; the angle AEF the complement of half the ſum of the angles at the baſe, and the angle ABF the complement of half their difference; hence putting AB+AE=2*p*; the radius of the inſcribed circle =*r*; the coſine of $\frac{1}{2}$ the ſum of the angles at the baſe =*m*, and of $\frac{1}{2}$ their difference, *y*; then will *p*—*r* be the baſe, and *p*+*r* the ſum of the other two ſides; but from the ſcheme it appears that *p*—*r* : *p*+*r* :: *m* : *y*. Therefore $y=\frac{p+r\times m}{p-r}$. Hence for all right-angled plain triangles we have this Theorem:

FIG. 12.

As half the ſum of the three ſides, minus *the radius of the inſcribed circle*: *to half their ſum* plus *the ſaid radius* :: *the coſine of half the ſum of the angles at the baſe* : *the coſine of half their difference.*

This queſtion was anſwered by Mr *W. Hart* very ingeniouſly.

QUEST. II. *By Mr* SAMUEL BAMFIELD, *anſwered by Mr* FARRER.

GIVEN the three perpendiculars being drawn from each angle of a plain triangle to its oppoſite ſide =95, 98, and 100 poles. Required the contents?

SOLUTION.

SOLUTION.

Becauſe the ſides of plain triangles are reciprocally as their reſpective perpendiculars, I aſſume a triangle whoſe three ſides are to one another as $\frac{1}{95}$, $\frac{1}{98}$ and $\frac{1}{100}$, *i. e.* as 1.05263, 1.02408 and 1.0 reſpectively, and thence find the perpendicular correſponding to the ſide 1.0, to be .90967359, *&c.* Then it will be, as .90967359 (the perpendicular in the triangle aſſumed,) : 1.0 (its reſpective ſide) : : 100 (the like perpendicular in the triangle required) : 109.9295, its reſpective ſide; or, more generally, as the perpendicular in the triangle aſſumed, is to any one of its ſides, ſo is the like perpendicular in the triangle required, to any one of its ſides; whence the other two ſides will be found equal to 112.5766 and 115.7151 reſpectively. Fig. 13.

QUESTION III. *By Mr* Tho. Williams; *anſwered by Mr* Farrer.

JUNE 10 in the morning 1742, lat. 51° 55′, I obſerved the ſhadow of an erect tree = 1½ its height; the tree immediately falling the ſame way to its ſhadow, I further obſerved, that when juſt half way down, its ſhadow was juſt 10 feet more than before. Quere, its length, and the time of the day it fell?

SOLUTION.

Let V F repreſent the tree when perpendicular to the plane of the horizon; C F the length of its ſhade; *v* F = V F, the poſition of the tree when half way down; A F the length of its ſhade; then will A C be = 10 feet. Now the length of the ſhade C F : the height of the tree V F : : 1½ : 1 : : rad. (1) : ⅔, the tangent of 33° 41′ 24″, the Sun's altitude, = alſo to the meaſure of the angle V C F; alſo in the triangle *v* C F are given C F : *v* F : : 1½ : 1, and the contained angle C F *v*, 45°; whence the angle *v* C F from thence may be found, and conſequently the angle A C *v*. Then in the triangle A C *v* are given all the angles, and the ſide A C (becauſe A *v* is parallel to C V); whence C *v* may be found, and conſequently C F and F *v*; the former of which gives 37.34590681 feet, for the height of the tree, and the latter 56.01886021 feet for the length of the ſhade. Now the Sun's altitude was found above = 33° 41′ 24″; the latitude of the place, and Sun's declination given by the queſtion; whence the time of the day it fell may eaſily be computed. Fig. 14.

This queſtion was alſo anſwered by Mr *William Hart*, of *Darlington*.

QUESTION IV. *By Mr* John Thompson; *anſwered by Mr* Farrer.

GIVEN the baſe of a plain triangle 43 chains, the angle oppoſite the baſe = 24° 30′; now if on the baſe produced a perpendicular be let fall, it will meaſure 28 chains; required the ſides?

CONSTRUCTION.

Biſect the given baſe A B with the perpendicular E F, and take therein E D equal to the given height of the triangle; make the angle E B C equal to the difference between the given angle and a right one, and from C as a centre with the radius C B let a circle be deſcribed; draw D G parallel to A B, Fig. 15.

AB, meeting the periphery of the circle in G, and from G upon AB produced let fall the perpendicular GH; join A, G, and B, G, and AGB will be the triangle required.

DEMONSTRATION.

The angle ACB at the periphery ſtanding upon the arc AB is equal to ECB, half the angle at the centre ſtanding upon the ſame arc (*Euc.* 20. 3.) but EBC is equal to the difference between the given angle and a right one, by conſtruction; therefore AGB, =ECB, is equal to the angle given; moreover DE is equal to the perpendicular given, by conſtruction, and DG parallel to AB, and GH parallel to DE; therefore GH=DE the perpendicular given. Q. E. D.

NUMERICAL SOLUTION.

In the triangle ECB are given all the angles, and the ſide EB; whence BC, =GC, will be found =51.8455; CE=47.1774, and conſequently DC=19.1774. Then in the triangle DCG right-angled at D, will be given the two ſides DC and CG; whence DG=EH, will be found = 48.1682; and conſequently BH=26.6682; BG=38.6677; and AG =75.0844. Q. E. I.

ALGEBRAIC SOLUTION.

Let AB=43=a; GH=28=b; the ſine of the angle AGB=s; the ſine and coſine of half the ſum of the angles GAB, ABG m and n; verſed ſine of their ſum p, and difference v. Then will $\sqrt{\frac{v}{2}}$ and $\sqrt{\frac{2-v}{2}}$ expreſs the ſine and coſine of half their difference; $\frac{m\sqrt{2-v}-n\sqrt{v}}{\sqrt{2}}$, and $\frac{m\sqrt{2-v}+n\sqrt{v}}{\sqrt{2}}$ the ſines themſelves; hence (by *Trig.*) it will be as $s:a::\frac{m\sqrt{2-v}-n\sqrt{v}}{\sqrt{2}}:\frac{am\sqrt{2-v}-an\sqrt{v}}{s\sqrt{2}}$=DC; and as $1:\frac{am\sqrt{2-v}-an\sqrt{v}}{s\sqrt{2}}::\frac{m\sqrt{2-v}+n\sqrt{v}}{\sqrt{2}}:b$. Therefore $\frac{am^2\times\sqrt{2-v}-an^2v}{2s}=b$, or $\overline{am^2+an^2}\times v=2m^2a-2sb$; wherefore $v=\frac{2m^2a-2sb}{m^2+n^2\times a}$; but $2m^2=p$, and $m^2+n^2=1$, conſequently $v=p-\frac{2sb}{a}$.

Mr *William Hart* anſwered this queſtion by conſtructing the figure, the ſame as above.

QUESTION V. *By Mr* ROBERT WRIGHT; *anſwered by Mr* FARRER.

LET the ſum of the ſides of an acute-angled plain triangle be equal 2470; the rectangle of the greateſt and leaſt ſides equal 626145; and the ſides of the greateſt inſcribed ſquare =380. Quere, the area and ſides of the triangle ſeverally?

SOLUTION.

SOLUTION.

Let the triangle be ACB; the inſcribed ſquare EFGH; then from the vertex C to AB, let there be drawn the perpend. CD, and put AC+CB+AB=2470=a; AC×CB=626145=b; EG=ID=c=380; CI=x, and AB=y; then by ſimilar triangles $x : c :: c+x : y$; thence $x=\frac{cc}{y-c}$; CD=$\frac{cy}{y-c}$, and the area $\frac{cy^2}{2y-2c}$; but $\frac{\overline{a\times a-2y}|^2}{4b}$=coſine of half the angle at the vertex, whence the ſine of the whole angle will be $\frac{\sqrt{4ba\times\overline{a-2y}-a^2\times\overline{a-2y}|^2}}{2b}$; and conſequently the area equal $\frac{\sqrt{4ba\times\overline{a-2y}-a^2\times\overline{a-2y}|^2}}{4}$=alſo to $\frac{cy^2}{2y-2c}$ from what has been ſaid above; whence it appears that y (expreſſing the meaſure of AB) may be found by an equation of four dimenſions. Fig. 16.

QUESTION VI. *By Mr* JOHN BADDER; *anſwered by the ſame.*

IN a plain triangle, ABC, there are given the ſides AC and BC equal to 24 and 30 poles reſpectively; and ſuppoſing a circle inſcribed in the ſame (ſo as to touch all its ſides) a line from the angle C to the centre thereof is found to meaſure 12 poles. *Quere*, the baſe of the triangle by a ſimple equation?

SOLUTION.

Let CH=12=a; CB=30=b; CA=24=c, and AB=x; by 3 *E.* 6. we have $b+c : x :: b : \frac{bx}{b+c}$=BI; $b+c : x :: c : \frac{cx}{b+c}$=IA; $b : \frac{bx}{b+c} :: a : \frac{ax}{b+c}$=HI; whence CI=$\frac{a\times\overline{b+c+x}}{b+c}$. But BC×AC $-$BI×IA=$\overline{\text{CI}}|^2$; that is, $bc-\frac{cbx^2}{\overline{b+c}|^2}=\frac{a^2\times\overline{b+c+x}|^2}{\overline{b+c}|^2}$; and $bc-\frac{cbx^2}{\overline{b+c}|^2}=\frac{bc\times\overline{b+c+x}\times\overline{b+c-x}}{\overline{b+c}|^2}$. Therefore $bc\times\overline{b+c-x}=a^2\times\overline{b+c+x}$, and thence $x=\frac{\overline{bc-aa}\times\overline{b+c}}{bc+aa}$=36, the baſe of the triangle. Fig. 17.

QUESTION VIII. *By Mr* JOHN ROBINSON; *anſwered by the ſame.*

GIVEN the abſciſſa of a parabola 50 chains, the ordinate 40; required a theorem to inſcribe the greateſt circle?

SOLUTION.

If the required circle touches the parabola, and from the centre to the point of contact, C, the right line CD be drawn, that line CD will be perpendicular to the tangent CT, by 3 *E.* 18. Moreover if from C be Fig. 18.

let fall a perpend. upon BG, the axis of the parabola at the point E, then *per Conics* the diſtance between that point, and the centre of the circle D, will be = ſemiparameter of the parabola. This being premiſed, put $50 = a$, $20 = b$, parameter $= 2p$, and DC the radius of the inſcribed circle $= x$; then $BE = a - p - x$; $CE = \sqrt{x^2 - p^2}$; and, from the nature of the parabola, $a - p - x : x^2 - p^2 :: a : b^2$. Therefore $ax^2 - ap^2 = ab^2 - pb^2 - b^2x$. Hence $x^2 + \frac{b^2}{a}x - p^2 = b^2 - \frac{pb^2}{a}$; but $\frac{b^2}{a} = 2p$, therefore $x^2 + 2px + p^2 = b^2$; whence $x + p = b$, and therefore $x = b - p = 16$, the radius of the inſcribed circle.

QUESTION IX. *By* GAMSTON RETFORD; *anſwered by the ſame.*

NINE men bought an elliptical meadow, containing 99 acres, whoſe diameters were in proportion as 7 to 5; the ſecond man takes 2 acres more than the firſt, the third 2 acres more than the ſecond, the fourth 2 acres more than the third, &c. in arithmetical progreſſion. Now the ſhares of the buyers are aſſigned by parallel lines to the conjugate diameter, the firſt beginning at the tranſverſe end, then the ſecond, and ſo on. Required the parallel diſtance, and from each end how far are the firſt and laſt?

SOLUTION.

If a circle be deſcribed on AB (168.0336) the tranſverſe axis of the ellipſe as a diameter, and lines drawn through both parallel to DE (120.024) the conjugate axis; then the areas of the circular ſegments will be to their reſpective elliptic ſegments, as the tranſverſe axis of the ellipſe, to the conjugate. This being premiſed, draw the lines as *per* figure, and put $AC = a$, $DC = b$, 0.78539, &c. $= p$, $DI = x$, and the arc ML of a circle whoſe radius is 1, $= z$; then the area of the ſector MCL will be $\frac{z}{2}$, and that of the ſimilar ſector FCI $\frac{a^2z}{2}$ (becauſe they are to one another as the ſquares of their radii); to which add $\frac{x\sqrt{a^2 - x^2}}{2}$, the area of the triangle CIN, and we ſhall have $\frac{a^2z + x\sqrt{a^2 - x^2}}{2}$ for the area of the ſpace FCIN. But FC : MC :: HI : KL $= \frac{x}{a}$; therefore the area of the circular ſegment IBGNI will be $2pa^2 - x\sqrt{a^2 - x^2} - a^2x$ by the circular arc whoſe radius is 1, and ſine $\frac{x}{a}$; conſequently the area of the elliptic ſegment RBS, from what has been ſaid above, will be $2pba - \frac{bx\sqrt{a^2 - x^2}}{a} - abx$ by the circular arc, whoſe radius is 1, and ſine $\frac{x}{a}$; which may ſerve as a general expreſſion

FIG. 19.

preſſion for finding the abſciſſa's correſponding to the ſeveral areas, cut off by the parallel ordinates. Q. E. F.

QUESTION X. *By Mr* A. CLARKE; *anſwered by the ſame.*

A Gentleman having an oblong garden plot, whoſe length is 640 feet, and breadth 477, would make a trench round it 16 feet deep, ſo as to raiſe the ſaid plot one foot higher; required the breadth of the trench?

SOLUTION.

Put $BC=640=a$; $DC=477=b$; $16=p$ the depth of the trench. FIG. 20. Then it is evident, by the figure, that $BC=FG+2IF$, and $BA=FE+2IF$. Therefore $BC-BA=FG-FE=163$; put this $=2d$, and $FG+FE=2x$; then will the length be $x+d$, the breadth $x-d$, and the area x^2-d^2. But the area of ABCD is $=ab$; therefore $\overline{ab-d^2-x^2}\times p$ will expreſs the content of the earth thrown out of the trench; which is $=\overline{x^2-d^2}\times 1$, *per* queſtion; wherefore $x=\frac{\overline{ab+1-p\times d^2}^{\frac{1}{2}}}{1+p}=542.1938$; the length of the garden incloſed $=623.6939$; the breadth 460.6939, and the breadth of the trench $=\frac{BC-FG}{2}=8.15305$ feet. Q. E. F.

QUESTION XII. *By Mr* JOHN MATHEWS; *anſwered by the ſame.*

THERE is a ſolid, whoſe ſuperficies is compoſed of four plain triangles, the angular points being A, B, C, D; the ſides $AB=17$, $BC=20$, $CA=24$, $AD=25$, $BD=26$, and $CD=30$ inches; the content thereof is required?

SOLUTION.

Let the four triangular planes that conſtitute the ſurface of this ſolid be FIG. 21. all parallel to the plane of the paper, as *per* figure and from the points *d* and D let two right lines be drawn cutting the lines AB, BC at right angles in P and Q, and their concourſe will be E; joyn EB, and upon B*d* deſcribe the ſemicircle B*T*d; apply therein BT=BE, and join *d*T; at E erect EG perpendicular to EB, and join GB. This done, conceive the planes EGB, A*d*B, ACD, and BCM to be turned up, and the points G, *d*, D, M, to unite; then will GE be the perpendicular altitude of this ſolid, and the planes in this poſition the repreſentation thereof.

NUMERICAL SOLUTION.

If we conſider the conſtituent planes, again parallel to the plane of the paper, we have in each of them all the ſides; whence, by *plain Trig.* the ſegment PB, made by a perpendicular falling from *d* will be found $=10$; the ſegment BQ made by a perpendicular falling from M=4.4, and the angle included between the two ſides AB and BC=80° 26′ 04″. Then in the quadrilateral *Fig.* EPBQ, which may be inſcribed in a circle, will be given the two ſides PB, BQ, and all the angles; whence, by *plain Trig.* the diagonal BE will be found $=10.3783857$, whoſe ſquare taken

from

from 676 the ſquare of BD, and the root extracted, gives 23.8388152, for the perpendicular altitude of the ſolid; which drawn into 55.8786005 $=\frac{1}{3}$ area of the triangle ABC at the baſe, gives 133.2796309541276, &c. ſolid inches for the content.

[*To be continued.*]

QUESTION 264. *Ladies Diary*, 1745.

TO determine the greateſt area, that can be incloſed by a parabolic curve of the ſecond kind, whoſe (equation is $ax^2=y^3$ and) length 100 feet, and an ordinate rightly applyed to its greateſt axis?

Anſwered by MERONES.

Given $ax^2=y^3$, curve $=c$; then $\frac{3y^{\frac{1}{2}}\dot{y}}{a^{\frac{1}{2}}}$ = fluxion of the area, and the area $=\frac{2}{7}\times\frac{y^{\frac{5}{2}}}{a^{\frac{1}{2}}}$; alſo the fluxion of the curve $=\dot{y}\sqrt{1+\frac{9y}{4a}}$, and the fluent is $\frac{8a}{27}\times\overline{\frac{1+9y}{4a}}^{\frac{3}{2}}-\frac{8a}{27}=c$, the curve. Now for the parameter a put v a variable quantity, and reducing the laſt equation, there is $y=\overline{cv^{\frac{1}{2}}+\frac{8}{27}v^{\frac{3}{2}}}^{\frac{2}{3}}-\frac{4}{9}v$. Put this for y in the area $\frac{2}{7}\times\frac{y^{\frac{5}{2}}}{v^{\frac{1}{2}}}$, and we ſhall have $\frac{2}{7}\times\frac{\overline{cv^{\frac{1}{2}}+\frac{8}{27}v^{\frac{3}{2}}}^{\frac{5}{3}}-\frac{4}{9}v}{v^{\frac{1}{2}}}^{\frac{5}{2}}$ = area = *maximum*; and therefore $\overline{cv^{\frac{1}{2}}+\frac{8}{27}v^{\frac{3}{2}}}^{\frac{5}{3}}-\frac{4}{9}v^{\frac{4}{3}}cv$ = *maximum*. This in fluxions, and reduced, gives $vv-\frac{2}{16}cv=\frac{22}{576}cc$, or $v=\frac{9+\sqrt{189}}{32}\times c$. From whence $y=.69785c$; and therefore if $c=50$ the double area $=1449$. If $c=100$, the double area $=5796$. This gentleman further obſerves that when the area is a *maximum*, the *abſciſſa*, *ordinate*, and *curve*, are reſpectively as $\frac{1}{\sqrt[4]{1.01823}}$, $\sqrt[4]{1.01823}$, and $\overline{1.01823}^{\frac{4}{4}}$.

The ſolutions to the remaining queſtions in the ſupplement in the Gentleman's Diary, *and* Gentleman's Magazine, *for want of room muſt be referred to our next.*

The End of NUMBER I.

Plate I.

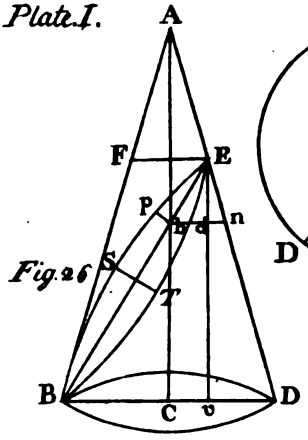
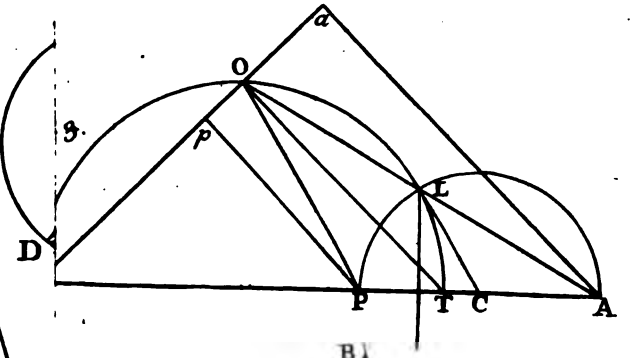


Fig. 26



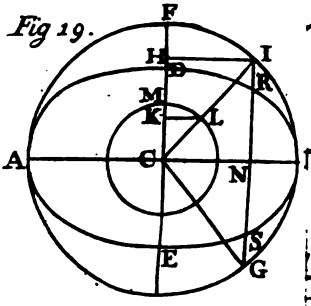
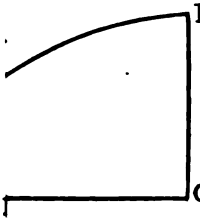


Fig 19.



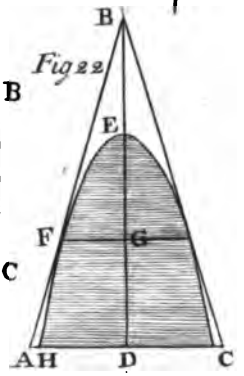
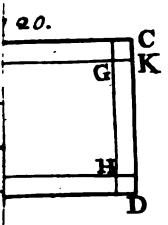


Fig 22

20.



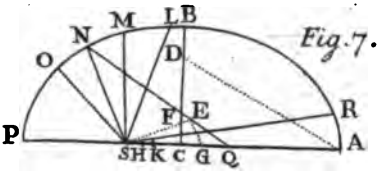


Fig. 7.

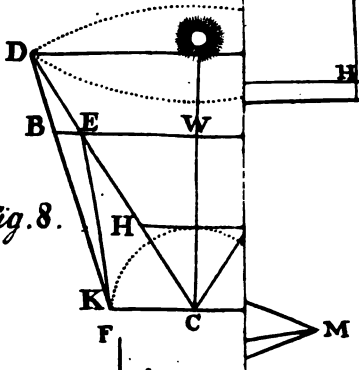


Fig. 8.

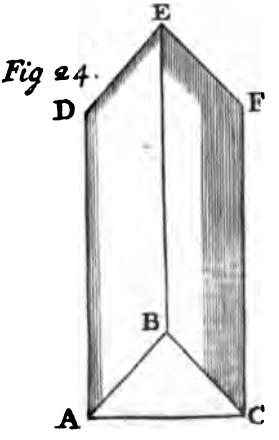


Fig 24.

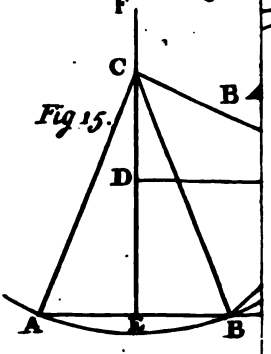


Fig 15.

Miſcellanea Curioſa, &c.

The ſolutions to the queſtions in the ſupplement to the Gentleman's Diary *continued. By Mr* SAMUEL FARRER, *of* London.

QUESTION XIII. *By Mr* W. MEESON; *anſwered by the ſame.*

OF all thoſe parabolic conoids which may be inſerted in that cone, whoſe altitude is $=a$, and ſemi-baſe $=b$, quere that whoſe ſolidity is a *maximum?*

SOLUTION.

Let 3.14159 &c. $=p$; BE$=x$; then DE$=a-x$; and from ſimilar triangles we have $a : b :: 2x : \frac{2bx}{a}=$FG. Alſo from the nature of the parabola $x : \frac{4b^2x^2}{a^2} :: a-x : \overline{DH}^2=\frac{4pb^2x\times\overline{a-x}}{a^2}$; whence the ſolidity of the conoid $=\frac{2pb^2x\times\overline{a-x}^2}{a^2}$; which *per* queſtion is a *maximum.* Therefore its fluxion $\frac{2pb^2\dot{x}\times\overline{a-x}^2-4pb^2x\dot{x}\times\overline{a-x}}{a^2}$ $=0$; wherefore $a-x-2x=0$. Conſequently $x=\frac{a}{3}$, and the greateſt ſolidity $\frac{8pb^2a}{27}$. W. W. D. Pl. 1. Fig. 22.

This was anſwered by *John Street*, Gent.

QUESTION XIV. *By Mr* PAUL SHARP; *anſwered by the ſame.*

A Gentleman having a piece of land, in the form of a ſemi-ellipſe, whoſe area is $=125.664$, and ſtraight ſide (or tranſverſe diameter) 20 chains, I have agreed with him for a four-ſide field to be taken out of it. Required ſo to place my fence, as I may have the moſt land poſſible, and how much?

SOLUTION.

It is very obvious, the greateſt quadrilateral figure that can be inſcribed in a ſemicircle, will have three of its ſides equal, each ſubtending Pl. 1. Fig. 23.

tending an angle at the centre of the entire circle of ſixty degrees, and the fourth ſide, the diameter thereof; wherefore drawing the line as *per Fig.* and puting the ſine of 60°=c, the ſemi-tranſverſe axe=a, the ſemi-conjugate =b; we ſhall have $\frac{ca^2}{2}$ for the area of one of the conſtituent triangles, as AFH; three of which will be $\frac{3ca^2}{2}$, the area of this quadrilateral; but this area is to that required in the ſemi-ellipſe as a to b; wherefore the area of the greateſt quadrilateral figure that can be inſcribed in the ſemi-ellipſe will be $\frac{3cab}{2}=\frac{3ab\sqrt{3}}{4}$ (becauſe the ſine of 60°=$\frac{1}{2}\sqrt{3}$) or in numbers 103.923048.

QUESTION XV. *By Mr* JOHN ASH; *anſwered by the ſame.*

THERE is a triangular priſm, whoſe angles are 50, 60, and 70°; alſo its whole ſuperficies is 100 Foot: the ſolidity (being a *maximum*) is required.

SOLUTION.

Pl. 1. Fig. 24. Let m, n, and p denote the ſines of the given angles, and x the greateſt ſide of the triangle at the baſe; then $p:x::m:\frac{mx}{p}$=BC; and $p:x::n:\frac{nx}{p}$=AB; wherefore double the baſe of the priſm = $\frac{nmx^2}{p}$, which deducted out of S, the whole ſurface, leaves $\frac{Sp-nmx^2}{p}$; this multiplied by the reciprocal of $\frac{px+mx+nx}{p}$ gives $\frac{Sp-nmx^2}{px+mx+nx}$, for the height of the priſm; hence $\frac{nm\times\overline{Spx-nmx^3}}{2p^2+2mp+2p}$ its ſolidity, being a *maximum* in fluxions, will be $Spx-3nmx^2\dot{x}=0$; wherefore $x=\sqrt{\frac{Sp}{3nm}}$, and the ſolidity from thence may be determined. Q. E. F.

QUESTION XVI.

OUR ingenious friend obſerves, that ſince the diſtances of the three towns are given; the poſitions from thence may be determined, and therefore this is only a particular caſe of a general method, given by Mr *Simpſon* in his *Treatiſe of Fluxions*, pag. 25. which the reader may conſult, &c.

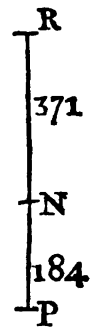
A new

A new geographical paradox, by GAMSTON RETFORD, *answered.*

SUPPOSE a ship sail from a port (whose longitude is 57° 23′ W.) first north 184 miles, then south 371 miles, and finds by observation, that she is 555 miles distance from whence she first sail'd: required the latitude of the place sail'd from, and the longitude and latitude of the place come to?

SOLUTION.

Let P be the port sail'd from, distance 184 miles from N the north pole, R the port arriv'd to 371 miles distance from N; then it is manifest the ship sails north 184 miles, then south 371 miles, and the distance of the two ports 555 miles, now 184 miles = 3° 4′, which taken from 90° gives 86° 46′ north for the latitude of the port P, and 90° *minus* 6° 11′ gives 83° 49′ south for the latitude of the port R; and consequently 180° *minus* 57° 23′ = 122° 37′ its longitude.



SAMUEL FARRER.

The two questions proposed in the Gentleman's Magazine, *Vol.* XIV. *p.* 588. *By Mr* JOHN TURNER *of* London; *answered.*

QUESTION I.

TO find the ratio of the axis and base of a cone, whose greatest ellipsis shall be exactly equal to the circle at the base.

SOLUTION. *The Proposer's answer.*

Let A B D represent the cone, and B S T E the ellipsis; put AC = a, C D = b, and C v = x; then D v = $b-x$, and, *per* sim. Triangles, D C : A C :: D v : E v = $\frac{ab-ax}{b}$; therefore $\overline{Ev}^2+\overline{Bv}^2=\overline{BE}^2=\frac{a^2\times\overline{b-x}^2+b^2\times\overline{b+x}^2}{b^2}$; and $\overline{ST}^2=4bx$; but $\overline{BE}^2\times ST^2=BD^4$, *viz.* $\frac{\overline{a^2\times\overline{b-x}^2+b^2\times\overline{b+x}^2}\times 4bx}{b^2}=4b^4$; which expression being a *maximum*, its fluxion $a^2b^2-4a^2bx+3a^2x^2+b^4+4b^3x+3b^2x^2=0$; whence $a^2=\frac{b^4+4b^3x+3b^2x^2}{4bx-3x^2-b^2}$. But the above equation reduced will give $a^2\times\overline{b^2x-2bx^2+x^3}+b^2\times\overline{b^2x+2bx^2+x^3}=4b^5$; and substituting for a^2 its

Pl. 1. Fig. 26.

its value, we have $b^2 \times \overline{b^2x+2bx^2+x^3} + \frac{b^2 \times \overline{b^2+4bx+3x^2}}{4bx-3x^2-b^2} \times \frac{b^2x-2bx^2+x^3}{4bx-3x^2-b^2} = 4b^5$, which reduc'd gives $x=b$ into $\sqrt{2}-1$. But to find a, let $b \times \sqrt{2}-1$ be ſubſtituted for x in the above value of aa; then we ſhall have $a^2=b^2$ into $11+8\sqrt{2}$; therefore $a=b \times \overline{11+8\sqrt{2}}^{\frac{1}{2}}$; whence this proportion $a : b :: \overline{11+8\sqrt{2}}^{\frac{1}{2}} : 1$; or $1 : \overline{11+8\sqrt{2}}^{\frac{1}{2}} :: b : a ::$ rad. tangent of the angle A D C.

The ſame anſwered by Mr TURNER *of* York.

MR *Simpſon*, in Ex. 15. *p.* 33. of his fluxions, has proved that when the area of the ellipſis B T E S B is a *maximum*, then $b^2 \times \overline{b^2+4bx+3x^2} = a^2 \times \overline{4bx-3x^2-b^2}$, or $\frac{b^4+4b^3x+3b^2x^2}{4bx-3x^2-b^2} = aa$ (putting A C$=a$; C D$=b$; C$v=x$;) but the ſquare of the area of the ſaid ellipſis is equal to $\frac{4a^2 \times b^2x-2bx^2+x^3 + 4b^2 \times \overline{b^2x+2bx^2+x^3}}{b} =$ $16b^4$ by the queſtion; this reduc'd gives $\frac{4b^5-b^4x-2b^3x^2-b^2x^3}{b^2x-2bx^2+x^3} =$ aa. Therefore $\frac{b^4+4b^3x+3b^2x^2}{4bx-3x^2-b^2} = \frac{4b^5-b^4x-2b^3x^2-b^2x^3}{b^2x-2bx^2+x^3}$, which reduc'd, we have $x^4-4b^2x^2+4b^3x=b^4$; and if we ſuppoſe b=unity, then $x^4-4x^2+4x=1$, and $x=.4142$; or $a=4.724$, and the angle B A D$=23°\ 54'\ 16''$.

CLEOPHILUS *has anſwered this queſtion in a method ſomething different from the two laſt.*

LET CD$=1$, C$v=x$, CA$=y$; and by a known property we have $\overline{\text{B D} \times \text{F E}}^{\frac{1}{2}} = \text{S T} = \sqrt{4x}$; and *per* ſim. triangles $v\text{E}=y-yx$; $\text{B E} = \overline{1+x^2+yy \times 1-x^2}^{\frac{1}{2}}$, and $\overline{1+x^2+yy \times 1-x} \times 4x$ will be a *maximum*; put into fluxions, *&c.* making y conſtant, we ſhall have $yy = \frac{4x+1+3xx}{4x-1-3xx}$. Again B E $\times$ S T $=$ B D; from which equation we ſhall have $yy = \frac{4-x-2xx-x^3}{x-2xx+x^3} = \frac{4x+1+3xx}{4x-1-3xx}$; which reduc'd is $xx+2x=1$. Therefore $x=.4142$; C A$=4.723$; and, conſequently, the ratio of the cone's baſe to its axis will be as 1, to 2.3615. Q. E. I.

We received ſolutions to the above from *Gamſton Retford*, *Philomathes*, and Mr *Badder*.

QUESTION

QUESTION II.

THERE are two places 462 miles aſunder, from which two travellers; A and B, ſet out at the ſame time to meet each other. A goes 1 mile the firſt day, 2 the ſecond, 3 the third, and ſo on in an arithmetical progreſſion increaſing each day 1 mile. B goes 1 mile the firſt day, 8 the ſecond, 27 the third, according to the natural numbers. It is required to know in how many days they will meet?

Anſwered by the propoſer Mr JOHN TURNER *of* London.

LET x be the required number of days; then $\frac{x^2+x}{2}$ will be the miles the firſt travels; and $\frac{\overline{x^2+x}^2}{4}$ the miles the other travels. Therefore $\frac{\overline{x^2+x}^2}{4}+\frac{x^2+x}{2}=462$ *(b;)* put $\frac{x^2+x}{2}=y$; then will $yy+y=b$; conſequently $y=\sqrt{b+\frac{1}{4}}-\frac{1}{2}=21$ the miles the firſt travelled. But becauſe $x^2+x=2y$, therefore $\sqrt{2y+\frac{1}{4}}-\frac{1}{2}=6$, the number of days they travelled.

The ſame anſwered by Mr JOHN TURNER *of* York.

LET $x=$ number of miles that A travels; then (*per Lemma* p. 226, *Ward*) $xx=$number of miles that B travels; and putting $c=462$, we have $x^2+x=c$, and $x=\sqrt{c+\frac{1}{4}}-\frac{1}{2}=21$, the miles A went; and $xx=441$, the miles B went. Laſtly making $a=1$ the firſt term, $e=1$ the common exceſs, $s=21$ the ſum of all the terms, $n=$ number of terms; then by the nature of arithmetical progreſſion $\frac{\sqrt{a^2+\frac{1}{4}e^2-ae+2es}-a}{e}+\frac{1}{2}=n=6$, exactly; the number of days that each man travelled. Q. E. I.

This was anſwered by *Gamſton Retford, Philomathefis, Cleophilus, S. W. J. W——s, W. Jephſon,* Mr *F. Cockſon,* Mr *J. Street,* Mr *J. Hurſt,* Mr *R. Davis.*

A queſtion propoſed in the Gentleman's Magazine *for* Auguſt, *by Mr* LANDEN.

LET a ſpherical ball of cork, whoſe diameter is one foot, be ſuppoſed to be projected upwards, in a direction perpendicular to the horizon, with a velocity ſufficient to carry it uniformly over a

ſpace of 500 feet in one ſecond of time. It is required to find the aſcent, the times of the aſcent and deſcent, with the velocity generated in falling?

Anſwered by Mr J. ANGLETWITCH.

Firſt then allowing the ſpecific gravity of water to that of air to be as 860 to 1, and that of cork to water to be as .237 to 1, (as I find it in the *Phil. Tranſ.*) the ſpecific gravity of cork to that of air will be as 203.82 to 1.

In the next place, if there were no reſiſtance from the medium, the ball would riſe to the height of 3886 feet, and the time of the aſcent would be 15.544 ſeconds, and in that time the ball would loſe all its motion by the counteraction of gravity alone.

Again, if there were no counteraction from gravity, but the reſiſtance of the medium only to be conſidered, and if we allow the ſpecific gravity of cork to that of air to be as above; then the ball projected upwards in air with a velocity that wou'd carry it uniformly 500 feet in a ſecond, or 7772 feet in 15.544 ſeconds, will be found by computation to loſe $\frac{93464}{100000}$ths of its firſt velocity in that time.

Since, therefore, the intenſities of powers are always as the whole effects they can produce in a given time; and ſince the force of gravity deſtroys all the motion of the ball in 15.544 ſeconds, and the reſiſtance of the medium deſtroys 93464 parts out of 100000 in the ſame time, the force of gravity will in this caſe be to the reſiſtance of the medium as 100000 to 93464. And, conſequently, if we ſay, *As the ſum of theſe forces : to the force of gravity only : : ſo the ſpace the ball would go thro' if gravity only acted : to the ſpace it will now go thro' when both forces act together*, we ſhall have the height required. By this analogy the ball will be found to riſe to the height of 2008.64 feet; and the time of the aſcent will be 8.034 ſeconds.

For the time of the deſcent; ſince gravity and reſiſtance may now be conſidered as acting againſt one another, and ſince the ball is to deſcend thro' the ſpace of 2008.64 feet, it will fall through this ſpace by the force of gravity alone in 11.17 ſeconds, and gain a velocity at the end of the fall, which would carry it uniformly 359.3 feet in a ſecond. But at this rate of velocity, the reſiſtance of the medium will be found (by a like calculation with the former) to be to the force of gravity, as 88072 to 100000. And, therefore, if we ſay, As 100000 : 188072 : : 11.17 : 21.0076, this will be the time it takes to deſcend in; and at the end of the fall will gain

gain a velocity that would carry it uniformly 191.169 feet in a ſecond.

N. B. *I have all along ſuppoſed that a heavy body will fall* $16\frac{1}{12}$ *feet in the firſt ſecond of time, by the force of gravity.*

As there was a miſtake in the application to the ſolution of the above problem in the Gentleman's Magazine *for* October, *Vol.* XIV. *p.* 533. *we ſhall inſert here its correction as promiſed, where we obſerve it is an inſtance of prob.* XIV. §. III. *of Mr* Emerſon's *fluxions, and there ſolv'd generally. We ſhall not trouble the reader with the proceſs at large, from whence the following corollaries are deduc'd, but refer him to the book itſelf, and ſhall only apply his method of ſolution to this ſingle caſe as follows.*

LET d = diameter of the ball = 1, q = its denſity = 203.82, p = denſity of the medium = 1, $s = 16\frac{1}{12}$ feet, the ſpace thro' which a heavy body deſcends by gravity in a ſecond, x = ſpace deſcribed from the beginning of the motion, t = time deſcribing x, v = velocity at the end of the time t, b = velocity the ball is projected upwards with = 500 feet; here I meaſure the velocity by the ſpace uniformly deſcribed in a ſecond. Then proceeding from the above-mentioned place we get this

COROLLARY.

When the ball aſcends, $t = \frac{2dq \times .01745}{\sqrt{p-p \times 3dsp}} \times$ degrees in the arch whoſe tangent is $\frac{b}{4}\sqrt{\frac{3p}{q-p \times ds}}$ = whole time of aſcent. Likewiſe the whole ſpace of aſcent is $x = \frac{4dq \times 2.30258}{3p} \times$ log. of $\frac{\overline{q-p} \times 16ds + 3pbb}{q-p \times 16ds}$,

Again, in *Corol.* 3. by putting $H = 4\sqrt{\frac{q-p \times ds}{3p}}$, and $G = q\sqrt{\frac{d}{q-p \times 3ps}}$, we have $x = 2.3025 \times \frac{4dq}{3p} \times$ log. $\frac{HH}{HH-vv}$, or, which is the ſame thing, $\frac{3px}{2.3025 \times 4dq} = \log. \frac{HH}{HH-vv}$, where x was found before; now call the log. of $\frac{3px}{2.3025 \times 5dq}$, R; then $\frac{HH}{HH-vv} =$ R, which reduc'd gives v the velocity it has when it has fallen to the earth again.

Laſtly,

Laſtly, by *Corol.* 2. we have $t=2.3025\mathrm{G}\times\log.\frac{\mathrm{H}+v}{\mathrm{H}-v}$, for the time of its deſcent.

The following eſſay, concerning the ſums of the powers of arithmetical progreſſions, was communicated by SPICILEGUS, *Gent.*

If the 1ſt term =0, common difference =1, and number of terms 1, 2, 3, 4, 5, or 6; the following tables will ſhew the ſums of their powers to the 12th,

	Powers.											
	1ſt	2d	3d	4th	5th	6th	7th	8th	9th	10th	11th	12th
	0	0	0	0	0	0	0	0	0	0	0	0
	1	1	1	1	1	1	1	1	1	1	1	1
	2	4	8	16	32	64	128	256	512	1024	2048	4096
	3	9	27	81	243	729	2187	6561	19683	59049	177147	531441
	4	16	64	256	1024	4096	16384	65536	262144	1048576	4194304	16737216
	5	25	125	625	3125	15625	78125	390625	1953125	9765625	48828125	244140625
Numb. Term	Sums of the number of terms under written of thoſe powers.											
3	3	5	9	17	33	65	129	257	513	1025	2049	4097
4	6	14	36	98	276	794	2316	6818	20196	60074	179196	535533
5	10	30	100	354	1300	4890	18700	72354	282340	1108050	4373500	17272754
6	15	55	225	979	4425	20515	96825	462979	2235465	10874275	53201625	261413379
	&c.			&c.				&c.			&c.	

Therefore if $n=$ number of thoſe terms,

The ſum of the 1ſt. powers $=\frac{1}{2}nn-\frac{1}{2}n$

2. $=\frac{1}{3}n^3-\frac{1}{2}nn+\frac{1}{6}n$

3. $=\frac{1}{4}n^4-\frac{1}{2}n^3+\frac{1}{4}nn$

4. $=\frac{1}{5}n^5-\frac{1}{2}n^4+\frac{1}{3}n^3-\frac{1}{30}n$

5. $=\frac{1}{6}n^6-\frac{1}{2}n^5+\frac{5}{12}n^4-\frac{1}{12}n^2$

6. $=\frac{1}{7}n^7-\frac{1}{2}n^6+\frac{1}{2}n^5-\frac{1}{6}n^3+\frac{1}{42}n.$ For

1. When $n=3$; $\frac{9}{2}-\frac{3}{2}=\frac{6}{2}=3$: when $n=4$; $\frac{16}{2}-\frac{4}{2}=\frac{12}{2}=6$.

$n=5$; $\frac{25}{2}-\frac{5}{2}=\frac{20}{2}=10$:

2. When $n=3$; $\frac{27}{3}-\frac{9}{2}+\frac{3}{6}=\frac{54-27+3}{6}=\frac{30}{6}=5$: $n=4$;

$n=4$; $\frac{64}{3}-\frac{16}{2}+\frac{4}{6}=\frac{128-48+4}{6}=\frac{84}{6}=14$:

$n=5$; $\frac{125}{3}-\frac{25}{2}+\frac{5}{6}=\frac{250-75+5}{6}=\frac{180}{6}=30$:

3. When $n=3$; $\frac{81}{4}-\frac{27}{2}+\frac{9}{4}=\frac{81-54+9}{4}=\frac{36}{4}=9$:

$n=4$; $\frac{256}{4}-\frac{64}{2}+\frac{16}{4}=\frac{256-128+16}{4}=\frac{144}{4}=36$:

$n=5$; $\frac{625}{4}-\frac{125}{2}+\frac{25}{4}=\frac{625-250+35}{4}=\frac{400}{4}=100$:

4. When $n=3$; $\frac{243}{5}-\frac{81}{2}+\frac{27}{3}-\frac{3}{30}=\frac{1458-1215+270-3}{30}=$
$\frac{1728-1218}{30}=\frac{510}{30}=17$:

$n=4$; $\frac{1024}{5}-\frac{256}{2}+\frac{64}{3}-\frac{4}{30}=\frac{6144-3840+640-4}{30}=$
$\frac{6784-3844}{30}=\frac{2940}{30}=98$:

$n=5$; $\frac{3125}{5}-\frac{625}{2}+\frac{125}{3}-\frac{5}{30}=\frac{18750-9375+1250-5}{30}$
$=\frac{20000-9380}{30}=\frac{10620}{30}=354$:

5. When $n=3$; $\frac{729}{6}-\frac{243}{2}+\frac{5\times81}{12}-\frac{9}{12}=\frac{1458-1458+405-9}{12}$
$=\frac{1863-1467}{12}=\frac{396}{12}=33$:

$n=4$; $\frac{4096}{6}-\frac{1024}{2}+\frac{5\times256}{12}-\frac{16}{12}=\frac{8192-6144+1280-16}{12}$
$=\frac{9472-6160}{12}=\frac{3312}{12}=276$:

$n=5$; $\frac{15625}{6}-\frac{3125}{2}+\frac{5\times625}{12}-\frac{25}{12}=\frac{31250-18750+3125-25}{12}$
$=\frac{34375-18775}{12}=\frac{15600}{12}=1300$: And

6. When $n=3$; $\frac{2187}{7}-\frac{729}{2}+\frac{243}{2}-\frac{27}{6}+\frac{3}{42}=$
$\frac{13122-15309+5103-189+3}{42}=\frac{18228-15498}{42}=\frac{455}{7}=65$:

$$n=4;\ \frac{16384}{7}-\frac{4096}{2}+\frac{1024}{2}-\frac{64}{6}+\frac{4}{42}=$$

$$\frac{98304-86016+21504-448+4}{42}=\frac{119812-86464}{42}=\frac{5558}{7}=794:$$

$$n=5;\ \frac{78125}{7}-\frac{15625}{2}+\frac{3125}{2}-\frac{125}{6}+\frac{5}{42}=$$

$$\frac{468750-328125+65625-875+5}{42}=\frac{534380-329000}{42}=$$

$$\frac{34230}{7}=4890:$$

And making m equal to the index of the power, alſo,

$$a=1$$

$$b=\frac{a}{2}$$

$$c=\left(b-\frac{a}{3}=\right)\frac{a}{6}$$

$$d=\left(\frac{4}{2}c-b+\frac{a}{5}=\right)\frac{a}{30}$$

$$e=\left(\frac{6\cdot5\cdot4}{2\cdot3\cdot4}d-\frac{6}{2}c+b-\frac{a}{7}=\right)\frac{a}{42}$$

$$f=\left(\frac{8\cdot7\cdot6\cdot5\cdot4}{2\cdot3\cdot4\cdot5\cdot6}e-\frac{8\cdot7\cdot6}{2\cdot3\cdot4}d+\frac{8}{2}c-b+\frac{a}{9}=\right)\frac{a}{30}$$

$$g=\left(\frac{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot4}{2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8}f-\frac{10\cdot9\cdot8\cdot7\cdot6}{2\cdot3\cdot4\cdot5\cdot6}e+\frac{10\cdot9\cdot8}{2\cdot3\cdot4}d-\frac{10}{2}c+b-\frac{a}{11}=\right)$$

$\frac{5a}{66}$, &c.

The ſum of the m^{th} powers will be,

$$\frac{1}{m+1}an^{m+1}-bn^m+\frac{m}{2}cn^{m-1}-\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{2\cdot3\cdot4}dn^{m-3}+$$

$$\frac{m\cdot\overline{m-1}\cdot\overline{m-2}\cdot\overline{m-3}\cdot\overline{m-4}}{2\cdot3\cdot4\cdot5\cdot6}en^{m-5}\ \text{&c.}$$

Laſtly, putting $\frac{1}{m+1}a=p$; $b=q$; $\frac{m}{2}c=r$; $\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{2\cdot3\cdot4}d=s$, &c. the ſum of the $\overline{m+1}^{th}$ powers will be,

$$\frac{m+1}{m+2}pn^{m+2}-qn^{m+1}+\frac{m+1}{m}rn^m-\frac{m+1}{m-2}sn^{m-2}+\frac{m+1}{m-4}tn^{m-4}$$ &c. wanting (if $\overline{m+1}$ be an even number) the laſt term; which if $\overline{m+1}=2$ will be cn; if $m+1=4$, dn; if $\overline{m+1}$ be 6, en, &c.

A table of the ſums formed as above follows, wnere A=n, or the ſum of the units; B= ſum of the firſt powers; C= the ſum of the ſecond powers; D= the ſum of the third powers, &c.

A=n

B=$\frac{1}{2}n^2-\frac{1}{2}n$

C=$\frac{1}{3}n^3-\frac{1}{2}n^2+\frac{1}{6}n$

D=$\frac{1}{4}n^4-\frac{1}{2}n^3+\frac{1}{4}n^2$

E=$\frac{1}{5}n^5-\frac{1}{2}n^4+\frac{1}{3}n^3*-\frac{1}{30}n$

F=$\frac{1}{6}n^6-\frac{1}{2}n^4+\frac{5}{12}n^3*-\frac{1}{12}n^2$

G=$\frac{1}{7}n^7-\frac{1}{2}n^6+\frac{1}{2}n^5*-\frac{1}{6}n^3*+\frac{1}{42}n$

H=$\frac{1}{8}n^8-\frac{1}{2}n^7+\frac{7}{12}n^6*-\frac{7}{24}n^4*+\frac{1}{12}n^2$

I=$\frac{1}{9}n^9-\frac{1}{2}n^8+\frac{2}{3}n^7*-\frac{7}{15}n^5*+\frac{2}{9}n^3*-\frac{1}{30}n$

K=$\frac{1}{10}n^{10}-\frac{1}{2}n^9+\frac{3}{4}n^8*-\frac{7}{10}n^6*+\frac{1}{2}n^4*-\frac{3}{20}n^2$

L=$\frac{1}{11}n^{11}-\frac{1}{2}n^{10}+\frac{5}{6}n^9*-n^7*+n^5*-\frac{1}{2}n^3*-\frac{5}{66}n$

M=$\frac{1}{12}n^{12}-\frac{1}{2}n^{11}+\frac{11}{12}n^{10}*-\frac{11}{8}n^8*+\frac{11}{6}n^6*-\frac{11}{8}n^4*+\frac{5}{12}n^2$, &c.

Note, A table of the ſums of the powers of the progreſſion 1, 2, 3, 4, &c. of this form is exhibited in Mr *De Moivre*'s *Appendix* to his *Miſcellanea Analytica*, p. 5. and is quoted from *Ja. Bernoulli*.

If the fractions above be reduced to common denominators, the above table will put on the following form:

A=n

B=$\frac{nn-n}{2}$

C=$\frac{2n^3-3n^2+n}{6}$

D=$\frac{n^4-2n^3+nn}{4}$

E=$\frac{6n^5-15n^4+10n^3-n}{30}$

F=$\frac{2n^6-6n^5+5n^4-n^2}{12}$

G=$\frac{6n^7-21n^6+21n^5-7n^3+n}{42}$

H=$\frac{3n^8-12n^7+14n^6-7n^4+2n^2}{24}$

I=$\frac{10n^9-45n^8+60n^7-42n^5+20n^3-3n}{90}$

K=$\frac{2n^{10}-10n^9+15n^8-14n^6+10n^4-3n^2}{20}$

L=

$$L=\frac{6n^{11}-33n^{10}+55n^{9}-66n^{7}+66n^{5}-33n^{3}+5n}{66}$$

$$M=\frac{2n^{12}-12n^{11}+22n^{10}-33n^{8}+44n^{6}-33n^{4}+10n^{2}}{24}$$

As theſe fractions are compoſed quantities, it may be often requiſite to uſe their component parts, which are here attempted to be expreſs'd as follows:

$$A=\frac{n}{1}$$

$$B=\frac{n.\overline{n-1}}{1\cdot 2}$$

$$C=\frac{n.\overline{n-1}.\overline{2n-1}}{1\cdot 2\cdot 3}$$

$$D=\frac{n^{2}.\overline{n-1}^{2}}{2\cdot 2}$$

$$E=\frac{n.\overline{n-1}.\overline{2n-1}.\overline{3n^{2}-3n-1}}{1\cdot 2\cdot 3\cdot 5}$$

$$F=\frac{n^{2}.\overline{n-1}^{2}.\overline{2n^{2}-2n-1}}{1\cdot 3\cdot 4}$$

$$G=\frac{n^{2}.\overline{n-1}^{2}.\overline{2n-1}.\overline{n^{2}-n-1}}{1\cdot 2\cdot 7}+\frac{n.\overline{n-1}.\overline{2n-1}}{2\cdot 3\cdot 7}$$

$$H=\frac{n^{3}.\overline{n-1}^{3}.\overline{3n^{2}-3n-4}}{2\cdot 3\cdot 4}+\frac{n^{2}.\overline{n-1}^{2}}{3\cdot 4}$$

$$I=\frac{n^{3}.\overline{n-1}^{3}.\overline{2n-1}.\overline{n^{2}-n-2}}{1\cdot 2\cdot 3\cdot 3}+\frac{n^{2}.\overline{n-1}^{2}.\overline{2n-1}}{1\cdot 2\cdot 5}+\frac{n.\overline{n-1}.\overline{2n-1}}{2\cdot 3\cdot 5}$$

$$K=\frac{n^{4}.\overline{n-1}^{4}.\overline{2n^{2}-2n-5}}{1\cdot 4\cdot 5}+\frac{3n^{3}.\overline{n-1}^{3}}{2\cdot 5}+\frac{3n^{2}.\overline{n-1}^{2}}{4\cdot 5}$$

$$L=\frac{n^{4}.\overline{n-1}^{4}.\overline{2n-1}.\overline{3n^{2}-3n-10}}{1\cdot 2\cdot 3\cdot 11}+\frac{17n^{3}.\overline{n-1}^{3}.\overline{2n-1}}{2\cdot 3\cdot 11}+\frac{15n^{2}.\overline{n-1}^{2}.\overline{2n-1}}{2\cdot 3\cdot 11}+\frac{5n.\overline{n-1}.\overline{2n-1}}{2\cdot 3\cdot 11}$$

$$M=\frac{n^{5}.\overline{n-1}^{5}.\overline{2n^{2}-2n-8}}{2\cdot 3\cdot 4}+\frac{17n^{4}.\overline{n-1}^{4}}{4\cdot 6}-\frac{20n^{3}.\overline{n-1}^{3}}{4\cdot 6}+\frac{10n^{2}.\overline{n-1}^{2}}{4\cdot 6}$$

From which expreſſions the following manner of writing the Sums of all the ſuperior powers in the terms of B or C is eaſily deduced.

A=

$$A=\frac{n}{1}$$

$$B=\frac{A-1}{2}A$$

$$C=\frac{2A-1}{3}B$$

$$D=B\overline{B}$$

$$E=\frac{6B-1}{5}C$$

$$F=\frac{4\overline{B-1}}{3}BB$$

$$G=\frac{2\overline{B-1}\times 6B+1}{7}C$$

$$H=\frac{3\overline{B-2}\times 2B+1}{3}BB$$

$$I=\frac{\overline{B-1}\times 40B^2+\overline{6B-1}\times 3}{15}C$$

$$K=\frac{4\overline{B-5}\times 4B^2+4\overline{B-1}\times 3}{5}BB$$

$$L=\frac{3\overline{B-5}\times 16B+68\times BB-\overline{6B-1}\times 5}{11}C$$

$$M=\frac{\overline{B-2}\times 8B+17\times 2B^2-\overline{4B-1}\times 5}{3}BB$$

Therefore in any arithmetical progreſſion (putting a = leaſt term, d = common difference, n the number of terms, and A, B, C, D, &c. as above) the ſum of the m^{th} powers of its terms will be

$$Aa^m+mBa^{m-1}d+\frac{m.\overline{m-1}}{1.2}Ca^{m-2}d^2+\frac{m.\overline{m-1}.\overline{m-2}}{1.2.3}Da^{m-3}d^3, \text{ \&c.}$$

For the powers of ſuch a progreſſion will be as under written:

a	aa	a^3	&c.
$a+d$	$aa+2\times 1ad+1dd$	$a^3+3\times 1a^2d+3\times 1ad^2+1d^3$	&c.
$a+2d$	$aa+2\times 2ad+4dd$	$a^3+3\times 2a^2d+3\times 4ad^2+8d^3$	&c.
$a+3d$	$aa+2\times 3ad+9dd$	$a^3+3\times 3a^2d+3\times 9ad^2+27d^3$	&c.

The conductor thinks it not improper to illuſtrate the above by an example; as, for inſtance,

QUESTION 258, *in* Ladies Diary, 1745, *answered.*

GIVEN $1+2+3+4$, &c. continued to x terms; to find $1+4+9+16$, &c. the sum of the squares of those numbers.

SOLUTION.

First here is given $1+2+3+4 \ldots . x$, to find the sum of the progression $1^1+2^1+3^1+4^1 \ldots . x^1$. Now it is prov'd above that if n be the number of terms,

The sums of the 1st. powers $=\frac{1}{2}nn+\frac{1}{2}n=\frac{n.\overline{n+1}}{2}$.

2d. $=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n=\frac{n.n+1.2n+1}{6}$.

3d. $=\frac{1}{4}n^4+\frac{1}{2}n^3+\frac{1}{4}n^2=\frac{n^2.\overline{n+1}^2}{4}$

&c. &c. &c.

Consequently it is evident, at first sight, that writing $x+1$ instead of n, the question in hand will be $=\frac{x.x+1.2x+1}{6}=\frac{1}{3}x^3+\frac{1}{2}x^2+\frac{1}{6}x$, the sum of the progression, which was to be found.

Also if you would find the sum of the progression $1+8+27+64 \ldots . x^3$; or, which is the same thing, to find the sum of the progression $1^3+2^3+3^3+4^3 \ldots . x^3$, it will be, by what is delivered above, $=\frac{x^2.\overline{x+1}^2}{2.2}$; also the sum of $1^4+2^4+3^4+4^4 \ldots . x^4=\frac{1}{5}x^5+\frac{1}{2}x^4+\frac{1}{3}x^3-\frac{1}{30}x$, &c. &c.

The 258th *question in the* Ladies Diary *answered by* SPICILEGUS.

PUT $x+1=n$; $1+2+3+4$, &c. to x terms $=B$; and $1+4+9+16$, &c. to x terms $=C$.

Then $\frac{2n-1}{3}B=C$ (*per form p.* 49.)

Therefore $\frac{2n-1}{3}=\frac{C}{B}$. And $n=\frac{3C+B}{2B}$

Also $\frac{nn-n}{2}=B$ (*per form p.* 47.)

Then $\overline{\frac{3C+B}{2B}}^2-\frac{3C+B}{2B}=2B$

Therefore $9C^2-BB=8B^3$

Then $C=\frac{1}{3}B\sqrt{8B+1}$.

QUE-

QUESTION I. *Answered by Mr* ROBERT ROBINSON, *teacher of the mathematics at* Stockton *upon* Tees.

LET AD=a=40, DB=q=90, sum of the angles CAB and CBA=80°, sine =n; and put x= sine <CAB, y= sine <ABC, and v=DE=DC. Then $x^2 : 4v^2 :: 1-x^2 : \frac{4v^2-4v^2x^2}{x^2} = \overline{AE}^2$; and by 47 *E.* 1. $aa=v^2+\frac{4v^2-4v^2x^2}{x^2}$. Therefore $v^2=\frac{a^2x^2}{4-3x^2}=\frac{q^2y^2}{y-3y^2}$. Therefore $q^2y^2-a^2x^2=\frac{3q^2-3a^2}{4}x^2y^2=dy^2x^2$; in which write ey for x, and it gives $y^2=\frac{q^2-a^2e^2}{de^2}$. But the sine of the angles CAB, ABC, is $\sqrt{1-x^2}\times\sqrt{1-y^2}-xy=n$; therefore $y^2+x^2+2nxy=1-n^2=m^2$; and for x putting ey, it is $y^2=\frac{m^2}{1+e^2+2ne}$. Consequently $\frac{m^2}{1+e^2+2ne}=\frac{q^2-a^2e^2}{de^2}$; which is by reduction $e^4+2ne^3+\frac{3dm^2-4d}{3a^2}e^2-\frac{2q^2n}{a^2}e=\frac{q^2}{a^2}$; in numbers is $e^4+.347296e^3-1.1075e^2-1.758188e=5.0625$; hence by converging series $e=1.765084$. Therefore $y=.4528849$, the sine of 26° 55′ 44″= <ABC; consequently AB=120.5345, AC=55.4304, BC=97.8393, and the area=2670.4385.

The same answered by Mr J. POWLE, *teacher of the mathematics at* Hereford.

PUT x=DE=DC, AD=a, DB=b, and s= sine of the angle ACB; then $AE=\sqrt{aa-xx}$, and $\sqrt{bb-xx}=EB$; also $\sqrt{aa+3xx}$=AC, $\sqrt{bb+3x^2}$=CB; and by Trigonometry we have (1) $\sqrt{aa-xx}+\sqrt{bb-xx}$ (=AB) : $s :: \sqrt{bb+3x^2} : \frac{s\sqrt{bb+3xx}}{\sqrt{aa-xx}+\sqrt{bb-xx}}$= sine <B. Again (2) $\sqrt{aa+3xx} : 1$ (=rad.) :: $2x$ (=CE) : $\frac{2x}{\sqrt{aa+3x^2}}$= sine <B too; consequently $\frac{2x}{\sqrt{aa+3x^2}}=\frac{s\sqrt{bb+3x^2}}{\sqrt{aa-xx}+\sqrt{bb-xx}}$, which reduc'd will produce an equation of the eighth power, from which it may be reduc'd to the fourth; when by converging series by form 12th of the *Syntagma Mathesios*, the value of x will be known.

The propoſer anſwers this in the ſame way as the laſt.

QUESTION II. *answered.*

Pl. 2. Fig. 28. LET ABC be the propoſed triangle, in which B denotes the right angle, AG and CF the two given lines, cutting off the equal ſegments FB and BG; then if FG be drawn, the angle GFB will appear to be equal to the angle FGB, the ſine of which put $=m$ (45°) and the meaſure of AG$=a$, CF$=b$, and BF$=$BG$=x$. Then (*per E.* 1. 47.) will FG$=x\sqrt{2}$, AB$=\sqrt{a^2-x^2}$, AF$=\sqrt{a^2-x^2}-x$; whence by *Trig.* $a : m :: x\sqrt{2} : \frac{mx\sqrt{2}}{a}$, the ſine of the angle FAG; $a : m :: \sqrt{a^2-x^2}-x : \frac{m\sqrt{a^2-x^2}-mx}{a}$, the ſine of the angle FGA. Therefore their correſponding coſines will be $\frac{\sqrt{a^2-2m^2x^2}}{a}$, and $\frac{\sqrt{a^2-m\sqrt{a^2-x^2}-mx|^2}}{a}$ reſpectively; but $b : m ::$ $x\sqrt{2} : \frac{mx\sqrt{2}}{b}$, the ſine of the angle FCG; and $b : m :: \sqrt{b^2-x^2}-x :$ $\frac{m\sqrt{b^2-x^2}-mx}{b}$, the ſine of the angle CFG; conſequently their correſponding coſines will be $\frac{\sqrt{b^2-2m^2x^2}}{b}$, and $\frac{\sqrt{b^2-m\sqrt{b^2-x^2}-mx|^2}}{b}$ reſpectively; whence by the elements of trigonometry, we have this equation,

$$\frac{mx\sqrt{2}\times\sqrt{a^2-m\sqrt{a^2-x^2}-mx|^2}+m\sqrt{a^2-x^2}-mx\times\sqrt{a^2-2m^2x^2}}{aa} =$$

$$\frac{mx\sqrt{2}\times\sqrt{b^2-m\sqrt{b^2-x^2}-mx|^2}+m\sqrt{b^2-x^2}-mx\times\sqrt{b^2-2m^2x^2}}{bb},$$

from the reſult of which x may be found.

There are perhaps other methods preferable to the above, by which this queſtion may be brought into algebraic terms; and if any ſuch ſhould occur to my readers, I ſhould be obliged to them for the diſcovery thereof, and it ſhall be inſerted in the next number.

Mr *Bamfield* attempted a ſolution to the above, but not with the deſired ſucceſs, which is the only one I have received.

QUESTION

QUESTION III. *answered by* INVOLUTOR.

PUT u, x, y, z= numbers ſought: then $uz=xy$; $u+x+y+z=b$; $u^2+x^2+y^2+z^2=c$; and $u^3+x^3+y^3+z^3=d$, *per* queſtion. Now $\frac{\overline{u+x+y+z}^2-\overline{u^2+x^2+y^2+z^2}}{2}=(=ux+uy+uz+xy+yz+xz=ux+uy+yz+xz+2xy=\overline{x+y}\times\overline{u+z}+2xy)\frac{bb-c}{2}$. And $\frac{\overline{u+x+y+z}^3-\overline{u^3+x^3+y^3+z^3}}{3}=\frac{b^3-d}{3}$. But $\frac{\frac{b^3-d}{3}\times 2}{bb-c\times 3}=\frac{\frac{1}{2}\times\overline{u+x+y+z}^3-\overline{u^3+x^3+y^3+z^3}}{\frac{1}{2}\times\overline{u+x+y+z}^2-\overline{u^2+x^2+y^2+z^2}}=u+x+y+z-\frac{uxy+uxz+uyz+xyz}{ux+uy+uz+xy+yz+xz}=b-\frac{uxy+x^2y+xy^2+xyz}{\frac{1}{2}\times bb-c}=b-\frac{\overline{u+x+y+z}\times xy}{\frac{1}{2}\times bb-c}\Big)=b-\frac{2bxy}{bb-c}$. Th. $(xy=)\ uz=\frac{b^3+2d-3bc}{6b}$; and (putting $x+y=b$, and $u+z=k$) $bk=\frac{b^3+3bc-4d}{6b}$; alſo ſince $b+k=b$ (*per* queſt.) $(k=)\ u+z=\frac{1}{2}\sqrt{\frac{b^3-6bc+8d}{3b}}+\frac{1}{2}b$; laſtly, if $(4uz=)\ \frac{2b^3+4d-6bc}{3b}=m)\ z=\frac{1}{2}\sqrt{kk-m}+\frac{1}{2}k$.

QUESTION 4. In which the propoſer made a miſtake, it ſhould run thus: "Given the equation $x^{x^x}=y$; required the value of x when y is a *minimum*?" The anſwer is Log. $x=-\frac{1}{e}$; or $x=.404$, &c. as Mr *Fearnſide* inform'd us; the ſolution of which our correſpondents may favour us with in their next. This ſhews how neceſſary it is for the propoſer always to ſend his ſolution at large with the queſtion, that it may be examined before it be inſerted, which we humbly deſire our correſpondents will obſerve, and we do aſſure them, that we will make it the moſt uſeful undertaking of this kind, by adapting it to all capacities, and by avoiding all partiality whatever.

QUESTION V. *answered by* Philofluentimecanalgegeomaſtrolongo.

THO' this may perhaps be eaſieſt ſolved by trials, aſſuming x as near as poſſible, and then by two of the neareſt values correcting the aſſumed value; yet I ſhall ſhew how the ſame may be performed

formed by converging ſeries, premiſing this, *viz.* that any compound exponential quantity as $\overline{n+v}|^{e+e}$, where v, e are very ſmall, is nearly equal to $n^e \times 1+\frac{le}{m} \times 1+\frac{cv}{n}$, where $l=$ Log. n, $m=.434294$; whence 1. Let $x=n+v$, v being very ſmall, then $x^x=\overline{n+v}|^{n+v}=n^n \times 1+\frac{lv}{m} \times \overline{1+v}=n^n \times 1+\frac{l}{m}+1 \times v$. Let $\frac{l}{m}+1=f$; $n^n f=g$, then

2. $\overline{x}|^{x^x}=\overline{n+v}|^{n^n}+n^n fv=\overline{n}|^{n^n} \times 1+\frac{ln^n fv}{m} \times \overline{1+\frac{n^n v}{n}}=\overline{n}|^{n^n} \times 1+\frac{gl}{m}+n^{n-1} \times v$.

3. $x^{-x}=\frac{1}{x^x}=\frac{1}{n^n \times \overline{1+fv}}=\frac{1-fv}{n^n}$.

4. $x^{\frac{1}{x}}=\overline{n+v}|^{\frac{1}{n+v}}=\overline{n+v}|^{\frac{1}{n}-\frac{v}{nn}}=n^{\frac{1}{n}} \times \overline{1-\frac{lv}{nnm}} \times \overline{1+\frac{v}{nn}}=n^{\frac{1}{n}} \times 1+\frac{1}{nn}-\frac{l}{n^2 m} \times v$.

5. Hence $\overline{n}|^{n^n} \times 1+\frac{gl}{m}+n^{n-1} \times v+n^n \times \overline{1+fv}-\frac{1}{n^n}+\frac{fv}{n^n}+n^{\frac{1}{n}} \times 1+\frac{1}{n^2}-\frac{l}{n^2 m} \times v=200$.

6. Let $200-\overline{n}|^{n^n}-n^n+\frac{1}{n^n}-n^{\frac{1}{n}}=A$; $\overline{n}|^{n^n} \times \frac{gl}{m}+n^{n-1}=B$; $n^{\frac{1}{n}} \times \frac{1}{nn}-\frac{l}{n^2 m}=C$.

Then $Bv+gv+\frac{f}{n^n}v+Cv=A$, and $v=\frac{A}{B+g+\frac{f}{n^n}+C}$.

Now v being had, take $n+v$ for a new value of n, and finding likewiſe new values for l, f, g, A, B, C, find a new v; and ſo the operation may be repeated to any degree of exactneſs.

The ſame anſwered by Mr WILLIAM CHAPPLE.

BY a trial or two, x is eaſily found to be more than 2, and leſs than 3. Th. put $x=2+y$, and $m=$ hyperbolic Log. thereof. Th. $2m+my=$hyp. Log. of x^x; then by the nature of hyp. Log. $1+2m+my+$

$my+\frac{2m^2+4m^2y+m^2y^2}{2}+\frac{2m^3+10m^3y+6m^3y^2+m^3y^3}{6}$ &c. $=x^x$.

Now m is $=\frac{1+y}{2+y}+\frac{1+2y+y^2}{8+8y+2y^2}+\frac{1+3y+3y^2+y^3}{24+36y+18y^2+3y^3}$ &c. and by writing this inſtead of m in the above equation, and reducing the fractions, &c. we have $5.4819+2.6498y+2.8333y^2+.55903y^3-.35245y^4$, &c. $=x^x$, which multiply'd by m gives the hyp. Log. of $\overline{x}|^{x^x}=4.00186+4.105y+2.1916y^2+1.4837y^3-.339y^4-.1666y^5+.1056y^6-.058y^7$, &c. which put $=z$: Then $1+z+\frac{z^2}{2}+\frac{z^3}{2\cdot 3}+\frac{z^4}{2\cdot 3\cdot 4}$, &c. $=\overline{x}|^{x^x}$. But as this will not converge, let $4.00186=$ hyp. Log. of 54.677 be deducted from z, and then we have $4.105y+2.1916y^2$ &c. $=z-4.00186$, which put $=n$: Then by nature of hyp. Log. $1+n+\frac{n^2}{2}+\frac{n^3}{6}+\frac{n^4}{24}$, &c. $=\frac{\overline{x}|^{x^x}}{54\cdot 677}$; reſtoring n, and writing the value thereof in the ſeries, &c. we have $1+4.10966y+10.2677y^2+19.8534y^3+33.0543y^4+45.173y^5+61.804y^6$, &c. $=\frac{\overline{x}|^{x^x}}{54\cdot 677}$. And by multiplication $54.677+224.704y+561.347y^2+1085.052y^3+1807.311y^4+2470.092y^5+3379.27y^6=\overline{x}|^{x^x}$.

x^x being already found as above, $x^{-x}=\frac{1}{x^x}$ is eaſily found, $=.1824-.08818y-.05166y^2+.05199y^3-.000785y^4-.0265y^5+.00577y^6$, &c. and by changing the ſigns, becomes $-x^{-x}$.

Again, (m being $=$ the hyp. Log. of x) $\frac{m}{x}=$ Log. of $x^{\frac{1}{x^x}}$, and (x being $=2+y$,) $1+\frac{m}{2+y}+\frac{m^2}{8+8y+2y^2}+\frac{m^3}{48+72y+36y^2+6y^3}$, &c. $=x^{\frac{1}{x^x}}$; that is, reſtoring m, and involving it, &c. $1.50686-.3086y+.18442y^2-.1519y^3+.10443y^4-.0667y^5+.0415y^6$ &c. $=x^{\frac{1}{x^x}}$.

And now we have four ſeries expreſſing the ſeveral values of x^x, $\overline{x}|^{x^x}$, $-x^{-x}$ and $x^{\frac{1}{x^x}}$, the ſum of which $+2+y=a$ $(=200)$; that is, $63.6656+228.045y+564.365y^2+1085.459y^3+1807.768y^4+2470.149y^5+3379.327y^6$, &c. $=200$; hence $y=.2681045$. Th. $x=2.2681045$. Q. E. I.

John Street, Gent. obſerves this is ſolved at p. 290. *Syntagma Mathefios*, from which this ſeems to have been taken.

QUES-

QUESTION VI: *anſwered by* Philofluentimecanalgegeomaſtrolongo.

LET X= hyp. Log. x, b= hyp. Log. a; then $x\mathrm{X}=\frac{by}{x}$, and $y=\frac{x^2\mathrm{X}}{b}$, and the fluxion of the area $y\dot{x}=\frac{x^2\mathrm{X}\dot{x}}{b}$, and fluent (by rule 8. p. 50. Emerſon's *doctrine of fluxions*) will be $\frac{x^3\mathrm{X}}{3b}+s$=area; whence $\dot{s}=\frac{-x^2\dot{x}}{3b}$, and $s=\frac{-x^3}{9b}$. Th. the area $=\frac{x^3\mathrm{X}}{3b}-\frac{x^3}{9b}$.

The ſame anſwered by CLEOPHILUS.

BY the nature of exponentials we have $x\mathrm{L}.x=y\mathrm{L}.a$ (L.x, L.a, denoting the hyperbolic Log. of x and a;) conſequently we ſhall have $x\mathrm{L}.x\times\frac{\dot{x}}{\mathrm{L}a}$ for the fluxion of the area of the curve, whoſe fluent will be found to be $\overline{\frac{1}{3}x^3\mathrm{L}.x-\frac{1}{9}x^3}+\frac{1}{9}\times\frac{\mathrm{I}}{\mathrm{L}a}$= (when x=10,, and a=100) 142.56, &c. Q. E. I.

The propoſer Mr J. POWLE'*s ſolution.*

LET A, X, Y, denote the hyp. Log. of a, x, y reſpectively; and put u= the required area. Then by the nature of exponentials $\mathrm{X}x^2=\mathrm{A}y$; Th. $y=\frac{\mathrm{X}x^2}{\mathrm{A}}$, and $y\dot{x}=\frac{\mathrm{X}x^2\dot{x}}{\mathrm{A}}=\dot{u}$, or adding $\frac{\dot{\mathrm{X}}x^3}{3\mathrm{A}}$ on both ſides, $\frac{\mathrm{X}x^2\dot{x}}{\mathrm{A}}+\frac{\dot{\mathrm{X}}x^3}{3\mathrm{A}}=\dot{u}+\frac{\dot{\mathrm{X}}x^3}{3\mathrm{A}}$; or (becauſe $\dot{\mathrm{X}}=\frac{\dot{x}}{x}$) $\frac{\mathrm{X}x^2\dot{x}}{\mathrm{A}}+\frac{\dot{\mathrm{X}}x^3}{3\mathrm{A}}=\dot{u}+\frac{x^2\dot{x}}{3\mathrm{A}}$; whoſe fluent is $\frac{\mathrm{X}x^3}{3\mathrm{A}}=u+\frac{x^3}{9\mathrm{A}}$. Th. by common algebra $\frac{\mathrm{X}x^3}{3\mathrm{A}}-\frac{x^3}{9\mathrm{A}}=\frac{3\mathrm{X}-\mathrm{I}}{9\mathrm{A}}\times x^3=u$=142.54.

Mr *W. Brown* has curiouſly anſwered this from *Ex.* 43. p. 51. of Mr *Emerſons* fluxions.

QUEST. VII. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

Pl. 2. Fig. 29. LET s=S$\frac{1}{2}$B; c=cof. $\frac{1}{2}$B; x, y= ſine and coſine of $\frac{1}{2}$ difference angles at B. Then $sy+cx$=S. greater $\angle$ CBZ, $sy-cx$=S. leſſer $<$ ABZ=z. Let S. AB=a, S. BC=b. Cof. AC=n, Cof. AZC=q, S. AZB=d, S. CZB=e. Then by trigonometry d: a :: z

$a :: z : \frac{az}{d} = AZ$; $b : e :: v : \frac{ve}{b} = CZ$; and by *Anderſon*'s theorem, $\frac{qaevz}{bd} + \sqrt{1-zz}\sqrt{1-vv} = n$; and thence $1 - nn = zz + vv - \frac{2aenq}{bd} \times v^2 + \frac{q^2a^2e^2}{b^2d^2}v^2z^2 - v^2z^2$; but $z^2 + v^2 = 2ssyy + 2ccxx$, and $vz = s^2y^2 - c^2x^2$; whence $1 - nn = 2s^2y^2 + 2c^2x^2 - \frac{2aenqss}{bd}yy + \frac{2ac^2nqe}{bd}$ $xx + \frac{qqa^2e^2}{b^2d^2} - 1 \times \overline{s^4y^4 - 2s^2c^2y^2x^2 + c^4x^4}$. Let $1 - nn = m^2$; $2ss - \frac{2aeqnss}{bd} = fss$; $2cc + \frac{2aenqcc}{bd} = fcc$; $\frac{q^2a^2e^2}{b^2d^2} - 1 = p$; and then $mm = fyyss + ccfxx + ps^4y^4 - 2ps^2c^2y^2x^2 + pc^4x^4$; and ſince $x^2 + y^2 = 1$, expunge y, and then $px^4 - 2ps^2x^2 = m^2$
$+fcc \quad -fs^2$
$-fss \quad -ps^4$.

QUESTION VIII. *anſwered by the Propoſer* ΦΙΛΟΣ.

LET AB$=a$; DC$=b$; EF$=c$; the weight at A$=$W; the weight at B$=w$; $b-c=d$; ſuppoſe I the place of the center of gravity of the fruſtum; and put $x=$IB. Then $a : d :: x : \frac{dx}{a}$; and $\frac{dx}{a} + c = \frac{dx+ac}{a} =$ HG; then the area of the ſection at HG $= \frac{d^2x^2 + 2adcx + a^2c^2}{aa}$; and if I B be very ſmall, $\frac{ddx^2 + 2dacx + a^2c^2x}{aa}$ $=$ ſolidity of the part HEFG, Th. $\frac{ddx^2\dot{x} + 2dacx\dot{x} + a^2c^2x\dot{x}}{aa} =$ fluxion of the moments of the fruſtum, whoſe fluent is $\frac{ddx^4}{4a^2} + \frac{2dacx^3}{3a^2} + \frac{a^2c^2x^2}{2a^2}$, and when $x = a$, the ſum of all the moments will be $= \frac{3ddaa + 8daac + 6aacc}{12}$; this divided by the ſolidity of the fruſtum $(= \frac{dda + 3bac}{3}$:) gives $\frac{3dda + 8dac + 6acc}{4dd + 12bc}$ for the diſtance of the center of gravity of the fruſtum from the point B.

Pl. 2. Fig. 30.

Let $s=$ ſolidity of the fruſtum; and $t=$ its weight. Now $t : s :: W : \frac{Ws}{t} = n$; and $t : s :: w : \frac{sw}{t} = m$. Suppoſe the Weights to

be hung on, and the fulcrum to be at the point B, then

$$\left(\frac{\frac{3ddaa+8daac+6aacc}{12}+an}{\frac{dda+3bac}{3}+m+n}=\right)\frac{3dda^2+8da^2c+6a^2c^2+12an}{4d^2a+12bac+12m+12n}$$

will be the distance of the place of the fulcrum from the point B.

If $b=c$, then $d=0$; and the frustum will become a parallelopipedon, and the center of gravity will be distant from B $\frac{a^2c^2+2an}{2ac^2+2m+2n}$ When W$=w$; or, which is the same, when $n=m$, and the beam is a parallelopiped, then $\frac{a}{2}$ will express the distance of the center of gravity from B.

When $c=0$, then $\frac{3d^2a^2+12an}{4d^2a+12n+12m}$ will be the distance of the center of gravity from the vertex of a square pyramid; and if the weights be taken away, then $\frac{3a}{4}$ will be the distance of the said center from the vertex. *Example*, If $a=20$, $b=7$, $c=5$, $d=2$, W$=12$, $w=8$, $t=12$; then $\frac{d^2a+3bac}{3}=726.66667$; $\frac{3d^2a+8dac+6ac^2}{4d^2+12bc}=11.1$; $n=726.66667$; $m=484.444$; and $\frac{3d^2a^2+8da^2c+6a^2c^2+12an}{4d^2a+12bac+12m+12n}=11.6628$.

When $d=0$; then $\frac{a^2c^2+2an}{2ac^2+2m+2n}=11.4$.

When W$=w$; then $\frac{a}{2}=10$

When $c=0$; then $\frac{3d^2a^2+12an}{4d^2a+12n+12m}=12.06$ with the weights on;

and $\frac{3a}{4}=15$ with the weights off.

QUESTION IX. *answered by the Proposer.*

Pl. 2. Fig. 31.

LET A and B represent the ports departed from, and C the place arrived in, according to *Wright*'s projection; here is given the latitude arrived in 54° M. parts $=$A, distance BG$=3059.7=m$, enlarged distance BC$=4256.5=n$. Put $y=$ arch lat. the southermost

moſt ſhip departed from, radius being 1; then will the diff. lat. BH=3240−3437.74y, which let be $a-by$; and the meridian diff. lat. BD=A min. $\frac{y+.1667y^3+.04167y^5+.0121y^7}{.000291}$, &c. which call q; then by ſimilar triangles $q : n :: a-by : m$; theref. $mq=na-nby$. In numbers $y-.4255y^3-.1065y^5-.0308y^7$, &c. $=.4775$, ſolved $y=.5585$; hence will be had the latitudes departed from = 32° and 78° north, difference of longitude CD=3840 miles, GH =2760.3, and EF =1425.2, her departure.

But ſince the ſhips ſailed from two ports under the ſame meridian to another port differing in longitude from the former, 'tis thence evident that both ſhips muſt have made the ſame departure, or meridian diſtance; whereas, if the ſaid diſtance be found by *Mercator*'s ſailing (the common way) as above, we ſhall find that one ſhip has made 1335 miles more meridian diſtance than the other, which is abſurd. The true meridian diſtance will be had by parallel ſailing =2257 miles.

QUESTION X. *anſwered by Mr* S. FARRER.

LET the binomial A+B be raiſed to the ſixteenth power, that is, to ſuch power as is denoted by the number of all the halfpence; and let the terms wherein the ninth power of A and the ſeventh power of B, and the ninth power of B and the ſeventh power of A are, be added together, and their ſum 22880 will expreſs the number of chances whereby nine heads or nine tails may exactly be thrown at any aſſigned trial, which let be denoted by a, and thoſe of the contrary by b. Then will $\frac{b^2}{a+b^2}$ repreſent the probability of the events failing twice together, and conſequently $\frac{a^2+2ab}{a+b^2}$ the probability of happening; whence the required odds will be as a^2+2ab to b^2, that is, as 2417415 to 1776889, or nearly as $1\frac{3}{10}$ to 1.

QUESTION XI. *anſwered by* CLEOPHILUS.

PUT $545\frac{13}{70}=b$, and $y=\frac{x}{z}$, and ſubſtitute in the given equation $x^3-y^4=ax^2y$, and we ſhall have $x=z^4-az^3$, and conſequently $\dot{y}\dot{x}=4z^6\dot{z}-7az^5\dot{z}+3aaz^4\dot{z}$, whoſe fluent $\frac{4z^7}{7}-\frac{7az^6}{6}+\frac{3aaz^5}{5}$ is

by

by the queſtion $=b$, from which equation x will be found nearly 4.

Note, If the Propoſer had made the area $556\frac{11}{70}$ inſtead of $545\frac{11}{70}$ (as it ſeems to have been intended ſo by his anſwer) x would be exactly 4, $x=64$, and $y=16$.

The ſame anſwered by Mr ROBERT ROBINSON.

LET $545\frac{33}{70}=A$, and here is given $x^3-y^4=ax^2y$; write vy for x, and it is $v^3-y=av^2$. Th. $y=v^3-av^2$, $x=v^4-av^3$, and the fluxion of the area $y\dot{x}=4v^6\dot{v}-7av^5\dot{v}+3a^2v^4\dot{v}$, whoſe fluent is the area $=\frac{120v^7-245av^6+126a^2v^5}{210}=A$; and when $a=3$, it is $v^5\times 120v^2-735v+1134=114549$, where v will be found $=3.99415$; hence $x=63.3477$, and $y=15.8601$.

QUESTION XII. *anſwered by Mr* S. FARRER.

Pl. 2. Fig. 32. LET AD$=t$, DF$=e$, LD$=x$, CL$=y$; then by the property of ellipſe $t^2:e^2::t^2-x^2:y^2$. Th. $y=\sqrt{e^2-\frac{e^2x^2}{t^2}}$, and $\dot{y}=\frac{-e^2x\dot{x}}{t\sqrt{t^2e^2-e^2x^2}}$; conſequently the fluxion of the external or internal convex ſuperficies, CF, will be $py\dot{z}$, or $\frac{pe\dot{x}}{t^2}\times\sqrt{t^4\mp d^2x^2}$; for the thickneſs of the caſk is not conſidered, p denoting the area of a circle whoſe ſemidiameter is unity, d^2 the difference between $t^2\mp e^2$, according as t happens to be greater or leſs than e; and the fluent thereof will be $\frac{pex}{2t^2}\sqrt{t^4+e^2-t^2\times x^2}+\frac{pet^2}{2\sqrt{t^2-e^2}}\times$ by the circular arc, whoſe radius is 1, and ſine $\frac{x\sqrt{t^2-e^2}}{t^2}$, when t is greater than e; or $\frac{pex}{2t^2}\sqrt{t^4+e^2-t^2\times x^2}+\frac{pet^2}{2\sqrt{e^2-t^2}}\times$ hyp. Log. $\frac{x\sqrt{e^2-t^2}+\sqrt{t^4+e^2-t^2\times x^2}}{t^2}$, when t is leſs than e; to each of which expreſſions add py^2, and their ſum will denote the ſuperficies of half the caſk. But, by the conditions of the queſtion, $\overline{y^2+2e^2}\times\frac{px}{3}=\frac{c}{2}$, half the ſolidity, (whence $y^2=\frac{3c-4pe^2x}{2px}$;) and $b^2:a^2::t^2-x^2:y^2$, Th. $y^2=\frac{t^2-x^2\times a^2}{b^2}=$

$3c-$

$\frac{3c-4pe^2x}{2px}$, by what has been already proved; wherefore $t^2=\frac{3cb^2+2pa^2x^3}{6pa^2x}$ and consequently $e^2=\frac{3cb^2+2pa^2x^3}{6pb^2}$. Now let the respective values of t^2, e^2 be substituted in the expression denoting the superficies of half the cask, and x thence made variable, its required value will be determined, and consequently the transverse and conjugate diameters of the generating ellipse. Q. E. F.

QUESTION XIII. *answered by Mr* ROBERT ROBINSON.

CONSTRUCTION.

ASSUME any right line, CD, for the Stationary distance; draw lines, making $\angle ACB=38°\ 40'$, $\angle ACD=118°\ 33'$, $\angle CDA=34°\ 20'$, and $\angle CDB=48°\ 27'$, which will intersect at A and B the place of the churches. Make AB = 5 miles on the sector, by which the rest of the lines may be measured, and thence the area ABDC obtained. Pl. 2. Fig. 33.

NUMERICAL SOLUTION.

Let fall the perpendiculars B*n*, C*m*, and suppose CD = unity. Then $\angle CAD : CD :: \angle ACD : AD = 1.9271$; and $R : CD :: \angle CDm : Cm = .564007$, $\angle CBD : CD :: \angle BCD : BD = 1.255013$; and $R : BD :: \angle BDn : Bn = .306094$. and $:: \angle nBD : nD = 1.217113$. Th. $An = .713032$; hence $\overline{An}^2+\overline{Bn}^2=\overline{AB}^2 = .597839$, and $\frac{1}{2}AD\times Bn+Cm=$ similar area $=.838405$. Then as $\overline{AB}^2=.597839$: sim. area :: $\overline{AB}^2$ (in miles) $=25$: area ABDC $=35.0598$ square miles.

ALGEBRAIC SOLUTION.

Let $m=$ sine $\angle BCD$, $d=$ sine $\angle ACD$, $f=$ sine, $p=$ cosine $\angle nDB$, $b=$ sine $\angle CAD$, $n=$ sine $\angle cDA$, and $a=AB=5$; also put $x=CD$. Then $b : x :: d : \frac{dx}{b}=AD$; and $r : x :: m : \frac{mx}{r} = BD$; also $1 : \frac{mx}{r} :: f : \frac{fmx}{r}=nB, =qx$; and $r : mx :: p : \frac{pmx}{r}=nD$.

Th. $An=\frac{d}{b}-\frac{pm}{r}\times x$, which let $=sx$; then by 47 *E.* 1. $q^2x^2+s^2x^2 = a^2$. Th. $x^2=\frac{a^2}{q^2+s^2}=41.817278$, and the area $=\frac{dq+dn}{2b}x^2=$ 35.0598 square miles.

The same answered by Mr WILLIAM CHAPPLE.

Pl. 2. Fig. 34. GIVEN AB = 5 miles = 400 chains *per* Gunter.

And the angles { ACB 38° 40′, ACD 118 33, CDA 34 20, CDB 48 27 } Hence the following angles are likewise known, *viz.* BCD 79° 53′, BDA 14° 7′, DBC 51° 40′, and DAC 27° 7′. Let AB = 400 chains = a.

The fines of the angles { DCA 118° 33′ = b = .8784, DAC 27 7 = c = .4551, DBC 51 40 = d = .7844, BCD 79 53 = e = .9844, BDA 14 7 = f = .2439 } Radius being put = 1, and DC = x. Then by plane trigonometry,

As $c : x :: b : \frac{bx}{c}$ = AD; and $d : x :: e : \frac{ex}{d}$ = BD. Produce DB to E, and let fall the perpend. AE; then the angle DAE is known, being the complem. of BDA, *viz.* 75° 53′, its fine .9698 which put = g. Then $1 : \frac{bx}{c} :: f : \frac{bfx}{c}$ = AE; and $1 : \frac{bx}{c} :: g : \frac{bgx}{c}$ = DE.

Th. EB = $\frac{bgx}{c} - \frac{ex}{d}$ that is, (putting $m = \frac{bg}{c} - \frac{e}{d}$) mx = EB; and *per* 47 *E.* 1. $\sqrt{\frac{b^2f^2x^2}{c^2} + m^2x^2} = a$, and by involution, &c. $x^2 = \frac{a^2c^2}{b^2f^2 + c^2m^2}$.

Th. $x = \sqrt{\frac{a^2c^2}{b^2f^2 + c^2m^2}} = 518.306$; hence the content of the figure is found = 22647.06 acres.

QUESTION XIV. *answered by Mr* WILLIAM JEPSON.

Pl. 1. Fig. 35. LET BCDE represent the plane of the billiard table, A the given point, and A 1 half the given diameter of the ball. Draw b, m parallel to B, C; m, n parallel to C, D; and n, p parallel to D, E; each at the distance A. 1; then thro' the point A, draw $s\,a$ parallel to E, B, making s, p = A p, and $a\,b$ = A b; draw $a\,d$ perpend. to $n\,m$ produc'd, and make $d\,c = a\,c$; then draw $s\,d$, and a 3, cutting the sides of the figure b, m, n, p; and A, 2, 3, 4, A, will be the path of the moving body A, because the angle of reflection is always equal to the angle of incidence, and the center of the ball, perpendicular to the point of contract; hence by drawing 2 F, 3 G and 4 H perpendicular to B, C; C, D; and D, E respectively; we shall have F, G and H, for the points where the ball will strike the several gins, thence reversing to the point A.

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QUESTION XV. *anſwered by Mr* ROBERT ROBINSON.

LET x=AE, y=DE, m=Pm=20, n=AP=30, p=3.14159; then *per* Queſt. $an=m^4$, $ax=y^4$, the cylinder's length PE=$n-y$, and its ſolidity $=py^2\times n-x=\frac{p}{a}\times any^2-y^6$ a *maximum*, whoſe fluxion $\frac{p}{a}\times 2any\dot{y}-6y^5\dot{y}=0$. Th. $an=3y^4=m^4$; hence $y=\frac{m}{3^{\frac{1}{4}}}=$ .75983568m=15.196713, the ſemidiameter of the cylinder's baſe, and its altitude PE=$\frac{2n}{3}$=20, which gives the ſolidity of the cylinder = 14510.3813. Pl. 2. Fig. 36.

Mr WILLIAM CHAPPLE's *Solution.*

THIS plain that the greateſt cylinder required will be that which is generated by the rotation of the greateſt parallelogram, that can be inſcribed in the given figure, about the axis BC. Pl. 2. Fig. 36.

Let DEFP be this parallelogram, and put Pm=20=b, AP=30 =c, DE=x, and EP=z; alſo let a= parameter =$\frac{b^4}{c}$=5333.33. Then xz= the area of the parallelogram, and its fluxion $x\dot{z}+z\dot{x}=0$, it being a *maximum*. Now $\frac{x^4}{a}$=AE, by the nature of the curve, and $c-\frac{x^4}{a}$=EP=z. Th. $\dot{z}=-\frac{4x^3\dot{x}}{a}$, and $x\dot{z}+z\dot{x}=c\dot{x}-\frac{5x^4}{a}\dot{x}=0$; Therefr. $\frac{ac}{5}=x^4$. But $ac=b^4$; Th. $\frac{b^4}{5}=x^4$. Hence $x=$ 13.3748= the ſemidiameter of the cylinder's baſe, and z=24 =its height.

This was ingeniouſly anſwered by the Propoſer Mr *John Aſh*, who makes the anſwer the ſame as Mr *Robinſon* above.

QUESTION XVI. *anſwered by the Propoſer, Mr* THOMAS DOD *of* Marchamley.

THIS queſtion depends upon this; that the acceleration of the deſcent of heavy bodies is as the ſquare of the times. The *reaſon* is, the action of gravity being continual in every ſpace of time, the falling body receives a new impulſe equal to what it had before in the ſame ſpace of time received from the firſt power; hence then the time the body will be in motion, in deſcending from the

the earth's atmoſphere, is eaſily found; for if we turn the miles into feet, it will be $47.12 \times 5280 = 248793.60$, and divided by $16 = 15549.97$, the ſquare of the ſeconds the body will be in falling, whoſe ſquare root $= 2'\ 4''\ 6'''$, the time the body is in motion.

In the ſolution to this queſtion I make the earth to be a right globe or ſphere; and according to the moſt accurate obſervations, one degree under the equator contains 69.5 miles *Engliſh*; then will the circumference of the earth be $=25020$ miles; conſequently thoſe inhabitants paſs thro' a ſpace every hour $=1042.5$ *Engliſh* miles. Then *per Fig.* EE$=7964.0563$, and CE$=3982.02815$ miles; hence we have this general theorem for all queſtions of this nature; as rad. is to theearth's ſemid. ($=$CA or CB) ſo is the coſ. of the given lat. ($=51°\ 32'$) to the ſemid. of the earth in that parallel ($=2476.36 =$ AI$\times 2 = 4952.72 =$ AB. For the circumference A*a* Bi, ſay $133 : 355 :: 4952.72 : 15562.085 =$ A*a* Bi, which divided by 360 gives 43.2801 for one degree of longitude in lat. $51°\ 32'$. Now ſay, if $1° : 43.2801$ miles $:: 15° : 649.2015$ *Engliſh* miles, that the *Tower of London* will paſs thro' in an hour by the earth's diurnal motion; then $2'\ 4''\ 6''' = 7446'''$, and one hour $= 216000'''$. Now ſay, if $216000''' : 649.2015$ miles $:: 7446 : 22.379$ *Engliſh* miles, that the body will fall from the *Tower of London* weſt.

Pl. 2. Fig. 37.

Note, *The Propoſer, Mr* Tho. Dod, *ſuppoſes the top of the earth's atmoſphere to have no diurnal rotation about the earth, and that the body has no motion, but towards the earth's center; otherwiſe the diſtance would be nothing near what he makes it above.*

QUEST. XVII. *anſwered by the Propoſer Mr* TURNER *of* York.

Pl. 2. Fig. 38.

LET AHB repreſent the orbit of the earth, CQD that of *Mercury*, HT is a tangent in the point *t*, and GHT the angle of elongation ſought.

From the two foci S and F draw F*m* and SE perpendicular to the tangent HET, and make F*v*, H*b* and H*p* perpendiculars to SE, AB and TG reſpectively. Put the ſemitranſverſe and ſemiconjugate of the leſſer ellipſis $= a$ and b, thoſe of the greater $= c$ and d; and the eccentricities OS, KS$= e$ and f; SE$= x$; and K*b*$= y$. Then becauſe by the nature of the curve ES$\times$F*m* is always $= bb$, we ſhall have $Fm = \frac{bb}{x}$, and $Sv = \frac{x^2 - b^2}{x}$, and $Fv = \sqrt{SF^2 - Sv^2} =$

$\sqrt{4ee}$

$\sqrt{4ee-\frac{\overline{x^2-b^2}^2}{x}}$; which, to facilitate the operation, we will call B; again, by the nature of the curve, we have $Hb=\frac{d}{c}\sqrt{c^2-y^2}$, and $HS=\frac{c^2-fy}{c}$; wherefore if the tangent of the given angle GSb be put $=n$, and its ſecant $=s$, we ſhall have as $1:n::\frac{d}{c}\sqrt{c^2-y^2}:\frac{nd}{c}\sqrt{c^2-y^2}=bn$, which taken from Sb $(y-f)$ leaves $Sn=y-f-\frac{nd}{c}\sqrt{c^2-y^2}$; but as $s:1::Sn:Sp=\frac{cy-cf-dn\sqrt{c^2-y^2}}{sc}$; alſo $1:n::Sb:Gb=ny-nf$; Th. $GH=\frac{d}{c}\sqrt{c^2-y^2}+ny-nf$; again $s:1::HG:Hp=\frac{d}{cs}\sqrt{c^2-y^2}+\frac{ny-nf}{s}$; and by ſimilar triangles $Sv:SF::ES:ST$; that is, in ſymbols, $\frac{x^2-b^2}{x}:2f::x:\frac{2fx^2}{x^2-b^2}$; hence $TS+Sp=Tp=\frac{2fx^2}{x^2-b^2}+\frac{cy-cf-dn\sqrt{c^2-y^2}}{sc}$. Th. becauſe $Fv:Sv::Tp:Hp$, by multiplying extremes and means already found we ſhall have $B\times\frac{d}{cs}\sqrt{c^2-y^2}+\frac{ny-nf}{s}=\frac{\overline{x^2-b^2}}{x}\times\frac{cy-cf-dn\sqrt{c^2-y^2}}{sc}+2fx$; and we likewiſe have $\frac{SE}{HS}=\frac{cx}{c^2-fy}$, which muſt be a *maximum*, or $\frac{c^2-fy}{cx}$ a *minimum*. Now let the value of y be found in the terms of the other quantities, conſidering x and B as known; then if in the reſulting equation $a+z$ be ſubſtituted for x, as alſo in the value of B, and the ſurds be all put into infinite ſeries, and the value of y, thus freed from ſurds, be ſubſtituted in $\frac{c^2-fy}{cx}$ or $\frac{c^2-fy}{a+z}$, and the reſulting expreſſion put into fluxions, we ſhall from thence have an equation by which the value of z may be determined, and by it the reſt of the values ſought.

QUESTION XVIII. *anſwered by Mr* J. Powle.

ASſume $\frac{x^{\frac{n}{1+n}}}{z}=y$; then $\frac{ax^{\frac{n+2n^2}{1+n}}}{z^n}=\frac{x^{\frac{n+2n^2}{1+n}}}{z^{1+2n}}+x^{2n+1}$, and dividing by $x^{\frac{n+2n^2}{1+n}}$, $\frac{a}{z^n}=\frac{1}{z^{1+2n}}=x^{\frac{1+2n}{1+n}}$; which gives $x^{\frac{1+2n}{n+2}}=az^{-n}-z^{-1-n}$. Th. in fluxions $\frac{1+2n}{1+n}\dot{x}x^{\frac{1}{1+n}}=-anz\dot{z}z^{-1-n}+1+2n\dot{z}z^{-2-n}$. But from the aſſumption $\frac{\dot{x}x^{\frac{n}{1+n}}}{z}=y\dot{x}=\dot{A}$, whence by ſubſtitution $\frac{1+2n}{1+n}\dot{A}z=-anz\dot{z}z^{-n-1}+1+2n.\dot{x}z^{-2-n}$, or $\frac{1+2n}{1+n}\dot{A}=-anz\dot{z}z^{-2-n}+1+2nz\dot{z}z^{-3-2n}$, whoſe fluent is $\frac{1+2n}{1+n}A=\frac{-anz^{-1-n}}{-1-n}+\frac{1+2nz^{-2-2n}}{-2-2n}$, which gives, when corrected, x and y, reſtored $\frac{any^{1+n}}{1+2n.x^n}-\frac{y^{2+2n}}{2x^{2n}}+\frac{a^2}{2+4n}=A$, which by means of the curve's equation will be reduc'd to $\frac{-ay^{1+n}}{2+4nx^n}+\frac{xy}{2}+\frac{a^2}{2+4n}=A$, and this to $\frac{xy}{2}+\frac{ax^{n+1}}{2+4ny^n}$.

N. B. If we had aſſumed $\frac{x^{\frac{1+2n}{n}}}{z}=y$, we ſhould have $\frac{yx}{2}-\frac{ay^{1+n}}{2+4x^n}$ an expreſſion ſerving to the ſame end as the former, but ſomewhat eaſier inveſtigated.

The ſame anſwered by the Propoſer, Mr S. Farrer.

BY writing vy inſtead of x in the equation of the curve, we have $av^ny^{2n}=y^{2n+1}+y^{2n+1}\times v^{2n+1}\therefore y=\frac{av^n}{1+v^{2n+1}}$, and $x=\frac{av^{n+1}}{1+v^{2n+1}}$; conſequently the fluxion of the area ſought, in terms of v and given quantities, will be $\frac{a^2\times n+1\times\dot{v}v^n-a^2n\dot{v}v^{4n+1}}{1+v^{2n+1}\Big|^3}$; the fluent whereof

may

may thus be determined. Let $1+v^{2n+1}=z$, then $v^{2n+1}=z-1$, and $\overline{2n+1}\times\dot{v}v^{2n}=\dot{z}$; therefore $\overline{2n+1}\times\dot{v}v^{4n+1}=z\dot{z}-\dot{z}$; consequently the former fluxion will, it is manifeſt, become $=\frac{a^2\dot{z}}{z^3}-\frac{a^2n\dot{z}}{\overline{2n+1}\times z^2}$, and the fluent thereof $=-\frac{a^2}{2z^2}+\frac{a^2n}{\overline{2n+1}\times z^2}+$ ſome conſtant quantity, as D, or $\frac{na^2}{\overline{2n+1}\times 1+v^{2n+1}}-\frac{a^2}{2\times\overline{1+v^{2n+1}}^2}+$ ſome conſtant quantity, as D; but, in order to determine D, let $v=0$, and $\frac{na^2}{2n+1}-\frac{a^2}{2}+D=0\therefore D=\frac{a^2}{4n+2}$; conſequently the expreſſion for the area becomes $\frac{na^2}{\overline{2n+1}\times 1+v^{2n+1}}-\frac{a^2}{2\times\overline{1+v^{2n+1}}^2}+\frac{a^2}{4n+2}$; wherein, if for $\frac{a}{1+v^{n+1}}$ be ſubſtituted $\frac{yx^n+1}{x^n}$, its value, deduced from the equation of the curve, it will become $\frac{nay^{n+1}}{\overline{2n+1}\times x^n}-\frac{y^{2n+2}}{2x^{2n}}+\frac{a^2}{4n+2}$; which alſo is $=\frac{2nax^ny^{n+1}-\overline{2n+1}\times y^{2n+2}+a^2x^{2n}}{\overline{4n+2}\times x^{2n}}$; and this by writing $2ny^{2n+2}+2nyx^{2n+1}$, inſtead of $2nax^ny^{n+1}$, will become $=\frac{a^2+2nyx}{4n+2}-\frac{y^{2n+2}}{\overline{4n+2}\times a^{2n}}$; wherefore if for the third term of this laſt expreſſion, *viz.* $-\frac{y^{2n+2}}{\overline{4n+2}\times x^{2n}}$, its equal $\frac{ay^{n+1}}{\overline{4n+2}\times x^n}-\frac{yx}{4n+2}$ be here ſubſtituted, the reſult, when contracted, will become $\frac{yx}{2}+\frac{a^2-ay^{n+1}x^{-n}}{4n+2}$; but $a^2-ay^{n+1}x^{-n}$ appears to be $=\frac{ax^{n+1}}{y^n}$, by multiplying the equation of the curve by $\frac{a}{y^nx^n}$; conſequently the former expreſſion for the area will at length be reduced to $\frac{yx}{2}+\frac{ax^{n+1}}{\overline{4n+2}\times y^n}$.

Q. E. F.

QUESTION XIX. *anſwered by Mr* R. ROBINSON.

HAving AB=5, BC=11, CD=9, DA=13, and the diagonal AC=12, project the quadrangle ABCD, divide each diago- Pl. 2. Fig. 39.

diago-

diagonal into two equal parts in the points M and H, join BM, DM, AH and CH, which lines interſect at ⊙ the center of gravity required.

TRIGONOMETRICAL SOLUTION.

In the Tr. ACD the three ſides being given find ∠ACD=74° 58′ 26″; In the triangle ABC having the ſides, find the ∠ACB=24 37 12; Th. ∠BCD=99° 35′ 38″. In the triangle BCD; BC, DC and ∠BCD are known, whence the ∠BDC will be found =45° 2′ 2″, and the ſide BD=15.3296, whoſe half is DH=7.6648. In the triangle MCD are given MC, CD, and ∠MCD, whence will be found the ∠DMC=67° 7′ 42″. In the triangle HCD there are given DH, DC and ∠HDC, whence will be found ∠DCH= 56° 32′ 41″. Th. ∠MC⊙=18° 25′ 45″; in the triangle MC⊙ having MC, and all the angles, find M⊙=1.9025, which determines the point of ſuſpenſion required.

The ſame anſwered by Mr WILLIAM CHAPPLE.

To anſwer this queſtion without algebra, I ſhall premiſe the following

LEMMA.

IF a line be drawn from any angular point in a triangle to the middle of the oppoſite ſide, the center of gravity of the triangle will be in that line, and diſtant two thirds thereof from the angular point, or $\frac{1}{3}$ thereof from the ſide of the triangle. This being premiſed, in the trapezium ABCD, we have given AB=5, BC=11, CD=9, DA=13, and AC=12. Biſect AC in M, and draw the lines BM, DM, and ſet off $\frac{2}{3}$ of each DG, Bg; then will G and g be the centers of gravity of the two triangles, and that of the whole trapezium will be ſomewhere in the line Gg. Let fall the perpendiculars BF, and DE; then by Trig. as (AC) 12 : (AB+BC) 16 :: (BC—AB) 6 : difference of the ſegments (AF and FC) 8 : hence AF=2, and FC=10, conſequently FM (AM—AF)=4. Then *per* 47 *E.* 1. BF is found =4.5825, and BM = 6.03276, conſequently g M = 2.02758. In like manner in the other triangle DE is found = 8.6922, DM=9.4341, and GM=3.1447.

Again by trigonometry as (BM) 6.08276 : radius :: (BF) 4.5825 : ſine of 48° 53′, the ∠BMF; and as (DM) 9.4341 : radius :: (DE) 8.6922 : ſine of 67° 8′, the ∠DME, and its ſupplem. to 180°, *viz.* 112° 52′, the ∠DMA; which added to BMF gives 161° 45′ for the ∠GMg; and now we have two ſides of the triangle GMg, and the angle included by them, whence Gg is found =5.11331.

The

The perpendiculars BF and DE being found as above, the areas of the triangles are alſo known, *viz.* ABC=27.495, and ADC=52.1532. Let ⊙ repreſent the center of gravity of the trapezium, then G⊙ and g⊙ are reciprocally as the areas of the triangles whoſe centers of gravity are at G and g; therefore

As the area of the trapezium 79.6482 { ABC 27.495 to G⊙=1.76514
Is to Gg, 5.11331, ſo is the area of △ { ADC 52.1532 to g⊙=3.34818
by which the point ⊙ is determined. Q. E. I.

QUEST. XX. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

LET PA be the axis of the ſolid, P the corpuſcle, and let the attractive force be as the n^{th} power of the diſtance to any point Q in the ſurface; draw PQ and QR perpendicular to the axis; then the force of a particle at A acting on P is as PA^n, and the force at Q, in direction PQ, as PQ^n; and PQ : PR :: PQ^n : $PR \times PQ^{n-1}$ the force of a particle at Q acting on P in direction PA. Now this force of the particle Q muſt be equal to that at A; for if the force at Q be greater or leſſer than that at A, the matter muſt be removed from A to Q, or from Q to A, and then the whole body will attract more than before; ſo that at laſt when the body attracts the moſt poſſible, the force at A and at all the points Q muſt be equal; whence $PR \times PQ^{n-1} = PA^n$. Let PR=$x$, QR=$y$, PA=$a$. Pl. 2. Fig. 40.

Then if $n=-2$, we ſhall have $a^2x=\overline{xx+yy}^{\frac{3}{2}}$, or $a^4x^2=\overline{xx+yy}^3$ for the nature of the curve P Q.

Note, If $n=-1$, the body PQA will be a ſphere; and if $n=0$, 1, 2, 3, &c. the body will be a long ſlender cylinder, whoſe axis is PA.

To find the force of attraction.

Let $\frac{2cfdd}{m}=A$, $c=3.1416$ then (by cor. 3. p. 277. of Mr *Emerſon*'s Doctrine of Fluxions) the force of the circle QR=$A \times 1-\frac{PR}{PQ}$ $=A \times 1-\frac{x}{\sqrt{x^2+y^2}}=A \times 1-\frac{x^{\frac{2}{3}}}{a^{\frac{2}{3}}}$, and flux. force $=A \times \dot{x}-\frac{x^{\frac{2}{3}}\dot{x}}{a^{\frac{2}{3}}}$, and the force of the ſolid $=A \times x-\frac{3x^{\frac{5}{3}}}{5a^{\frac{2}{3}}}$, and the force of the whole ſolid $=A \times \frac{2}{5}a$.

Let d= diameter of the ſphere equal to the ſolid, the flux. force of the ſphere $=A \times \dot{x}-\frac{x\dot{x}}{\sqrt{dx}}$, and the fluent $=A \times x-\frac{2x^{\frac{3}{2}}}{3\sqrt{d}}$, and the

force of the whole ſphere $=A\times\frac{1}{3}d$.

For the content of the ſolid, its fluxion $=cy^2\dot{x}=ca^{\frac{4}{3}}x^{\frac{2}{3}}\dot{x}-cx^2\dot{x}$, and the ſolid $=\frac{3}{5}ca^{\frac{4}{3}}x^{\frac{5}{3}}-\frac{cx^3}{3}$, and the whole ſolid $=\frac{4ca^3}{15}$, likewiſe the content of the ſphere $=\frac{1}{6}cd^3$. Therefore $\frac{4ca^3}{15}=\frac{cd^3}{6}$, and $d=a\times\sqrt[3]{\frac{8}{5}}$. Th. the force of the body to the force of the ſphere is as $A\times\frac{2}{3}a$ to $A\times\frac{1}{3}d$, or as $\frac{2}{3}a$ to $\frac{1}{3}d$; that is, as $\frac{2}{3}a$ to $\frac{1}{3}a\times\sqrt[3]{\frac{8}{5}}$, or as 3 to $\sqrt[3]{25}$.

New QUESTIONS *to be anſwered in our next.*

QUESTION XXI. *By* Geminus Rhodanus Hoffman, *from* Holland.

THERE is a certain number conſiſting of three places, whoſe digits are in arithmetical progreſſion; if this number be divided by the ſum of its digits the quotient will be 26, and if to the number you add 198 the digits will be inverted. Quere the number?

QUESTION XXII. *By Mr* J. LANDEN.

Pl. 2. Fig. 41. LET ABC be a curve of ſuch a nature, that (calling AC, a; AE, x; DE, y;) the value of the ordinate y is expreſſed by this fraction, $y=\frac{\sqrt{2a^3x-x^4}-\sqrt{ax^3}}{a-\sqrt{ax}}$, the numerator and denominator of which do each of them become 0, when $x=a$, that is, when the point E coincides with the point C. It is required to find what will then be the value of the ordinate BC?

QUESTION XXIII. *By Mr* THO. ATKINSON.

IN an oblique-angled triangle there is given the angle oppoſite the baſe $=112°$, the line which biſects the ſaid angle $=40$, and the ſum of its ſides $=277$; to find each ſide ſeparately?

QUESTION XXIV. *By Mr* WILLIAM HART.

IN a right-angled plain triangle ABC, right-angled at B, there is given a line that biſects the right angle, falling upon the hypotenuſe $=24.24$; and if a line be drawn from the angle A to the middle of the perpendicular BC, the angle at A adjoining to the hypotenuſe will be $=16°\ 19'$. Required the ſides and area?

QUESTION XXV. *By Mr* JOHN ROBINSON.

THERE is a ſpherodical caſk whoſe leſſer head, bung and greater head diameters are in proportion of 1, 2 and 3; alſo the two diagonals are 35 and 40 inches; required the dimenſions with the content in ale gallons?

QUESTION XXVI. *By Mr* THO. ATKINSON.

IN an oblique-angled triangle, given the angle oppoſite the baſe =94° 12′, the line which biſects the ſaid angle =78°, dividing the baſe into two ſegments in proportion as 4 to 7. Required the ſides?

QUESTION XXVII. *By* ΦΙΛΟΣ.

ON the 8th of *April*, P. M. in the Latitude of 50° 40′ N. walking with a brother *Philomath* near the ſea ſhore; we ſaw from a ſhip in the offing, the flaſh of one of her guns, before we heard the ſound, 20 beats of the artery at the wriſt, allowing (as is generally ſuppos'd) 75 to a minute: and at that inſtant, obſerving a body floating from the ſhip towards us, keeping equal pace with the waves, we ſtuck up a ſtick perpendicular to the ground, its length was 3 feet, and its ſhadow $3\frac{1}{2}$ feet; when the ſhadow was projected E. N. E, the body juſt then arrived aſhore; we found it to be the fruſtum of a cone of the leaſt reſiſtance, the greater diameter 4 feet, the leſſer 2 feet; it was full of liquor, reckon'd worth 6*s.* a gallon wine meaſure: now our ſhares are to be proportioned to each other, as are the diſtances of the end from the center of gravity: the value of the ſhare of each is required, the rate it ſwam *per* hour, and the breadth of each wave it paſſed over?

N. B. That the anſwers to this queſtion may not differ much, as they certainly will, if it is left to the choice of thoſe who ſolve it to uſe different numbers, for the velocity of ſound, for the greateſt declination, and for the length of a royal pendulum, it is thought proper to add, that from the beſt obſervations yet made,

Sound moves 1106 feet in one ſecond of time
The greateſt declination is 23° 28′ 27″.
The length of a royal pendulum 39,13 inches.

QUESTION XXVIII. *By Mr* N. OATES.

REquired a general theorem for inveſtigating the convex ſuperficies of the *ungula* $gBrq$, made by a plane gr, cutting the cone ABV, parallel to its ſide AV? Pl. 2. Fig. 42.

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QUESTION XXIX. *By Mr* HOLLIDAY.

THREE market towns, ſuppoſe *Hertford*, *St Albans*, and *Hitchin* to be 10, 12 and 14 miles diſtant from each other reſpectively, (in a triangular manner) 'tis required to know where a tradeſman ſhall fix his houſe, ſo as to ride the feweſt miles poſſible in keeping each market?

QUESTION XXX. *By Mr* W. BROWN.

GIVEN $y^2+x^4-x=2yx$, an equation expreſſing the nature of a curve, to find its area, when $y=260$, and $x=16$?

QUESTION XXXI. *By Mr* ROBERT ROBINSON.

BEING at ſea in ſummer 1744, obſerved the ſun's altitude at 10 o'clock in the forenoon $=48°\ 6'$, and his meridian altitude $=54°\ 48'$. It is thence required to determine the latitude of the place, and day of obſervation?

QUESTION XXXII. *By* SPICILEGUS, *Gent.*

TO inveſtigate a method of finding the reciprocal of any number n (that is to expreſs the fraction $\frac{1}{n}$ decimally) more expeditiouſly than by diviſion?

QUESTION XXXIII. *By* SPICILEGUS, *Gent.*

REquired that arithmetical progreſſion whoſe number of terms n, ſum of the terms s, and ſum of the cubes of the terms z; by a ſimple equation?

QUESTION XXXIV. *By Mr* WILLIAM CHAPPLE.

IF a piece of paper be bound round a cylinder, a ſweep taken thereon with a pair of compaſſes or callipeers, will, when the paper is taken off and ſtretched out, form a curvilinear figure, ſomewhat like an ellipſis (but not having the properties of one;) now there is given the tranſverſe or longeſt diameter of ſuch a figure $=16$, and the conjugate or ſhorteſt diameter $=12$, to find the content of the leaſt cylinder on the ſuperficies of which the ſaid figure may be deſcribed, and alſo the equation of the curve?

QUESTION XXXV. *By Mr* J. POWLE.

LET an arc of 30° of a circle whoſe diameter $=100$, revolve round its ſine; required the curve's ſuperficies of the generated ſolid?

QUES-

QUESTION XXXVI. *By Mr* J. POWLE.

SUPPOSE two balls to move from the point A, one along the diameter AB, one mile the firſt day, two the ſecond, three the third, four the fourth, &c. in arithmetical progreſſion; and the other (from A) along the circumference ACB, the ſquares of thoſe numbers, and they both meet at B in one and the ſame time; required the area of the ſemicircle?

QUESTION XXXVII. *By Mr* WILLIAM BROWN.

IN a northern latitude, on board a ſhip riding at anchor, in a calm ſea, exactly at ſix o'clock P. M. the ſun's altitude was taken by Mr *Hadley's Octant* upon deck, which (by allowance for refraction, &c.) was 14° 32′ 46″, and the ſun was nicely obſerved that evening to ſet 59° 19′ 55″ from the true north point of the horizon; from whence the latitude of the place where, the declination of the ſun, and the day of the month when theſe obſervations were made, may be found by a ſimple equation, and are hereby required; with a general *theorem* for all queſtions of this nature?

QUESTION XXXVIII. *By* CLEOPHILUS.

LET there be an infinite number of curves repreſented by this equation $y=\overline{\sqrt{x}+\sqrt{x+a}}^{n}$, n denoting any whole number poſitive or negative; it is required to find the area, or, which is the ſame thing, to find the fluent of $\dot{x}+\overline{\sqrt{x}+\sqrt{x+a}}^{n}$?

QUESTION XXXIX. *By* CLEOPHILUS.

LET ALC be an horizontal plane, and EA perpendicular thereto, of equal height with the ſemidiameter of the wheel LOG, to which is fix'd the rope EGOBP; it is plain, that if the wheel move along the plane from A to C, the weight P will be thereby carried in a curve MM; it is required to find the nature of the curve ſo deſcribed, having the diameter of the wheel given equal to two feet, and length of the rope equal to ten feet; as alſo to find the length of its greateſt ordinate and abſciſſa, and, if it be not too much, its quadrature and rectification? Pl. 2. Fig. 43.

QUESTION XL. *By* CLEOPHILUS.

LET $x^3+y^3=1000xy$ expreſs the nature of the curve ACDE with reſpect to the diameter AB; it is required to find the area of the greateſt iſoſceles triangle, ACE, that can be inſcribed in the ſaid curve, whoſe baſe CE ſhall cut the diameter AD at right angles? Pl. 2. Fig. 44.

The QUESTIONS *in the* Gentleman's Diary *answered.*

QUESTION I. *By Mr* R. Robinſon; *anſwered by Mr* J. Waine.

BEING on an eminence near Giſbrough *in* Yorkſhire, *I obſerved a field in form of an oblong, the proportion of its ſides was as* 2 *to* 3; *the field lyeth horizontal, its longeſt ſide was in a direct line with my ſtation, from whence if two right lines be drawn to each end of the ſaid ſide, the angle contained between them will be* 2° 22′ 30″. *Likewiſe the center of gravity of the ſaid field was* 678.4 *yards diſtant from me, and the height of my ſtation above the level of the ſaid field was* 96 *yards. Required the content.*

SOLUTION.

Pl. 2. Fig. 45. Given LO =678.4, whence by 47 *E.* 1. LI =671.5732=*b*, OI =96=*d*; put *x* and *y*= ſine and coſine of GIL, and by Trig. as 1 : *x* :: *b* : *bx*=GL. Theref. by queſtion 1.5 *bx*=DG=GC, and 1 : *y* :: *b* : *by*=GI, whence *by*+1.5*bx*=DI, and *by*−1.5*bx* =CI. Again, as *d* : 1 :: *by*+1.5*bx* : $\frac{by+1.5bx}{d}$=tangent of DOI, and by the ſame rule $\frac{by-1.5bx}{d}$= tangent of COI; but DOI−COI=DOC, which is given =2° 22′ 30″, whoſe tangent call *t*; then will $\frac{3dbx}{d^2+b^2y^2-2.25b^2x^2}=t$. Theref. $3dbx=d^2t+b^2ty^2-2.25tbbx^2$; but $1-xx=yy$, theref. $xx+\frac{3d}{3.25tb}x=\frac{d^2+b^2}{3.25b^2}$; put $2n=\frac{3d}{3.25tb}$, and by compleating the ſquare $x=\sqrt{\frac{dd+bb}{3.25bb}+nn}-n=$ .09578, ſine of $5^\circ_{\text{Ag.}}\ 29'_{\text{R.}}\ \frac{3}{4}''_{\text{P.}}$. Theref. DC=192.96. DA=128.64, and content 5 0 20.

The ſame anſwered by the Propoſer, Mr R. Robinſon.

LET F repreſent the, ſtation ADCB the field, and L the center of gravity thereof. Then here is given ∠AFB=2° 22′ 30″, ſine =*n*, EF=96=*a*, FL=678.4=*d*; then $\overline{EL}^2=d^2-a^2=$ 451010.56=*m*. Let *x*=AP=PB, *y*=EP; then *x*+*y*=EB, *y*−*x*=EA, and by *queſtion* 3 : 2 :: *x* : $\frac{2x}{3}=qx$=PL, and by 47 *E.* 1. we have $\overline{FA}^2\times\overline{FB}^2=a^2+\overline{y-x}^2\times a^2+\overline{y+x}^2$; but n^2 : 1 ::

$4a^2x^2$

$4a^2x^2 : \frac{4a^2x^2}{n^2} = \overline{FA}^2 \times \overline{FB}^2$, hence $a^2 + \overline{y-x}|^2 \times a^2 + \overline{y+x}|^2 = \frac{4a^2x^2}{n^2}$, which being reduced gives $n^2a^4 + 2a^2n^2y^2 + 2a^2n^2x^2 + n^2y^4 + n^2x^4 - 2n^2x^2y^2 - 4a^2x^2 = 0$; but $m = q^2x^2 + y^2$, theref. $y^2 = m - q^2x^2$, which substituted above, we have $x^4 + \frac{2n^2a^2 - 4a^2 - 2mn^2 - 2q^2n^2a^2 - 2mn^2q^2}{n^2 \times q^2 + 1|^2}$ $x^2 + \frac{a^2+m}{q^2+1}|^2 = 0$. This may be better expressed, since $d^2 = a^2 + m$, thus; $x^4 + \frac{2n^2 \times a^2 - m - d^2q^2 - 4a^2}{n^2 \times q^2+1|^2} x^2 + \frac{d^4}{q^2+1|^2} = 0$, solv'd $x = 96.5162$; hence AB = 193.0324, BC = 128.6883, and the area =

A. R. P.
5 0 21.19.

QUESTION II. *By Mr* Wm Butcher; *answered by Mr* R. Robinson.

BEING at a certain place, I observed the Sun to rise 10 *minutes past* 4, *and his altitude at noon* 58° 40′; *required the latitude of the place, and day of observation.*

SOLUTION.

Here is given the time of sun's rising 1^h 50′ before six = 27° 30′ = AB sine = q, sun's meridian altitude = 58° 40′ sine = m; let x and y be the sine and cosine of the latitude, u and w = sine and cosine sun's declination, then the sine of the meridian altitude is $wy + ux = m$; Theref. $ux = m - wy$. Again Rad. : tan. BO = $\frac{u}{w}$ :: co-tang. $\angle A = \frac{x}{y}$: sine AB = q. Theref. $qwy = m - wy = ux$, and $wy = \frac{m}{q+1}$, hence $ux = \frac{qm}{q+1}$, then will $wy - ux = m \times \frac{1-q}{1+q} = .3141521$ the sine of 18° 19′ 55″ = difference sun's declination, and co-latitude, hence latitude = 51° 30′, sun's declination = 20° 10′ both north or both south, consequently, if the latitude be north, the day of observation is *May* 9, or *July* 11. But if the latitude be south, the observation was made, *January* 8, or *November* 11.

Pl. 1. Fig. 46.

This was answered by Mr *J. Waine*, Mr *Wm Hart*, Mr *Rob. Williams*, *John Street*, Gent.

QUESTION III. *By Mr* T. Atkinson; *answered by Mr* R. Robinson.

IN an oblique-angled plain triangle, is given the difference of the sides which include an angle of 71° 10′, *equal to* 11, *and the line that*

that biſects the ſaid angle is equal to 24, *from whence is required a theorem that will determine the baſe and ſides of the triangle.*

SOLUTION.

Pl. 2. Fig. 47. Let CD=24=m, AC−BC=11=2n, ∠ACD=∠DCB=35° 35′, ſecant =r; let 2x=AC+BC, then BC=$x-n$, AC=$x+n$, and we have 1 : mr :: x : AC×BC=x^2-n^2. Theref. $x^2-mrx=n^2$; hence $x=\frac{mr+\sqrt{4n^2+m^2r^2}}{2}$=30.5022; hence the ſides are BC=25.0022, AC=36.0022, and AB=36.5909.

This was anſwered by Mr *J. Waine*, very ingeniouſly, alſo *Gamſton Retford*, Mr *John Hurſt*, *John Street*, Gent. and Mr *Wm Hart*.

QUESTION IV. *By Mr* SAMUEL FARRER.

SUPPOSE an annuity to continue for ever payable one pound the firſt year, eight pounds the ſecond year, twenty ſeven pounds the third year, &c. *(that is, as the cubes of the numbers* 1, 2, 3, 4, 5, &c. *due at the end of* 1, 2, 3, 4, 5, &c. *years reſpectively) at the rate of four* per cent. *it is required to find the preſent value of this annuity?*

Mr FARRER's *anſwer.*

Let a be the preſent value of one pound due at the end of one year, then will a^2 be the preſent value of the ſame due at the end of two years, a^3 the preſent value of the ſame due at the end of three years, &c. whence it appears that if 1+8+27+64+125, &c. be reſpectively multiplied into the terms of the geometric ſeries $a+a^2+a^3+a^4$, &c. expreſſing the preſent value of one pound, to be received at the end of 1, 2, 3, 4, &c. years, the ſum of the product thence ariſing, *viz.* $a+8a^2+27a^3+64a^4+125a^5$ &c. will, it is manifeſt, expreſs the preſent value of this annuity. Now by taking the difference of the coefficients, as in the margin, it appears that their laſt differences are all =0; wherefore multiplying the given ſeries by the binomial $1-a$, actually raiſed to that power whoſe index denotes the order of thoſe laſt differences, the product will be $a+4a^2+a^3$; this divided by $\overline{1-a}^4$, gives $\frac{a+4a^2+a^3}{\overline{1-a}^4}$ = 2535650 *l.* for the preſent value of the annuity required.

	1+8+27+64+125+216, &c.
1ſt dif.	7+19+37+61+91, &c.
2d dif.	12+18+24+30, &c.
3d. dif.	6+6+6, &c.
4th dif.	0+0, &c.

The End of NUMBER II.

Plate II.

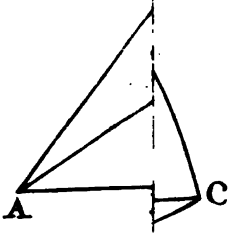


Fig. 43

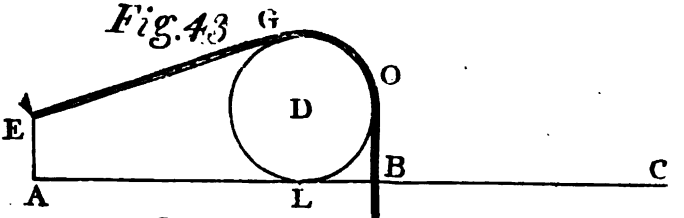
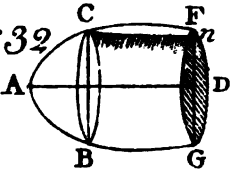


Fig. 32



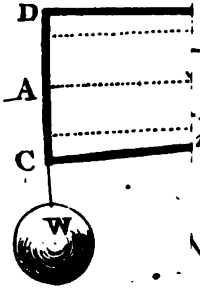


Fig. 33

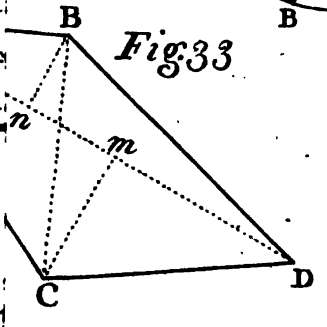
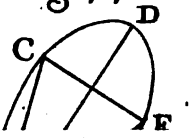
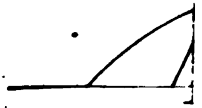
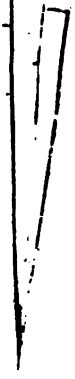
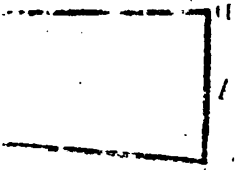


Fig. 44









NUMBER II.

Miſcellanea Curioſa, &c.

NUMBER III.

The ſolutions to the queſtions in the Gentleman's Diary *continued.*

QUESTION V. *By Mr* Paul Sharp; *anſwered by Mr* R. Robinſon.

TWO gameſters, A and B (both equally ſkill'd in game) agree to play at cards, till ſuch time as A ſhall be a winner of five ſtakes, or B of eight. Now there is a ſpectator C, that lays 50 guineas with another ſpectator D, that the play will be ended in 20 games; but the gameſters conſidering the wager, agree not to play, upon condition that C and D will part the ſaid 50 guineas in proportion to their reſpective chances; it is required to find each of their ſhares.

SOLUTION.

By *Prob.* 25. Mr *Simpſon's Chances*, $\overline{1+1}\vert^{20}=1+20+190+1140+4845+15504+38760+77520$, &c. The ſum of theſe 8 terms is 137980, which being divided by $\overline{2}\vert^{19}$ is $\frac{137980}{524288}$ = the probability of the play's ending to the advantage of A in 20 games.

Again, $20+190+1140+4845+15504+38760=60439$; to the double of this, *viz.* 120878, add $+20-38760$, and the reſult is 82138, which being divided by $\overline{2}\vert^{20}$ is $\frac{41069}{524288}$, the probability of the ſame thing happening in the favour of B, and the ſum $\frac{179049}{524288}$ is the probability that the play ends in 20 games; hence C's part of the 50 guineas is 17*l.* 18*s.* 0*d.* $\frac{767}{16384}$, and D's part 34*l.* 11*s.* 4*d.* 39$\frac{15617}{16384}$.

Gamſton Retford ſays this is a particular inſtance of Mr *Simpſon's Laws of Chance*, at page 62, and from thence may eaſily be ſolved.

QUEST. VI. *By* Vulpes Dunelmenſis; *anſwered by the ſame.*

TWO ſhips ſet ſail from a certain port, in latitude 50° 00′ *north, making an angle (between them) of* 29° 8′; *and after they had*

ſailed for ſome time (each of them on the arch of a great circle) they arrive at two different ports, both lying under the equinoctial, whoſe difference of longitude is 30° 0′. Quaere *the diſtance ſailed by each ſhip?*

SOLUTION.

Pl. 2. Fig. 48. Let A repreſent the port departed from, CD a portion of the equator, and C, B, the ports arrived at; where is known AD = 50°, ſine = q, BC = 30°, ſine = m, co-ſine = n, ∠CAB = 29° 8′, cotang. = d; let x = ſine ∠ABD, z = ſine AB. Then *per Spherics* $nz+m\sqrt{1-z^2}\times\sqrt{1-x^2}=dmx$; Th. $dmx-nx=m\sqrt{1-z^2}\times\sqrt{1-x^2}$. Again, $x:q::1:z$, Th. $x=\frac{q}{z}$, which being ſubſtituted above gives $d^2m^2+m^2\times\frac{q^2}{z^2}+z^2=2dmnq+m^2+m^2q^2=2A$; let $B=d^2m^2+m^2$, and we have $z^4-2Az^2+B=0$. Th. $z=\sqrt{A-\sqrt{AA-B}}=.830785$, the ſine of 56° 10′ 46″ = AB; or $z=\sqrt{A+\sqrt{AA-B}}=.946991$, the ſine of 71° 15′ 39″ = AC, the diſtances ſailed by each ſhip.

The ſame anſwered by Mr J. Waine.

GIVEN the latitude AD = 50°, whoſe ſine call s; let CBD repreſent a portion of the equator, where CB = 30°, whoſe tangent let be n; put t = tangent of the angle CAB = 29° 8′, and x = the tangent of BAD; then will $\frac{t+x}{1-tx}$ = tangent of CAD, *per Spherics*, $1:s::\frac{t+x}{1-tx}:\frac{ts+sx}{1-tx}$, the tangent of CD; again $1:s::x:sx$ = BD; but CB + BD = CD, theref. $\frac{n+sx}{1-nsx}=\frac{ts+sx}{1-tx}$; Th. $xx+\frac{tn-tnss}{ts-nss}x=\frac{n-ts}{ts-nss}$, by compleating the ſquare, &c. $x=\sqrt{\frac{n-ts}{ts+nss}+rr}^{\frac{1}{2}}-r=$ (where $2r=\frac{tn-tnss}{ts-nss}$) $.75409=37°\ 1'\ \frac{1}{6}$, whence CA = 71° 15′ 40″, and AB = 56° 10′ $\frac{3}{4}$.

Queſtion 7. *unlimited.*

QUEST. VIII. *By Mr* J. Robinſon; *anſwered by Mr* R. Robinſon.

A *Parabola, whoſe focus is* 22 *yards from its vertex, in which is a triangle containing* 1048 *ſquare yards, one angle of the triangle touches*

touches the vertex of the parabola, *another the extreme point of the ordinate, and the third meets the ſaid ordinate in that point which is cut by an inſcribed ſquare, next the other extremity; required the axis, area, and ordinate.*

SOLUTION.

Let parameter $=a=88$, area $\triangle AVC=A=1048$, and $y=DE$; then by the nature of the curve $VD=\frac{yy}{a}$, $VB=\frac{yy}{a}+2y$, $AB=\sqrt{yy+2ay}$, and $AC=y+\sqrt{y^2+2ay}$; then will $\frac{y^2+2ay}{a}\times y+\sqrt{y^2+2ay}=2A$, which being reduced is $y^5+4ay^4+\overline{4a^2+2A}\times y^3+4Aay^2=2A^2a$; this in numbers is $y^5+352y^4+38072y^3+368896y^2=193301504$; hence by converging ſeries $y=14.46432=DE$, whence $VB=31.3061$, $AB=52.4875$, and the area of the parabola $=2190.9048$. Pl. 2. Fig. 49.

The ſame anſwered by Mr J. Waine.

LET ALVRD repreſent the curve of the parabola, whoſe focal diſtance, *viz.* VF, is given $=22=\frac{1}{4}n$. Then will $\frac{1}{4}n=LF$; put $x=GD$, and by the nature of the parabola, as $\overline{LF}^2=\frac{1}{4}nn : VF=\frac{1}{4}n :: \overline{GD}^2=xx : \frac{xx}{n}=VD$; whence $\frac{xx+2nx}{n}=VB$; and by the reverſe of the former analogy $\frac{1}{4}n : \frac{1}{4}nn :: \frac{xx+2nx}{n} : xx+2nx=\overline{AB}^2$. Th. $\sqrt{xx+2n}+x=AC\times\frac{VB}{2}=A=1048$, whence $\sqrt{xx+2nx}=\frac{2an-x^3-2nxx}{xx+2nx}$, order'd is $x^5+4nx^4+4n^2x^3+2Ax^3+4Anx^2=2A^2n$, which in numbers is exactly the ſame as that above, and conſequently $x=14.464$, and ſo $AD=104.98$, and the area as above.

QUEST. IX. *By Mr* James Elgar, *jun. anſwered by Mr* R. Robinſon.

THERE is a triangular garden, the length of whoſe ſides are 200, 198, *and* 178 *yards; now there is a dial ſo placed in the garden, that if walks be made from each of the three angles to the dial, they will exactly divide the ſaid garden into three equal parts; from whence is required the length of each walk.*

SOLU-

SOLUTION.

Pl. 2. Fig. 50. Having given the ſides AB=200, AC=178, and BC=198, the ∠CAB will be found =62° 51′ 33″, and the area =15840; one third is =5280= area of the △ADB=BDC=CDA. Then twice area △ADB divided by AB=52.8=DG; and dividing the ſaid double area by AC, gives FD=59.3258. Draw DH parallel to AC, and ED parallel to AB; then in the △HDG, there is given DG, and all the angles; whence will be found HD=59.3333, HG=27.0666. Alſo in the △EFD there is given FD, and all the angles, whence will be found ED=66.6666, EF=39.41197; hence AF=89.7453, AG=93.7333, GB=106.2667, FC=88.2547, and by 47 *E.* 1. AD=107.5814, BD=118.6611, and CD=106.3412.

The ſame anſwered by Mr Robert Williams.

DIvide each ſide of the triangle into two equal parts, *viz.* AL=LC=89, AP=PB=100, BM=MC=99; and from M, P, L draw lines to their oppoſite angles A, B. C. Then will the three walks be repreſented by BD, AD, CD; let fall the perpendicular C*r*, put AB=200=2*a*, BC=198=*b*, AC=178=*c*, P*r*=*x*; then is *a*−*x*=*r*A, and *a*+*x*=B*r*, and $x=\frac{bb-cc}{4a}=18.8$; Th. B*r* =118.8, and *r*A=81.2; then *per* plane Trigonometry BC : R :: B*r* : ſine comp. ∠CB*r*=∠CBP. Then $\overline{BC+BP} : \overline{BC-BP}$ :: tang. $\frac{CPB+BCP}{2} : \frac{CPB-BCP}{2}=33° 20' 2''$, and the angles BPC=96° 46′ 8″, and BCP=30° 6′ 4″=BCD. Again, $\overline{BC+CL} : \overline{BC-CL}$ :: tang. $\frac{CLB+CBL}{2}$: tang. $\frac{CLB-CBL}{2}=$ 31° 17′ 8″; whence CLB=89° 16′ 48″, and CBL=26° 42′ 32″ =CBD, and 180°−∠CLB−∠CBL=CDB=123° 11′ 24″; then BDC : BC :: BCD : BD=118.6. Again BCD : BD :: CBD : DC=106.3. Then BA+BM : BA−BM :: tangent $\frac{BMA+BAM}{2}$: tang. $\frac{BMA-BAM}{2}$; whence BAM=BAD. Again BAD : BD :: ABD : AD; and the required ſides are BD=118.6, CD=106.3, and AD=107.6.

QUEST-

QUEST. X. *By Mr* John Aſh; *anſwered by Mr* R. Robinſon.

IN the triangle A B C *there is given the ſide* A B=2, B C=3, *and the included angle* =60°; *required to find the true poſition of the ſaid triangle with the right line* E D, *ſo that the ſum of the two perpendiculars* A E *and* C D *may be a* maximum. Pl. 2. Fig. 51.

SOLUTION.

Here is given A B=2=a, B C=3=b, ∠A B C=60°, m=ſine, n= coſine of its half; let x and y= ſine and coſine of half diff. angles E A B, B C D, then $ny-mx$=coſine ∠E A B, $ny+mx$= coſine ∠B C D; and by *Trig.* 1 : a :: $ny-mx$: $any-amx$=A E; and 1 : b :: $ny+mx$: $bny+bmx$=C D, whoſe ſum is $pny+dmx$, (where $p=b+a$, and $d=b-a$) and writing for y its value $\sqrt{1-x^2}$, it is $pn\sqrt{1-x^2}+dmx$ a *maximum*, which put into fluxions is $dm\dot{x}-\frac{pnx\dot{x}}{\sqrt{1-xx}}=0$. Theref. $x=\frac{dm}{\sqrt{p^2n^2+d^2m^2}}=.1147078$, the ſine of 6° 35′ 12″; hence the angle E B A=53° 24′ 48″, the required poſition of the line E D.

This was likewiſe ingeniouſly anſwered by Mr *Thomas Cowper*, Mr *J. Waine*, *Gamſton Retford*, Mr *W. Dent*, Mr *Thomas Atkinſon*, and Mr *S. Bamfield*.

The Demonſtrations of four Theorems mentioned in the Gentleman's Magazine, *Vol VII. p.* 611.

Number 3, 4, 5 *and* 6, *rendered compleat, by Mr* FACIO.

1. SINCE by means of thoſe theorems the parallax of the ſun, and even its diſtance from the earth in feet may be found, as well as the ſituation of the center of the ſolar ſyſtem; both the importance of this ſubject, and its connexion with ſome other diſcourſes which I may further publiſh, require that I render thoſe demonſtrations compleat; which I do as follows.

2. I repeat here from N° 3 and N° 4 my definition of a *ſtereographic ſphere*. If a common ſtereographic map (where a terreſtrial or celeſtial hemiſphere is projected upon a plane parallel to a meridian, the eye being in the ſurface of the ſphere; and the line drawn from the eye thro' the center of the ſphere being perpendicular to the plane of the projection) if ſuch a map, I ſay, be extended on all ſides *in infinitum*, ſo that it may contain a repreſentation of the whole ſphere; and if about each pole all the proper circles parallel to one another be projected in it by intire circles from minute to minute, or

from ſecond to ſecond: and the whole be turned about the infinite axis paſſing thro' the poles of the projection: I call any of the ſpherical ſurfaces thus formed by an intire revolution, *a ſtereographic ſphere.*

THEOREM I.

Pl. 1. Fig. 5. 3. In any common ſtereographic projection of the ſphere OAC, deſcribed upon a plane CEI tangent of the ſphere; the eye O being ſuppoſed in the ſurface, and the line OC, drawn from the eye thro' the center of the ſphere, being perpendicular to the plane of the projection: I ſay, that all the angles formed upon the ſphere (where any circles great or ſmall or their tangents interſect each other) are equal to the angles repreſenting them in the projection.

DEMONSTRATION.

4. For upon the plane that touches the ſphere in A, and whoſe ſection with the plane of the projection is perpendicular to AE and to CE tangents of the circle OAC, take the angle BAD formed by AB and AD, two tangents of the ſphere piercing the plane of the projection in the determined points B and D. And I ſay firſt, that the point A being projected in I by the right line OAI, the tangent AE will be projected by the line EI equal to AE. For let the tangent OT be drawn parallel to CEI, and the angle EAI will be meaſured by half the arc AO. Likewiſe the angle AIE or its equal AOT will be meaſured by half the arc AO: and ſo the triangle AEI will be equicrural. But the points B and D being in the very plane of the projection, they do project themſelves. And ſo the tangent AB is projected by its equal BI; and the tangent AD is projected by its equal DI. And the baſis BD remaining common, the triangle BAD is projected by its equal triangle BID. And the angle BAD is projected by its equal angle BID. Which was to be demonſtrated.

THEOREM II.

Pl. 1. Fig. 1. 5. In the right line SPTA, let SP to PT be as SA to AT. And upon the diameter PA deſcribe the *ſtereographic* ſphere ALPA, having its poles in S and T. From any point L in that ſphere draw the lines LS, LP, LT. And I ſay that SL is always to LT, in the determined proportion of SP to PT.

DEMONSTRATION.

6. For to the angle TLP make the angle PLE equal. And prolong

prolong the line L E till it cuts the prolonged line A P. And becauſe of the equality of thoſe angles, the interſection will fall upon the very point S. For thus, and thus only, will L S to LT be, as S P to P T.

7. I have called the ſphere A L PA *ſtereographic*, becauſe in the common ſtereographic projections of the ſphere of the world, ſuppoſing S and T to be the poles of the projection, and the circle A L PA any one of the circles parallel to the equator, the nature of the projection itſelf requires that S P be to P T, as S A to AT.

8. For ſuppoſing that S and T placed upon the line S P TA be the poles of a common ſtereographic projection of the ſphere TLOS; and A P to be the diameter of a circle A L P parallel to the equator; I ſay that S P is to P T, as S A to AT. Pl. 1. Fig. 3.

9. Let O repreſent the eye placed in the equator; and draw the line S *p* O *a*: and upon it draw the perpendicular lines P *p*, T O and A *a*. And if S *p* be to *p* O, as S *a* to *a* O; then, by conſequence, S P will be to P T, as S A to AT. But I ſay that S *p* is indeed to *p* O, as S *a* to *a* O.

10. For let the lines O P and O A be drawn: and, by the nature of the projection, the angles P O T and T O A will be equal. And to them alſo will the angles *p* P O and *a* A O be equal. Now taking P *p* or its equal S *p* for radius, then *p* O is tangent of a certain determined angle. And taking A *a* or its equal S *a* for radius; then is *a* O tangent of the ſame determined angle. From whence it follows that really S *p* is to *p* O, as S *a* to *a* O.

11. And hence it will be evident reciprocally, that if A and P be made the poles of another common ſtereographic projection A L P, then will the circle T L O S be a parallel to the equator thereof, and repreſent what I have called a ſtereographic ſphere. For S P will be to P T, as S A to AT.

THEOREM III. *But more particularly expreſſed.*

12. In the ſecond figure, let S O P F G C A, and L G T, and L F *y* Y be three right-lines S, L and T, the centers of the ſun, of the moon and of the earth: G the common center of gravity of the earth and the moon. Let the lines G *y* and T Y be parallel to L S. And let S and G be the poles of a ſtereographic ſphere A L P whoſe center is C. And let S and G be ſuppoſed to remain at reſt, while the center of the moon deſcribes its orbit, without departing from the ſurface of the ſtereographic ſphere A L P. And I ſay that the two united gravitations of the moon toward the earth and toward the Pl. 1. Fig. 2.

ſun

ſun will be directed toward one determined focus F placed upon the line PG. About which focus the line FL will by conſequence deſcribe equal areas in equal times.

DEMONSTRATION.

13. For SL will be to LG in the determined proportion of SP to PG, becauſe ALP is ſuppoſed to be a ſtereographic ſphere. And the proportion of LG to LT will alſo be determined in nature, as it depends upon the unchangeable maſſes of the earth and of the moon. And ſuppoſing t to be the ſemidiameter of the earth, and D the exponent of her denſity; and s to be the ſemidiameter of the ſun, and d the exponent of his denſity; the moon will be attracted toward the earth and toward the ſun, in the determined proportion of $\frac{t^3 D}{LT^2}$ and $\frac{s^3 d}{SL^2}$; as is well known to aſtronomers. Let LG be to Gy in that very proportion: and ſo the line LFy, cutting PG in F, will give the direction of the two united gravitations of the moon, toward the ſun and toward the earth. And thus the proportion of SL to its parallel Gy being determined in nature; and being the ſame with the proportion of SF to FG: by conſequence the ſituation of the focus F will be determined alſo.

14. The revolution of the center T about G will produce, for any eye placed upon the ſurface of the earth, an apparent luxation of the ſun in longitude and latitude, perceptible in the paſſages of the ſun thro' the meridian; and ſtill more indiſputably perceptible, in proper years, in the meridian altitudes of the ſun at the times of the ſolſtices. Which luxation, in many of the calculations for finding the longitude and the latitude of a ſhip at ſea, or even of an obſervator by land, ought by no means to be neglected: and particularly in the obſervations of the eclipſes of the ſatellites of *Jupiter*. This I did ſhew diſtinctly to Mr profeſſor *Hodgſon*, in a letter dated *May* 2, 1741, and directed to him; whereof I have a ſolemn copy atteſted by * * * *Pickering*, Eſq; who delivered my letter into Mr *Hodgſon*'s hand.

15. I have not Mr *Flamſteed*'s *Hiſtoria Coeleſtis*. But I believe that the great parallax of the ſun may be fully confirmed even by the meridian altitudes of the ſun, obſerved by him or by other aſtronomers at the times of the ſolſtices. For the annual variations, of thoſe meridian altitudes, will afford ſuch arguments in favour of my ſyſtem,

ſyſtem, as cannot be eluded, by aſcribing univerſally thoſe variations to the refractions of the rays of light in our atmoſphere.

THEOREM IV.

16. The fourth theorem may be conceived in theſe terms. And in general, if the gravitations of the moon toward the ſun be as $s^3 d \times SL^n$, and her gravitation toward the earth be as $t^3 D \times LT^n$, taking n for any given index whatſoever, affirmative or negative: The direction of thoſe two united gravitations will tend towards or *in oppoſitum* to one and the ſame focus F, placed ſomewhere upon the axis SG, or rather upon the line PG. But that index n has its proper limit; beyond which no focus F can be found; as it has been proved by Sir *Iſaac Newton*.

DEMONSTRATION.

17. This fourth theorem is demonſtrated in the ſame manner as the third. For t^3 to s^3, and D to d, and (LT to LS and by conſequence) LT^n to SL^n being ſo many proportions determined in nature; their compounds $t^3 D \times LT^n$ and $s^3 d \times SL^n$ will be to one another in a determined proportion. In which proportion taking LG to Gy, the direction of the line LFy will be determined; and the ſituation of the focus F will be determined alſo.

18. Theſe demonſtrations being premiſed, let us now ſee *how the excentricities* CG *and* GF *may be found, for inſtance by obſervations made when the center of the moon does revolve, without departing from the ſurface of a ſtereographic ſphere.*

I ſpeak in this diſcourſe only of the times, when the earth in ſummer is in her aphelie, or in winter is in her perihelie; and the node aſcending or deſcending of the moon's orbit is nearly in oppoſition to the ſun, ſo that there may be in about one month's time, three eclipſes of the luminaries, as nearly central as may be convenient or poſſible. This diſpoſition of the heavens does happen with a ſufficient exactneſs, a certain number of times in eighteen years. But in ſo difficult and intricate a ſubject, I ſhall at preſent imitate Sir *Iſaac Newton*, and ſuppoſe the ſun, and the common center of gravity of the earth and of the moon, to be at reſt.

19. Let the times of the oppoſition and conjunction of the luminaries, in reference to the center of the earth, be obſerved exactly. And let the times of the paſſage of the center of the moon thro' the meridian be obſerved as often as poſſible during the whole ſynodical revolution of the moon. But let theſe times be meaſured, not by

the hours of the ſun; but by a clock fitted to the motion of the ſphere of fixed ſtars. However, three or four proper obſervations may ſuffice.

Pl. 3. Fig. 52. 20. Upon the line SPCA, paſſing thro' S the center of the ſun, and thro' C the center of a ſtereographic orbit of the moon, which orbit is A*abcd*P, let G the common center of gravity of the earth and of the moon be taken by eſtimation, or as aſtronomers have determined it. Sir *Iſaac Newton* makes, at thoſe times, the excentricity C G of $66\frac{1}{4}$ parts, whereof the radius of the orbit of the moon is 1000: and that, both in ſummer and in winter: tho' that excentricity does vary; being greater in winter, and in ſummer leſs.

21. Let F, taken upon the line GP, repreſent the focus about which the line drawn from F to the center of the moon deſcribes equal areas in equal times. The excentricity GF is alſo greater in winter than in ſummer. But Sir *Iſaac* makes it always equal, to wit, of 41.964 parts, whereof the radius of the moon's orbit contains ten thouſand.

22. But whereas by ſtating exactly theſe excentricities CG and GF the ſun's parallax may be found, and likewiſe the ſituation of the center of the ſolar ſyſtem, as well as the true theory of the motion of the moon; I intend to ſhew here how both the ſaid excentricities might be exactly found at once, by obſervations made at the times aforeſaid. Dr *Halley* himſelf did maintain, that, without an exact knowledge of the motions of the moon, navigation cannot come to that perfection which it is capable of.

23. Let the lines G*A*, G*B*, G*C*, G*D*, if their inclination to GA, give, among the fixed ſtars and upon the moon's orbit, the apparent places of the center of the moon, at the times of the obſervations of its paſſage thro' the meridian. Theſe lines being prolonged to the center of the earth, will neceſſarily be in the meridian of the obſervator. Let the ſemicircle A*abcd*P be the exponent or meaſure of the time between the oppoſition and conjunction of the luminaries. And let the given points *a*, *b*, *c*, *d*, be determined accordingly, by the times or moments at which the obſervations were made. And taking a ſtraight line equal to the arc *A a*, at that diſtance from the line C*A* draw a parallel to C*A* cutting the ſemidiameter CP in F. And if the excentricity CG be well taken, the point F thus found will give the true focus about which the line drawn from F to the center of the moon deſcribes equal areas in equal times. For the area *A*C*a* will be equal to the area C*A*F.

24. In like manner, for inſtance in reference to the obſervation made

made at the moment of time expreſſed by the arc A*b*, the center of the moon having then appeared in the meridian G*B* paſſing thro' a given point among the fixed ſtars; take a ſtraight line equal to the arc *Bb*; and at that diſtance from the line C*B*, draw a line parallel to C*B*, and cutting the ſemidiameter CP in the ſame or another point F. If, both theſe points F be coincident; then was the excentricity CG well choſen; and the other excentricity GF well determined.

25. But if the two points F are not coincident; then, upon a long abſciſſe repreſenting CG, erect an ordinate G 1, repreſenting how much the firſt F is nearer to C or to P than is the ſecond F.

26. After which, upon the ſame abſciſſe, take a new line C 2, to repreſent again by it the excentricity CG. And, by means of the ſame couple of obſervations, the points *a* and *b* remaining the ſame as before, find upon the orbit of the moon two new points *A* and *B*, thro' which places of the center of the moon the meridian did paſs at the times of the obſervations. And, by the like proceſs as in the Paragraphs 23 and 24, find a ſet of two new points F. If both theſe new points F be coincident; then the line C 2 gives the true excentricity CG; and the excentricity GF becomes known alſo. But if the *two new points* F be not coincident; then upon the abſciſſe C 2 erect an ordinate 22 repreſenting how much *the firſt of them* is nearer to C or to P than is *the ſecond*. Underſtand always that as you change the place of the point G; all the lines drawn from it, according to their proper inclinations ſhewing the places of the moon, do change their places likewiſe.

27. Thus expreſſing again the excentricity CG by another abſciſſe C 3, you may find a new ſet of two points F, and their diſtance from one another expreſſed by the ordinate 33. And if you will you may take the abſciſſe C4, to try alſo by it what ordinate 44 would then expreſs the diſtance between two new points F found by the ſame method as before. And placing the ordinate 44, as well as each of the others, on its proper ſide of the abſciſſe C4; then an uniform curve 1 2 3 4 cutting the abſciſſe CG in *G* will give you CG for the excentricity of the orbit of the moon. And at the ſame time the other excentricity GF will be known alſo.

28. Whereas two proper obſervations, taken within two days before or after the oppoſition of the luminaries, might be ſufficient for the method here ſet forth; if you have a greater number of proper obſervations, you may by them many ways confirm, or even correct, your firſt concluſions. But tho' a figure may help to find theſe excen-

excentricities nearly; yet your operations being made in numbers will be much more accurate.

29. It may be perceived, by my former diſcourſes, that (if the excentricity C G be known, either in feet or in parts of the radius of the orbit of the moon) the diſtance of the center of the ſun from C the center of the moon's ſtereographic orbit, or from G the common center of gravity of the earth and of the moon, will be known alſo, either in feet or in parts of the radius of the moon's orbit, by this proportion; The variable excentricity CG, is to the radius of the orbit of the moon; As that radius, is to the diſtance betwixt the center of the ſun and the common center of gravity of the earth and the moon. And accordingly the excentricity CG will be found greater in winter than in ſummer.

30. But as the ſecond excentricity GF, expreſſed either in parts of the radius of the orbit of the moon, or in feet, does enable us to determine very nearly how much both the centers of the earth and of the moon, near the quadratures and in the ſyzygies, do fall toward the ſun in two minutes time, or toward the center O of the ſolar ſyſtem: ſo we may find, by that means, the diſtance betwixt the center of the ſolar ſyſtem, and the common center of gravity of the earth and of the moon. And at the ſame time we may allow what ſhall appear proper, upon account of the *vis centrifuga* ariſing from the circular motion of the whole ſolar ſyſtem, about the true center of that ſyſtem itſelf. For, As the verſed-ſine of the *arc* of the great orb *deſcribed* in two minutes in winter or in ſummer; is to the radius; So is the fall of G (the common center of gravity of the earth and of the moon) toward the ſun or toward the center of the ſolar ſyſtem; to the diſtance of G from the center of the ſolar ſyſtem; whether the calculation be made in feet, or in parts of the ſemidiameter of the orbit of the moon. Aſtronomers need not be told that from this does depend the true knowledge or diſcovery of the ſyſtem of the world; and of the monſtrous errors and abſurdities of the vulgar ſyſtem; to which even the ſyſtem of *Tycho Brahe* will be found preferable.

31. The greatneſs of the orbit of the moon, and the ſwiftneſs of the revolution of that orbit about the center of the ſolar ſyſtem, and the nearneſs of that center to the orbit of the moon, are attended with ſome conſiderable tho' not inſuperable difficulties; which in a great meaſure may be avoided, by making uſe of a ſatellite of *Jupiter* or of *Saturn*, in order to find both the parallax of the ſun and the ſituation of the center of the ſolar ſyſtem. For we may ſafely

ſuppoſe

ſuppoſe that the center of the ſatellite revolves at any time in a ſtereographic orbit. This curious and definitive diſquiſition is reſerved to be treated of more fully hereafter.

NIC. FACIO DUILLIER.

QUESTION II. N° I. *anſwered by Mr* R. ROBINSON.

HERE is given AG=43.5182=m, CF=34.5522=n. Let $m^2+n^2=a$, and $m^2-n^2=d$. Draw DH perpendicular, and DE parallel to AB, and put x=BC, y=AB, s=AC, and z=BD. Pl. 3. Fig. 53.

Then $s:z::x:\frac{zx}{s}$=HB; and $s:z::y:\frac{zy}{s}$=BE. Th. CE=$\frac{sx-yz}{s}$, AH=$\frac{sy-zx}{s}$; and AH : HD :: AB : BG=$\frac{zy^2}{sy-zx}$, CE : DE :: CB : BF=$\frac{zx^2}{sy-zy}=\frac{zy^2}{sy-zx}$. Theref. $z=\frac{syz}{y^2+yx+x^2}$; hence BG=BF=$\frac{xy}{x+y}$. Then *per* 47 *E.* 1. $y^2+\frac{x^2y^2}{\overline{x+y}^2}=m^2$, and $x^2+\frac{x^2y^2}{\overline{x+y}^2}=n^2$; in which for $y+x$ write $2v$, and $2e$ for $y-x$; and it will be $\overline{v+e}^2+\frac{\overline{v^2-e^2}^2}{4v^2}=m^2$, and $\overline{v-2}^2+\frac{\overline{v^2-e^2}^2}{4v^2}=n^2$; the ſum is $2v^2+2e^2+\frac{\overline{v^2-e^2}^2}{2v^2}=a$; in which writing vw for e, we have $v^2\times 5+2w^2+w^4=2a$: again, the difference is $4ve=d=4v^2w$; Th. $v^2=\frac{d}{4w}$, which ſubſtituted in the above equation gives $w^4+2w^2-\frac{8a}{d}w+5=0$; by converging ſeries w=.1428559727: hence $v=\frac{1}{2}\sqrt{\frac{d}{w}}$=34.99962346, $e=vw$=4.99990526, and the ſides AB =y=39.99952872, and BC=x=29.9997182. Q. E. I.

N. B. If w^4 be rejected, by reaſon of its ſmallneſs, we have $w=\frac{2a-\sqrt{4a^2-2.5d^2}}{d}$=.14284, vaſtly near the truth.

TABLES *of Chances for any Number of Points thrown with* 1, 2, 3, *&c. to* 12 *Dice inclusive, and the Method of Computation. By Mr* R. GIBBONS.

1 Di.		2 Di.		3 Di.		4 Dice.		5 Dice.		6 Dice.		7 Dice.		8 Dice.		9 Dice.		10 Dice.		11 Dice.		12 Dice.	
N° Points.	Chances.	N° Points.	Chances.	N° Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.	Number of Points.	Chances.
1	1	2	1	3	1	4 or 24	1	5 or 30	1	6 or 36	1	7 or 42	1	8 or 48	1	9 or 54	1	10 or 60	1	11 or 66	1	12 or 72	1
2	1	3	2	4	3	5 23	4	6 29	5	7 35	6	8 41	7	9 47	8	10 53	9	11 59	10	12 65	11	13 71	12
3	1	4	3	5	6	6 22	10	7 28	15	8 34	21	9 40	28	10 46	36	11 52	45	12 58	55	13 64	66	14 70	78
4	1	5	4	6	10	7 21	20	8 27	35	9 33	56	10 35	84	11 45	120	12 51	165	13 57	220	14 63	286	15 69	364
5	1	6	5	7	15	8 20	35	9 26	70	10 32	126	11 38	210	12 44	330	13 50	495	14 56	715	15 62	1001	16 68	1365
6	1	7	6	8	21	9 19	56	10 25	126	11 31	252	12 37	462	13 43	792	14 49	1287	15 55	2002	16 61	3003	17 67	4368
= 6		8	5	9	25	10 18	80	11 24	205	12 30	456	13 36	917	14 42	1768	15 48	2994	16 54	4995	17 60	7997	18 66	12364
		9	4	10	27	11 17	104	12 23	305	13 29	756	14 35	1667	15 41	3368	16 47	6354	17 53	11340	18 59	19327	19 65	31680
		10	3	11	27	12 16	125	13 22	420	14 28	1161	15 34	2807	16 40	6147	17 46	12465	18 52	23760	19 58	43032	20 64	74646
		11	2	12	25	13 15	140	14 21	540	15 27	1666	16 33	4417	17 39	10480	18 45	22825	19 51	46420	20 57	89232	21 63	163592
		12	1	13	21	14	146	15 20	651	16 26	2247	17 32	6538	18 38	16808	19 44	39303	20 50	85228	21 56	173745	22 62	336336
Sum = $6^2 = 36$				14	15		721	16 19	735	17 25	2856	18 31	9142	19 37	25488	20 43	63999	21 49	147940	22 55	319683	23 61	653016
				15	10		+ 575	17 18	780	18 24	3431	19 30	12117	20 36	36688	21 42	98979	22 48	243925	23 54	558613	24 60	1203632
				16	6		$6^4 = 1296$		3888	19 23	3906	20 29	15267	21 35	50288	22 41	145809	23 47	383470	24 53	930743	25 59	2115048
				17	3				+ 3888	20 22	4221	21 28	18327	22 34	65808	23 40	205560	24 46	576565	25 52	1483548	26 58	3555564
				18	1				$6^5 = 7776$	21	4332	22 27	20993	23 33	82384	24 39	277464	25 45	831204	26 51	2268332	27 57	5734664
Sum = $6^3 = 216$											25494	23 26	22967	24 32	98813	25 38	359469	26 44	1151370	27 50	3334474	28 56	8895393
											+ 21162	24 25	24017	25 31	113688	26 37	447669	27 43	1535040	28 49	4721574	29 55	13297284
											$6^6 = 46656$		139968	26 30	125588	27 36	536569	28 42	1972630	29 48	6450275	30 54	19188950
													+ 139968	27 29	133288	28 35	610056	29 41	2446300	30 47	8513105	31 53	26771316
													$6^7 = 279936$	28	135954	29 34	689715	30 40	2930455	31 46	10866995	32 52	36154767
															907785	30 33	740619	31 39	3393610	32 45	13429405	33 51	47315840
															+ 771831	31 32	767394	32 38	3801535	33 44	16079570	34 50	60060936
															$6^8 = 1679616$		5038848	33 37	4121260	34 43	18665799	35 49	74005152
																	+ 5038848	34 36	4325310	35 42	21018470	36 48	88573343
																	$6^9 = 10077696$	35	4305455	36 41	22967626	37 47	103027860
																			32430816	37 40	24362481	38 46	116523342
																			+ 28035360	38 39	25090131	39 45	128184068
																			$6^{10} = 60466176$		181398528	40 44	137194629
																					+ 181398528	41 43	142891320
																					$6^{11} = 362797056$	42	144840476
																							116081406
																							+ 101597093
																							$6^{12} = 217678233$

(90)

SIR,

SIR, *Totnes, July 21, 1745.*

IN the year 1737, I made theſe tables, and for the eaſe of computation may challenge all other methods hitherto known; the whole operation being performed by common addition, in fewer hours than can be done in days by any other method I know of as yet; I think, I may juſtly claim them as my own. The firſt table is obvious, the ſecond is made from the firſt, the third from the ſecond, *&c.* thus, *

* The chances for

With 2 dice is equal to the ſum of the chances, for the following number of points with one die.		
2	1	= 1
3	1, 2	= 2
4	1, 2, 3	= 3
5	1, 2, 3, 4	= 4
6	1, 2, 3, 4, 5	= 5
7	1, 2, 3, 4, 5, 6	= 6
8	2, 3, 4, 5, 6	= 5
9	3, 4, 5, 6	= 4
10	4, 5, 6	= 3
11	5, 6	= 2
12	6	= 1

And the chances for

With 3 dice is equal to the ſum of the chances for the following number of points with two dice,			&c. for computing the chances with 4, 5, 6, &c. dice. †
3	2	=	1
4	2, 3	=	3
5	2, 3, 4	=	6
6	2, 3, 4, 5	=	10
7	2, 3, 4, 5, 6	=	15
8	2, 3, 4, 5, 6, 7	=	21
9	3, 4, 5, 6, 7, 8	=	25
10	4, 5, 6, 7, 8, 9	=	27
11	5, 6, 7, 8, 9, 10	=	27
12	6, 7, 8, 9, 10, 11	=	25
13	7, 8, 9, 10, 11, 12	=	21
14	8, 9, 10, 11, 12	=	15
15	9, 10, 11, 12	=	10
16	10, 11, 12	=	6
17	11, 12	=	3
18	12	=	1

† In the tables great care muſt be taken, that no more nor leſs than 6 ſteps, (ſo I term them for want of a better term) muſt be added together at any one time, except in the firſt and laſt five ſteps in each table. The beſt way is to make each table throughout, as the three firſt are; for if the chances equally diſtant from the firſt and laſt are equal, or alike, the operation is then right; otherwiſe falſe. Your publiſhing theſe tables and method of computation may enable ſome gentlemen of your correſpondents to handle the nature and laws of chance, with reſpect to dice, in a plainer and eaſier manner than has hitherto appeared RICH. GIBBONS.

Solutions

Solutions to the queſtions in Number II.

QUESTION XXI. *anſwered by Mr* T. SIMPSON.

LET the digits be repreſented a, e and y; then will $100a+10e+y$ be the number ſought. Now $\frac{100a+10e+y}{a+e+y}=26$; and if to the number be added 198, the digits will be inverted, *i. e.* $100a+10e+y+198=100y+10e+a$. Th. $198=99y-99a$, or $y=a+2$; conſequently a, $a+1$, $a+2$, will be the three digits, which ſubſtituted in the equation where the number is divided by the ſum of the digits, $a=2$, $a+1=3$, $a+2=4$; hence 234 the number.

The ſame anſwered by DIFFERENTIALIS.

LET a, $\overline{a+d}$, $\overline{a+2d}$, be the digits required; and that a general ſolution may be obtained, put $b=26$, $c=198$. Then ($100a+10a+10d$, $a+2d=$) $111a+12d=$ number required; alſo $\left(\frac{111a+12d}{3a+3d}=\right)\frac{37a+4d}{a+d}=b$. Th. $\frac{37-b}{b-4}a=d$, and $111a+12d+c=$ ($100a+200d+10a+10d+a=$) $111a+210d$. Th. $12d+c=210d$, and $\frac{c}{198}=d$; alſo $\frac{37-b}{b-4}a=\frac{c}{198}$, Th. $a=\frac{\overline{b-4}\times c}{37-b\times 198}$ $(=\frac{22}{111}\times\frac{198}{198})=2$, and the number ſought is 234.

QUESTION XXII. *anſwered by Mr* J. LANDEN, *the Propoſer.*

IN *L'Hoſpital's Fluxions*, p. 191. it is found, that if the fluxion of the numerator be divided by the fluxion of the denominator, after having made $x=a$, the quotient will be the value of the ordinate ſought; the fluxion of the given numerator is $\frac{a^3\dot{x}-2x^3\dot{x}}{\sqrt{2a^3x-x^4}}$, $\frac{2a^3x^2\dot{x}}{\sqrt{ax^3}}$, of the denominator is $-\frac{a\dot{x}}{2\sqrt{ax}}$; but making $x=a$, the former fluxion is $-\frac{5a\dot{x}}{2}$, the latter $-\frac{\dot{x}}{2}$, and the required value of y is $5a$.

QUESTION XXIII. I muſt obſerve, was taken in a different ſenſe, moſt of our correſpondents taking the ſum of the ſides to ſignify the ſum of all the three ſides, others the ſum of two ſides; which laſt

Pl. 3. Fig. 54.

in

in my humble opinion is the true meaning: for when the three ſides are ſignified, it is (or at leaſt always ought to be) expreſſed the *perimeter*; however, as we have now the problem reſolved into two diſtinct ones, and to do juſtice to our correſpondents, we ſhall inſert both methods of ſolution.

Firſt, By making the ſum of the ſides, Mr *S. Bamfield* anſwers it thus.

Let AC+CB=277=a, CD=d=40, s= ſine ∠ACD=∠DCB, c = its coſine, x= ſine ∠ at D, and y its coſine. Then $cx+sy$= ∠CBD, and $cx-sy$=∠CAD. Now $cx+sy : d :: x : \frac{dx}{cx+sy}$ = CB; by the queſtion $a-\frac{dx}{cx+sy}$ = AC. Again, $cx-sy : d :: x : \frac{dx}{cx-sy}$ = AC; whence we have $a-\frac{dx}{cx+sy}=\frac{dx}{cx-sy}$, and reduc'd $2cdxx=a\times c^2x^2-s^2y^2$. For yy ſubſtitute its value, $(1-xx)$ and we have $2cdx^2=ac^2x^2-as^2+as^2x^2$. Therefore $x=.90536$ = nat. ſine of 64° 52′ 20″, whence we have CB=42.1927, and AC= 234,807, and the Baſe AB=253.6801. Pl. 3. Fig. 54.

In this ſenſe it was ingeniouſly anſwered by Mr *John Williams*, junior, *Mr John Todd*, and Mr *John Hurſt*.

The ſame anſwered by Mr W. Jepson.

Let CD=b=40, AC+BC=2c=277, ſecant of the ∠ACD= ∠DCB=s, and AC−CB=2x. Then will AC=$c+x$, and CB =$c-x$; and we ſhall by *P.* 76. *Numb.* 2. have this analogy 1 : bs :: $c : bcs=c^2-x^2$. Th. $x^2=c^2-bsc$, and $x=\overline{c^2-bsc}^{\frac{1}{2}}$=96.3074; Theref. AC=234.3074, and CB=42.1906.

In the other ſenſe, by making it the perimeter, it was anſwered by Mr Robert Robinson thus:

HERE is given the ∠ACD=∠DCB=56°, ſine =n, coſine= m, CD=40=a, 277=d. Let y and x = ſine and coſine ∠D, then ſine ∠A=$my+nx : a :: y : \frac{ay}{my+nx}$ = BC, ſine ∠A= $my-nx : a :: y : \frac{ay}{my-nx}$ =AC. Again, $my-nx : \frac{ay}{my+nx}$ :: ſine ∠ACB=$2mn : \frac{2mnay}{m^2y^2-a^2x^2}$ = AB; hence $\frac{ay}{my+nx}+\frac{ay}{my-nx}+\frac{2mnay}{}$

$\frac{2mnay}{m^2y^2-n^2x^2}=d$, and by reduction $y=\frac{nd}{d-2am}=.9887148$=fine of 81° 23′ 2″=∠BDC; hence AB=126.334, BC=58.41, and AC=92.256.

The ſame anſwered by Mr NICH. FARRER.

LET CD=40=a, 277=d, ∠ACD=DCB=56, fine=s, coſiné =q. Circumſcribe the triangle ABC with a circle, and put x=AF=FB, y=FD. Then $q:x::1:\frac{x}{q}$=AE=EB, $q:x::s:\frac{sx}{q}$=EF. Th. $\frac{s^2a^2x^2}{q^2}+y^2a^2=\overline{x^2-y^2}|^2$. Again, $x+y:\frac{x}{q}::a:\frac{ax}{qx+qy}$=BC, $x-y:\frac{x}{q}::a:\frac{ax}{qx-qy}$=AC. Th. $\frac{2ax^2}{qx^2-qy^2}+2x=d$; Theref. $y^2=x^2-\frac{2ax^2}{qd-2qx}$, which being ſubſtituted in the firſt equation, and reduc'd, $x=\frac{d^2-2daq}{4d-4aq}$, and $2x=\frac{dd-2daq}{2d-2aq}$= 126.3337, the baſe AB, whence the other ſides are eaſily found as above.

In this ſenſe it was anſwered very curiouſly by Mr *Chapple*, Mr *J. Waine*, Mr *J. Croſley*, Mr *W. Kingſton*, Mr *Paul Sharpe*, Mr *T. Simpſon*, Mr *Chagfordian Hill*, Mr *Samuel Smith*, Mr *Robert Williams*, &c.

QUESTION XXIV. *anſwered by Mr* ROB. ROBINSON.

Pl. 3. Fig. 55. LET a= tangent ∠EAC, y= tang. ∠BAE; then tang. ∠BAC $=\frac{a+y}{1-ay}=2y$. Th. $y=\frac{1\pm\sqrt{1-8a^2}}{4a}$=1.3329, and .3751219; hence ∠BAC=69°. 26′ 16″, or 36° 52′ 43″. Then by *Trig.* having FB=24.24.

Find AB=23.569977, BC=62.832683, AC=67.108047,
Or AB=40, BC=30, AC=50.

The ſame anſwered by Mr J. WAINE.

LET s denote the fine and coſine of half the right angle, n and m of CAE=16° 19′, x and y=ſines of the leſſer and greater angles in the triangle ſought reſpectively. Then we get $sy+sx$ for fine of either angle at F, and $mx-ny$=ſine of DAB; as $y:sy+sx::$ EB (=24.24=2e) : $\frac{sy+sx\times 2e}{y}$=CB. Theref. $\frac{sy+sx\times e}{y}$=CE, or EB;

but

but $n : y :: \overline{sy+sx\times e} : \overline{sy+sx\times e}$ $\frac{}{y}$ $\frac{}{n}$ $= AE$, alſo $mx-ny : 1 :: \overline{sy+sx\times e}$ $\frac{}{y}$ $: \overline{sy+sx\times e}$ $\frac{}{mxy-nyy}$ $= AE$; hence $2nyy+nx^2=mxy$, ſolv'd $\frac{x}{y}=\pm\overline{\frac{mm}{4n^2}-2}\Big|^{\frac{1}{2}}+\frac{m}{2n}=.75$, the tang. 36° 52′ 11″; whence the ſides are 30, 40 and 50 reſpectively.

The ſame anſwered by Mr N. FARRER.

HAVING given FB=24.24, find BE=EF=FG=GB=a, ∠CAE=16° 19′, ſine=s; let y=AB, and x=BC; then $y : 2x :: a : \frac{2ax}{y}=EC=2x-a$; Th. $y=\frac{2ax}{2x-a}$; and $x : s :: AC=\sqrt{y^2+4x^2} : \frac{s}{x}\sqrt{y^2+4x^2}=$ ſine ∠E; and ſine ∠E : $y :: 1 : \frac{yx}{s\sqrt{y^2+4x^2}}=AE$; hence *per* 47 *E.* 1. $\frac{y^2x^2}{s^2y^2+4s^2x^2}=y^2+x^2$; and ſubſtituting for y its value, we have $a^2\times\overline{2x-a}\Big|^2=s^2\times 4a^4+\overline{5a^2\times 2x-x}\Big|^2+\overline{2x-a}\Big|^4$; let $z=\overline{2x-a}\Big|^2$, then $z=\frac{5a^2s^2-a\pm a\sqrt{9a^2s^4+1-10as^2}}{2s^2}=165.374$; hence $2x=a+\sqrt{z}=30$. Th. y=40, and the hyp. AC=50.

This was ingeniouſly anſwered by Mr *W. Kingſton*, Mr *Paul Sharp*, &c. See Catalogue.

QUESTION XXV. *anſwered by Mr* N. FARRER.

LET d=CE=40, b=CB=35, x=FG, $2x$=AL, $3x$=CH; then $\sqrt{d^2-16x^2}$=EM, $\sqrt{b^2-25x^2}$=BK, and *per Conics*, $8x^2 : d^2-16x^2 :: 9x^2 : \frac{9d^2-144x^2}{8}=\overline{HG}^2$. Again, $5x^2 : b^2-25x^2 :: 9x^2 : \frac{9b^2-225x^2}{5}=\overline{HL}^2$; hence $\frac{9d^2-144x^2}{8}=\frac{9b^2-225x^2}{5}$. Th. $x=\frac{\sqrt{8b^2-5d^2}}{2\sqrt{30}}=3,873$; hence leſſer head =7.746, the greater head 15.492, bung =23.238, content =74.76 ale gallons. Pl. 3. Fig. 56.

The ſame anſwered by Mr R. ROBINSON.

LET BC=a=35, CE=n=40, y=FG, $2y$=AL, and $3y$=CH, then CM=$4y$, CK=$5y$; hence *per* 47 *E.* 1. $\overline{BK}^2=a^2-25y^2$, $\overline{ME}^2$

$ME^2 = n^2 - 16y^2$, and by *Conics* $4y \times 2y : n^2 - 16y^2 :: 5y \times y : a^2 - 25y^2$. Th. $y^2 = \frac{8a^2 - 5n^2}{120} = 15$, and $y = 3.872983$; hence HG = 36.87817, HL = 29.154759, leſſer head diameter = 7.745966, greater 15.491932, bung = 23.237898, and content = 74.7598 A.G.

This was curiouſly ſolv'd by Mr *T. Simpſon.*

QUESTION XXVI. *anſwered by Mr* T. SIMPSON.

Pl. 3. Fig. 57. IN the triangle ACB, put m = ſine of the $\angle ACB = 94° 12'$, n = ſine of half the above angle, CD = 780 = b, the leſſer ſegment = $4x$ and the greater = $7x$ by the queſtion. Then by *Trig.* $7x : n :: b : \frac{bn}{7x} = \angle CBA$. Again, $m : 11x :: \frac{nb}{7x} : \frac{11nb}{7m} = AC = 900.306$; whence BC = 1575.53579.

CONSTRUCTION.

Fig. 58. Deſcribe a ſegment of a circle, that ſhall contain the given $\angle ACB$ (by *E.* 3. 33.) and produce the line biſecting the angle ACB to the periphery, as to Q, and from thence draw AQ and QB. Then AQB will be an iſoſceles triangle; for the $\angle QAB = \angle QCB$, and $\angle ABQ = \angle ACQ$ (by *E.* 26. 3.) Th. $\angle QAB = \angle ABQ$.

CALCULATION.

As the leſſer ſegment is in proportion to the greater as 4 to 7; call the leſſer 4, and the greater 7, and let fall the perpendicular QW; then will WB = 5.5. Theref. in the triangle QWB the angles, and the baſe WB are given; find QB the hypotenuſe; then in the triangle QBD we have QB and DB, with the angle included, to find DQ; then it will be as DQ : QB :: CD : AC = 900.30617.

The ſame anſwered by Mr ROBERT ROBINSON.

Fig. 57. THE ratio of AC to BC is the ſame as AD to DB; Th. in a triangle aCb, where Ca = 4, Cb = 7, and contained angle = 94° 12', find $\angle a = \angle A = 57° 7' 16''$. Then

As ſine $\angle A$: CD = 780 :: ſine $\angle D$: AC = 900.3

As ſine $\angle B$: CD :: ſine $\angle D$: BC = 1575.5

As ſine $\angle A$: BC :: ſine $\angle ACB$: AB = 1871.

Or, algebraically, thus,

Let n = ſine $\angle ACD$, m = ſine $\angle ACB$, a = CD = 780, x = AD. Then AD : DB :: 1 : 1.75 :: 1 : r; then rx = DB; $x : n :: a : \frac{an}{x}$ = ſine $\angle A$, and $rx : n :: a : \frac{an}{rx}$ = ſine $\angle B$. Th.

Th. $m : \overline{r+1 \times x} :: \begin{cases} \frac{na}{rx} : AC = \frac{\overline{r+1 \times na}}{mr} = 900.30614. \\ \frac{na}{x} : BC = \frac{\overline{r+1 \times na}}{m} = 1575.53577. \end{cases}$

My method of solving this question is thus:

AS the sum of the ratios (11) is to their difference (3), so is the tangent of half the sum of the angles at the base to the tangent of half their difference; whence all the angles are known, and consequently the triangle.

Mr *J. Todd* ingeniously has solved this, also Mr *N. Farrer*, Mr *J. Powle*, Mr *Kingston*, and Mr *T. Cowper*, &c.

QUESTION XXVII. *answered by Mr* ROBERT ROBINSON. Pl. 3. Fig. 59.

LET DECA represent the conic fruſtum which will be of the leaſt reſiſtance, when BF=FG, and FC=FV, *per Sir* Iſaac Newton's *Principia, Vol. II. p.* 119. Put n=BC=2, a=GE=1, v and e= ſine and coſine $\angle$ BVC; then $2ve$= ſine $\angle$ BFC, and $v : n :: e : \frac{ne}{v}$ =BV; $2ve : n :: 1 : \frac{n}{2ve}$ =FC,; hence FB=FG= $\frac{ne}{v} - \frac{n}{2ve}$, VG=$\frac{n}{ve} - \frac{ne}{v}$. Again, $\frac{n-ne^2}{ve} : a :: \frac{ne}{v} : n$; theref. $e^2 = \frac{n}{n+a}$, and e=.8163128, the ſine of 54° 43′ 2″=$\angle$BCV; hence the fruſtum's length =1.413251 feet, and its content =77.495717 W. G. whoſe value is 23*l.* 4*s.* 11*d.* $\frac{3}{4}$.

Now let x=GI, y=IK; then $q : n-a :: x : y-a$; Th. $x = \frac{q \times y-a}{n-a}$, hence $yyx = \dot{A} = \frac{qy^2\dot{y}}{n-a}$; Th. $A = \frac{q}{3n-3a} \times y^3 - a^3$; and $\propto \dot{A} = \dot{B} = \frac{q^2}{n-a|^2} \times y^3\dot{y} - ay^2\dot{y}$. Th. $B = \frac{q^2}{n-a|^2} \times \frac{3y^4-4ay^3+a^4}{12}$; then $\frac{B}{A} = \frac{q \times 3y^4-4ay^3+a^4}{4 \times n-a \times y^3-a^3}$, and when $y=n$, it is $\frac{q \times 3n^3-an^2-a^2n-a^3}{4 \times n^3-a^3}$ $= \frac{q \times 3n^2+2na+a^2}{4 \times n^2+na+a^2} = .858045$, the diſtance of the center of gravity from the leſſer end; hence 23*l.* 4*s.* 11*d.* $\frac{3}{4}$, divided in proportion of 1 to 1.54546, gives 9*l.* 2*s.* 8*d.* and 14*l.* 2*s.* 3*d.* $\frac{3}{4}$, their required ſhares.

The time of the ſound's motion, 16 ſeconds, multiplied by 1106, gives the diſtance of the ſhip =17696 feet =3.35151 miles; as

3.5 : Rad. : : 3 : tang. ſun's altitude $=40° 36' 4''$, *April* 8th, the ſun's declination $=11° 18' 11''$ N. Hence in the lat. 50° 40′ N. the time of the day is $2^h 26' 15''$, and the time when the ſun was W.S.W. is $3^h 34' 6''$; conſequently the time the body was ſwimming is $1^h 7' 51''$, which is at the rate of 2.9637 miles *per* hour, or 3129.7273 inches $=q$, *per* minute.

Again, let v = the breadth of the waves, = the length of a pendulum which makes one vibration in the time they run thro' their breadth, $a=39.13$ inches, the length of a pendulum which makes $60=d$ vibrations *per* minute. Then it will be $v : d^2 : : a : \frac{ad^2}{v}$;

Th. $\sqrt{\frac{ad^2}{v}}$ = number of vibrations *per* minute of the pendulum whoſe length is v; hence $\sqrt{\frac{v}{ad^2}}$ = min. in one vibration, and $\sqrt{\frac{v}{ad^2}} : v : : 1 : q$. Th. $v = \frac{q^2}{ad^2} = 69.5345$ inches, $= 5.7945$ feet, the breadth of the waves.

The ſame anſwered by the Propoſer, ΦΙΛΟΣ, *Gent.*

Pl. 3. Fig. 60.

1*ſt.* As 75 : 60 : : 20 : 16 ſeconds, the time of the ſound's flight.

2*d.* As 1 : 1106 : : 16 : 17696 feet = 3.351 miles, the ſhip's diſtance.

3*d.* As 3,5 : 3 : : rad. tang. 40° 36′ 4″.

And 40° 36′ 4″ — 52″ (for refraction) leaves 40° 35′ 12″ = ſun's altitude, at the time of ſetting up the ſtick.

4*th.* The ſun's longit. on the 8th of *April* 1745 may be taken at

	29°	28′	7″
And the greateſt declination being taken at	23	28	27
Therefore the preſent declination will be	11	18	3

5*th.* From the latitude, declination, and altitude; the hour will be found $2^h 26' 22''$ P. M.

6*th.* From the latitude, declination, and azimuth; the hour will be found 3° 24′ 7″ P. M.

Therefore the time the body took to come aſhore was $1^h 7' 45''$

7*th.* The velocity of the body *per* ſecond was 4,35326 feet; or 2,9682 miles *per* hour, and the breadth of each wave was 5,8113 feet.

8*th.* In the fruſtum (A E F B) of a cone of the leaſt reſiſtance;

Given A B $=2a=4$; E F $=2b=2$; required the ſolidity.

Let C G $=2x$; then (*per* the *Newtonian* conſtruction) D C $=x+\sqrt{aa+xx}$.

Th.

Th. (D C=) $2x+\frac{bx+b\sqrt{aa+xx}}{a}=x+\sqrt{aa+xx}$.

Th. $a-b=\frac{ax+bx}{\sqrt{aa+xx}}$.

Th. $x=\frac{a-b}{2}\times\sqrt{\frac{a}{b}}=0{,}707$; and C G=1,414 feet.

And $\overline{aa+bb+ab}\times 8x\times 0{,}2618=10{,}3652$ feet, the ſolidity.

9*th*. In the fruſtum (A E F B) of a cone;

Given A B=D=4, E F=*d*=2, C G=*b*=1,414; required in C G, the place of the center of gravity.

Let H be that place, and *x*=C H. Then $b:\overline{D-d}::x:\frac{\overline{D-d}\times x}{b}$.

And $\frac{\overline{D-d}\times x}{b}+d,=\frac{\overline{\overline{D-d}\times x}+bd}{b},=$I K, $=\frac{cx+bd}{b}$(putting $c=$ D$-d$.)

Then $\frac{accxx+2achdx+abhdd}{bb}=$ area at I K (putting a=0,7854.)

And $\frac{accx^3\dot{x}+2achdx^2\dot{x}+abhddx\dot{x}}{bb}=$ fluxion of all the moments;

Whoſe fluent is $\frac{3cchha+8chhda+6bhdda}{12}$ (when $x=b$.)

Th. $\frac{3cc+8cd+6dd}{cc+3Dd}\times\frac{b}{4}=0{,}8586$, the diſtance of H from G.

10*th*. The ſolidity of the part E F K I is 3,8955 feet, and of A B K I is 6,4697.

11*th*. One ſhare was 48,3968 gallons, and worth 14*l*. 10*s*. 4*d*.$\frac{3}{4}$.
The other ſhare was 29,1404 gallons, and worth 8 14 10$\frac{1}{4}$.

QUEST. XXVIII. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

DRAW V C, *r*D, perpendicular to A B, and ſuppoſe right lines to be drawn from every point of the parabola *qrp* perpendicular to the baſe of the cone; then the parabola *qrp* will be projected into the parabola *q*D*p*; after a like manner the conic ſurface *p*B*qr* will be projected into *p*B*q*D. Let A C, or C B, =*r*, V C=*a*, V A, or V B, =*s*, B*r*, or *gr*, =*d*, *pq*=*b*. Then, by ſimilar triangles, *s*: *r*:: *d*: $\frac{dr}{s}$ =*g*D, and $\frac{2dr}{s}$ =*g*B. The circular area *q*B*p* is eaſily found, Pl. 3. Fig. 6.

found, which put $=A$, and the parabolic area qDp is $=\frac{2bdr}{3s}$, and the area $qBpD=A-\frac{2bdr}{3s}$. Then ſince any part of the convex ſurface is to the correſpondent part of the baſe it is projected into, as V B to C B; therefore it will be $r:s::A-\frac{2bdr}{3s}:\frac{sA}{r}-\frac{2bd}{3}=$ convex ſurface $qrBp$.

Mr *William Brown* has curiouſly anſwered this by a ſeries, and further obſerves that if $Ag=a$, $gB=c$, $e=gr=rB$, and $A=$ circular ſegment's area pqB, then we have this *Theorem*, for the curve ſurface of any *ungula*, $pqrB$, *viz.* $\frac{2eA}{c}-\frac{4e\sqrt{ac}}{3}$.

Mr *N. Oates* has given the ſame theorem as the laſt.

Pl. 3. Fig. 62.

QUESTION XXIX. *anſwered by the Propoſer Mr* HOLLIDAY.

LET ABC be the ſuppoſed triangle; draw OE, and OG, ſo that the angles OEC, and OGC may be each 120°; and make OH and DOF perpendicular to BC and AC.

Let $AC=a=14$, $BC=b=12$, $AB=10=c$, nat. ſine $\angle ACB=m.=.7$, coſine $=.714=n$, tangent $\angle OEG$, or $\angle OGH=60°$, $=r$, tang. $\angle DCO=y$, and $CD=x$.

Then rad. : $\angle DCO::CD:DO=xy$; and *per* 47 *E.* 1. $xx+x^2y^2=CO^2$; and $n:x::m:\frac{mx}{n}=DF$. Th. $\frac{mx}{n}-xy=OF$; and $n:x::1:\frac{x}{n}=CF$; Then $1:\frac{mx}{n}-xy::n:mx-nxy=OH$; and $1:\frac{mx}{n}-xy::m:\frac{m^2x}{n}-mxy=FH$. Th. $\frac{x}{n}-\frac{m^2x}{n}+mxy=CH$. By reaſon of ſimilar triangles CAO, COE, it will be $AC:CO::CO:CE$, that is, $a:\sqrt{x^2+x^2y^2}::\sqrt{x^2+x^2y^2}:\frac{x^2+x^2y^2}{a}=CE$. Likewiſe $b:\sqrt{x^2+x^2y^2}::\sqrt{x^2+x^2y^2}:\frac{x^2+x^2y^2}{b}=CG$. Th. $x-\frac{x^2+x^2y^2}{a}=ED$, and $\frac{x}{n}-\frac{m^2}{n}x+mxy-\frac{x^2+x^2y^2}{b}=GH$; for $CH-CG=GH$. Alſo $1:r::x-\frac{x^2+x^2y^2}{a}:xy=$ DC,

DC, and $1 : r :: \frac{x}{n} - \frac{m^2}{n}x + mxy - \frac{x^2+x^2y^2}{b} : mx - nxy = OH$.

From these two last analogies, arise the following equations;

1st. $xy = rx - \frac{rx^2+rx^2y^2}{a}$.

2d. $mx - nxy = \frac{rx}{n} - \frac{rm^2x}{n} + rmxy - \frac{rx^2+rx^2y^2}{b}$; now extermi-

nate x, and you will find the value of $y = \frac{mb - \frac{rb}{n} + \frac{brm^2}{n} + ar}{nb+brm+a}$ $= .479 =$ nat. tang. 25° 35′ the ∠ A C O. Then in the triangle A O C will be given all the angles, and one side A C; find AO= 6.98. Then in the triangle B O C will be given B C, and all the angles; find O B=4.473; and after the same manner O C will be found =9.137 miles.

CONSTRUCTION.

Make a triangle whose three sides are given; bisect A C and C B, by the perpendiculars D F and G I; make the angles D C E and G C H, each equal to 30°; from the centers E and H, with the radii E C and C H, describe two circles cutting each other in the point P, which is the place required. Pl. 3. Fig. 63.

CALCULATION.

Draw H P, E P, P C and E H. In the triangle A B C, are given all the three sides, whence the angle A C B will be known. Also in the triangles C D E, and C H G, all the angles and one side are given, whence C E, and C H will be known. Then in the triangle H C E will be given two sides, H C, C E, and an included angle E C H, whence the angle C E H, or C E O will be known. Then in the right-angled triangle C E O will be given all the angles and the side E C, whence C O=½ C P will be known.

Therefore the house is from { *Hertford* 6.98 miles. / *St Albans* 4.473 / *Hitchin* 9.137

Mr ROBERT ROBINSON's *Solution.*

LET a=AB=14, AC=12=m; find CAB=44° 24′ 55″; but *p.* 31. *Simpson*'s Fluxions the ∠AHC=∠AHB=120°, sine =q; then 120°− ∠CAB=∠ACH+∠HBA=75° 35′ 5″, Pl. 3. Fig. 64.

ſine $=d$, coſine $=w$; put $x=$AH. Then $m:q::x:\frac{qx}{m}=$ſine $\angle$ ACH, and $a:q::x:\frac{qx}{a}=$ ſine $\angle$ HBA; but the coſine of their ſum is $\frac{\sqrt{m^2-q^2x^2}\times a^2-q^2x^2-q^2x^2}{am}=w$, hence $x=$ $\frac{dma}{q\sqrt{m^2+a^2+2amw}}=9.122565=$AH.

CONSTRUCTION.

Upon the middle of any two ſides AC, AB, raiſe perpendiculars; make $\angle$ DAE $=\angle$ FAG $=30°$; upon E with the radius EA ſtrike an arc, and upon G with the extent GA cut the former arc in H, which will be the place where the houſe muſt be fixed. Then in the triangle ABC, having the ſides, find the angle CAB $=44°\ 24'\ 55''$. In the triangle ADE, having AD $=7$, and the angles, find AE $=8.082903$; AE $+$ AG : AE $-$ AG :: 13 : 1 :: tang. $\frac{1}{2}$ ſum $\angle$ AGE, AEG : tang. $\frac{1}{2}$ their difference $=3°\ 24'\ 49''$. Th. $\angle$ AEG $=34°\ 22'\ 43''$; $\angle$ BAH $=25°\ 37'\ 17''$; hence AH $=9.128168$, HB $=6.99045$, HC $=4.46404$ miles, the diſtances required.

Mr *Kingſton*, Mr *J. Waine*, Mr *W. Jepſon*, &c. anſwered this queſtion very well.

QUESTION XXX. *anſwered by the Propoſer.*

BY tranſpoſing the terms, and ſolving a quadratic equation, we ſhall get $y=x^{\frac{1}{2}}+x^2$; Th. $\dot{x}y$ (the fluxion of the variable area) is $=\dot{x}x^{\frac{1}{2}}+\dot{x}x^2$, whoſe fluent is $\frac{2}{3}x^{\frac{3}{2}}+\frac{x^3}{3}=1408$.

Note, y cannot be taken at pleaſure, when x is aſſigned.

QUESTION XXXI. *anſwered by Mr* THO. COWPER.

PUT $a=$ ſine $60°=$ coſine of the hour from noon, $d=$ ſine $48°\ 6'$ the ſun's altitude at 10, $s=$ ſine, $c=$ coſine of $17°\ 36'=$ half the difference of co-lat. and co-declination $=$ half the complement of the ſun's altitude x, and $y=$ ſine and coſine of half the ſum of the co-lat. and co-declination.

Then $\left\{\begin{matrix}cy-sx=\text{ſine}\\cx+sy=\text{coſine}\end{matrix}\right\}$ ſun's decli. and $\left\{\begin{matrix}cy+sx=\text{ſine}\\cx-sy=\text{co-ſine}\end{matrix}\right\}$ of the lat.

and, by a known theorem, we have $ac^2x^2-as^2y^2+c^2y^2-s^2x^2=d$; now putting $1-x^2$ for y^2, the equation will be $ac^2x^2-as^2-as^2x^2+c^2-$

$c^2-c^2x^2-s^2x^2=d$; but $c^2+s^2=1$, Th. $ax^2-as^2+c^2-x^2=d$; and $ax^2-x^2=d+as^2-c^2$; Th. $x=\frac{\sqrt{c^2-as^2-d}}{1-a}=.796909=$ fine 52° 50′ 10″; conſequently the lat. is 54° 45′ 50″ N. and ſun's declination 19° 33′ 50″, anſwering to *July* 14th. Q. E. I.

The ſame anſwered by Mr PAUL SHARP.

GIVEN the ſun's meridian altitude $=54°\ 48'=$ ſum of the complement of the lat. of the place and ſun's declination, the natural ſine and coſine of which put $=s$ and q, and that of half their difference x and y, rad. $=s$. Then will $sy+qx$, and $qy+sx$, expreſs the coſines of the required latitude and declination; alſo put the exceſs of the natural ſine of the ſun's meridian altitude above that of its altidude at 10 o'clock $=a$, and the verſed ſine of the hour from noon $=c$; then, by the elements of *ſpherical Trigonometry*, we have this analogy; as

$s : sqy^2+q^2xy+s^2xy+sqx^2 :: c : a$; Th. $xy+sq=\frac{a}{c}$ (becauſe x^2+y^2, s^2+q^2 each $=1$;) this multiplied by 2 and tranſpoſed gives $2yx=\frac{2a}{c}-2qs=.27012=15°\ 40'\ 16''=$ the difference betwixt the complement of the place's latitude and ſun's declination, whence the latitude is $=54°\ 45'\ 52''$ N. and ſun's declination $=19°\ 33'\ 52''$, anſwering the 7th of *May* or *July* 14th. Hence this

THEOREM.

Divide twice the exceſs of the natural ſine of the ſun's meridian altitude above that of his altitude at 10 *o'clock, by the natural verſed ſine of the hour from noon; from the quotient deduct the natural ſine of the meridian altitude, the remainder is equal the natural ſine of the difference betwixt the complement of the latitude of the place and ſun's declination.*

This was ingeniouſly anſwered by Mr *T. Simpſon*, Mr *J. Todd*, Mr *Kingſton*, Mr *S. Smith*, &c.

QUESTION XXXII. *anſwered by the Propoſer* SPICILEGUS, *Gent.*

LET $a+x=\frac{1}{n}$, then $x=\left(\frac{1-an}{n}=\right)\sqrt{1-an}\times\sqrt{a+x}$.

Let $x'=\sqrt{1-an}\times a$ $(=1-A'\times q'$, putting $an=A'$, and $a=q')$ $x''=(\sqrt{1-a+x'}\times n\times\sqrt{a+x'}=)\ \sqrt{1-A'-x'n}\times q'+x'$ $(=\sqrt{1-A''}\times q''$; where

where $A^{I}+x^{I}n=A^{II}$, and $q^{I}+x^{I}=q^{II}$) $x^{III}=\overline{(1-\overline{a+x^{I}+x^{II}}\times n\times \overline{a+x^{I}+x^{II}}=)\ 1-A^{II}-x^{II}n\times q^{II}+x^{II}}$ $(=1-A^{III}\times q^{III})$ &c. Then $(a+x^{I}+x^{II}+x^{III}$ &c.$=)\ q=\frac{1}{n}$. Q. E. F.

E X A M P L E.

Required the reciprocal of 2.30258, 50929, 94045, 68401, 79914, 54684$=n$.

1.00000, 00000, 00000, 00000, 00000, 00000 (,4$=a=q^{I}$

($an=$) $A^{I}=$	92103 40371 97618 27360 71965 81874	
$1-A^{I}$	7896 59628 02381 72639 28034 18126	789$=1-A^{I}$
		$-434=a+x^{I}$
nx^{I}	6907 75527 89821 37052 05397 43640	31 | 56
	921 03403 71976 18273 60719 65819	2 | 37
	46 05170 18598 80913 68035 98291	31
($1-A^{I}-nx^{I}=$) $1-A^{II}=$	21 75526 21985 36399 93881 10376	3424$=x^{I}$
	20 72326 58369 46411 15616 19231	
	92103 40371 97618 27360 71966	
$nx^{II}=$	9210 34037 19761 82736 07197	21755262$=1-A^{II}$
	1842 06807 43952 36547 21439	$3449\cdot2434=q^{II}+x^{II}$
	23 02585 09299 40456 84018	8702 | 1048
	20 72326 58369 46411 15616	652 | 6579
($1-A^{II}-nx^{II}=$) $1-A^{III}=$	7487 60987 44752 90990	87 | 0210
$q^{III}+x^{III}$ inverted	56728 15230 91844 92434	4 | 3510
	29950 43949 71011 63630	1 | 9580
	2246 28296 23425 87273	870
	299 50439 49790 11636	87
	14 97521 97489 50582	17
	6 73884 88870 27761	94481901$=x^{II}$
	2995 04394 79012	
	599 00878 99580	
	7 48760 98745	
	6 73884 88870	
	2246 28296	
	149 75220	
	37 43804	
	74876	
	59901	
	1498	
	524	
	45	
	3	

$3251827651128918916 3=x^{III}$.

Therefore .43429, 44819, 03251, 82765, 11289, 18916 $=\frac{1}{n}$.

Note, In each multiplier the figures to the left of the point are the sum of so much of the product as is separated from the rest by the line.

QUEST-

QUESTION XXXIII. *answered by Mr* N. FARRER.

LET x= leaſt term, y=X; then we ſhall have theſe equations, $nx+\frac{n^2-n}{2}y$, and $x^3+\frac{3n-3}{2}x^2y+\frac{2n^2-3n+1}{2}xy^2+\frac{n^3-2n^2+n}{4}y^3=\frac{z}{n}$. The firſt gives $x+\frac{n-1}{2}y=\frac{s}{a}$, whoſe cube ſubtracted from the ſecond leaves $\overline{n^2-1}\times xy^2+\frac{n^3-n^2-n+1}{2}y^3=\frac{4z}{n}-\frac{4s^3}{n^3}$; multiply $x+\frac{n-1}{2}y=\frac{s}{n}$ by $\frac{x^2-1}{4}y^2$, and it is $\overline{n-1}\times xy^2+\frac{n^3-n^2-n+1}{2}y^3=\frac{sy^2}{n}\times n^2-1=\frac{4z}{n}-\frac{4s^3}{n^3}$. Th. $y^2=\frac{4n^2z-4s^3}{sn^4-sn^2}$, and $x=\frac{s}{n}+\frac{y-ny}{2}$. Q. E. D.

The ſame anſwered by the Propoſer, SPICILEGUS, *Gent.*

Put a= firſt term, and d= common difference; then

$$\left.\begin{matrix}1\\2\end{matrix}\right\}\ per\ \text{Prob.}\begin{cases}s=na+\frac{n\cdot\overline{n-1}}{1\cdot2}d.\\ z=na^3+3aa\times\frac{n\cdot\overline{n-1}}{1\cdot2}d+3a\times\frac{n\cdot\overline{n-1}\cdot\overline{2n-1}}{1\cdot2\cdot3}dd+\frac{n^2\cdot\overline{n-1}^2}{1\cdot4}d^3.\end{cases}$$

$$s^3=n^3a^3+3a^2\times\frac{n^2.n.\overline{n-1}}{1\cdot2}d+3a\times\frac{n^3\cdot\overline{n-1}^2}{1.4}d^2+\frac{n^3\cdot\overline{n-1}^3}{1\cdot8}d^3.$$

$$n^2z=n^3a^3+3a^2\times\frac{n^3.\overline{n-1}}{1\cdot2}d+3a\times\frac{n^3.\overline{n-1}.\overline{2n-1}}{1\cdot2\cdot3}d^2+\frac{n^4.\overline{n-1}^2}{1\cdot4}d^3.$$

$$n^2z-s^3=a\times\frac{n^3.\overline{n-1}.\overline{n+1}}{2.2.}d^2+\frac{n^3.\overline{n-1}^2.\overline{n+1}}{1\cdot2\cdot4}d^3.$$

$$\frac{n^2.\overline{n-1}.\overline{n+1}}{2.2.}d^2s=a\times\frac{n^3.\overline{n-1}.\overline{n+1}}{2.2.}d^2+\frac{n^3.\overline{n-1}^2.\overline{n+1}}{1\cdot2\cdot4}d^3.$$

$$\frac{n^2.\overline{n-1}.\overline{n+1}}{4}sd^2=n^2z-s^3.$$

Th. $dd=\frac{\overline{nnz-s^3}\times4}{n^2.\overline{n-1}.\overline{n+1}.s}$

If n=10, s=185, and z=104525; then n^2z=10452500, s^3=6331625,

6331625, $n^2z-s^2=4120875$, and $\overline{n^2z-s^2}\times 4=16483500$; alſo $nn.\overline{n-1}.\overline{n+1}=9900$, and $n^2.\overline{n-1}.\overline{n+1}\times s=1831500$, laſtly $\frac{16483500}{1831500}=q=dd$. Th. $d=3$, and $a=5$, ſo the progreſſion required will be 5.8.11.14.17.20.23.26.29.32.

Philo-Whimſicolos obſerves, if $a=$ ſum of the cubes of the terms, and $z=$ Number of terms $=6$, then the following equation ſolves the problem, *viz.*

$$a=zz\times\overline{\frac{zz}{4}+\frac{z^3}{2}+\frac{z^2}{4}}=441.$$

The ſums of the cubes of 1, 2, 3, &c. a (in arithmetical proportion) are ſquare numbers, the roots of which ſums increaſe every term as many as the number of terms, *viz.*

Terms.	Cubes.	Sum of cubes.	Square root of ſums.	Increaſe of roots.
1	1	1	1	1
2	8	9	3	2
3	27	36	6	3
4	64	100	10	4
5	125	225	15	5
6	216	441	21	6
&c. a	&c. a	&c. a	&c. a	&c. a

If $z=$ number of terms, then will $z\times\overline{\frac{z}{2}+\frac{z}{2}}=a^{\frac{1}{2}}=21$, the rectangle of which is 441, as above.

Mr *John Williams*, jun. has anſwered this queſtion after the ſame manner.

QUESTION XXXIV. *anſwered by the Propoſer, Mr* CHAPPLE.

Pl. 3. Fig. 65.

IT is plain that in all curves ſo deſcribed, the ſemitranſverſe will be always equal to the arc of a circle, whoſe chord is equal to the ſemi-conjugate, and the circle itſelf equal to the baſe of the cylinder; therefore the *ſame* figure cannot be deſcribed on *another* cylinder of *different* dimenſions; for the tranſverſe and conjugate diameters, when given, will perfectly limit it; and the finding the baſe of the cylinder will amount to no more than this, viz. *from an arc of a circle and its chord line given, to find the circle*; which circle will be the cylinder's baſe ſought: and the height of the leaſt cylinder,

linder, on which the figure can be deſcribed, muſt be equal to the conjugate diameter.

By *page* 178, of the *Syntagma Matheſeos*, if y be put = the ſine of an arc, z = the length of that arc, and rad =1; $y=z-\frac{z^3}{2.3}+\frac{z^5}{2.3.4.5}$ &c. Th. if rad. be more than 1, put $=r$, it will be $\frac{y}{r}=\frac{z}{r}-\frac{z^3}{2.3.r^3}+\frac{z^5}{2.3.4.5.r^5}$ &c. (the values of y and z being here r times their values in the former caſe when rad. =1, and conſequently $\frac{y}{r}$ and $\frac{z}{r}$ muſt be written inſtead of y and z in that ſeries) and multiplying both ſides by r, we have $y=z-\frac{z^3}{2.3.r^2}+\frac{z^5}{2.3.4.5.r^4}-\frac{z^7}{2.3.4.5.6.7r^6}$ &c.

To apply this to the caſe in hand, put $a=\frac{1}{4}$ of the conjugate $=3$, and $b=\frac{1}{4}$ of the tranſverſe $=4$; then we have $a=b-\frac{b^3}{2.3r^2}+\frac{b^5}{2.3.4.5.r^4}$ &c. that is, $a=b-\frac{b^3r^{-2}}{2.3}+\frac{b^5r^{-4}}{2.3.4.5}$, &c. and by tranſpoſing b, and changing the ſigns, and putting $b-a=d$, we have $d=\frac{b^3r^{-2}}{2.3}-\frac{b^5r^{-4}}{2.3.4.5}+\frac{b^7r^{-6}}{2.3.4.5.6.7}$, &c. and putting $\frac{2.3.d}{b^3}=e$, and reverting the ſeries, $r=e^{-\frac{1}{2}}-\frac{2b^2e^{\frac{1}{2}}}{4.5}-\frac{\frac{13}{16}b^4e^{\frac{3}{2}}}{4.5.6.7}$, &c. =3.102, the radius of the circle which is the baſe of the cylinder, and its altitude being =12, its content is found =362.7568.

Now, in order to find the equation of the curve, it muſt be conſidered that every ſemi-ordinate, as NL (taking the abſciſſa in the conjugate diameter) is an arc of a circle whoſe chord line is equal to the correſponding ordinate ND in a circle, whoſe diameter is equal to the conjugate, that is in the inſcribed circle. This will plainly appear, if we conſider that perpendiculars let fall from D on the ſtretch'd paper, and L on the curve ſuperficies of the cylinder (or from any other point in the line NL) will be equal, *i. e.* DF will be =LG; and that the point D on the ſtretch'd out paper muſt correſpond with L on the curve ſuperficies, for the foot of the compaſſes, when at the point L, muſt ſtand at the diſtance CD, taken in a right

right line from the center C, and at the diſtance LG=DF, from the tranſverſe.

Put r= radius of the ſweep =CB= the ſemi-conjugate or ſemi-diameter of the inſcribed circle, and x= any abſciſs or verſed ſine in the inſcribed circle as BN; then *per* property of the circle its correſponding ſine or ſemiordinate (ND) $=\sqrt{2rx-x^2}$ = chord of an arc (NL) which is = the ſemi-ordinate in the curve. Put $\sqrt{2rx-x^2}=c$, R= rad. cylinder's baſe, and y=NL; now if y be taken for half the arc only, c may be taken for the ſine or half the chord thereof; which will make no difference here, as both muſt come into the equation, and the halves of two quantities being as their wholes, then by the foregoing rules we have $c=y-\frac{y^3}{2.3.R^2}+\frac{y^5}{2.3.4.5.R^4}-\frac{y^7}{2.3.4.5.6.7.R^6}$, and reverting the ſeries $y=c+\frac{c^3}{2.3.R^2}-\frac{11c^5}{2.3.4.5.R^4}+\frac{225c^7}{2.3.4.5.6.7.R^6}$, &c. that is, reſtoring c, $y=\overline{2rx-x^2}^{\frac{1}{2}}+\frac{\overline{2rx+x^2}^{\frac{3}{2}}}{2.3.R^2}-\frac{11\times\overline{2rx-x^2}^{\frac{5}{2}}}{2.3.4.5.R^4}+\frac{225\times\overline{2rx-x^2}^{\frac{7}{2}}}{2.3.4.5.6.7R^6}$, &c. or $y=\overline{2rx-x^2}^{\frac{1}{2}}+\frac{\overline{2rx-x^2}^{\frac{3}{2}}\times R^{-2}}{2.3}$, &c. which is the equation of the curve, putting x= any abſciſs, &c.

Alſo by Philofluentimecanalgegeomaſtrolongo.

LET CB=r=6, x= any arc CM, v= chord of it, y=PM. Then $y^2=r^2-v^2$ for the nature of the curve, which therefore is a mechanical curve. Now there is given the ratio of the arc AC and its chord CB as 8 to 6; let 1=rad.=a arc, s=ſine; find the arc and its ſine that have the ratio of 4 to 3, thus; by ſeries $a=s+\frac{s^3}{3}+\frac{s^5}{40}$, &c. therefore $3:4::s:s+\frac{s^3}{3}+\frac{s^5}{40}$, &c. whence $s^2+\frac{6}{40}s^4+\frac{6}{112}s^6$ &c. =2, and by reverſion s=.95677, and $\frac{6}{.95677}=6.2711=D$, the diameter of the cylinder, and .7854 DD × 12 = ſolidity.

QUEST. XXXV. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

Pl. 3. Fig. 66. LET CR=r=100, CF=a, FP=b, CA or FE=x, DE=y, BD=z, c=2×3.1416. Then the fluxionary ſurface =cyz; but

but $\dot{z}=\frac{r\dot{x}}{\sqrt{rr-xx}}$, and $y=\sqrt{rr-xx}-a$; Th. the fluxionary ſurface $=cr\dot{x}-ca\dot{z}$, and ſurface BD$=crx-caz$, and the whole ſurface BDP$=crb-ca\times$BP.

The ſame anſwered by Mr W. BROWN.

PUT BC=CR=CD$=r$, FE$=x$, DE$=y$, and FC=EA$=b$; then *per Circle's Prop.* $y=\sqrt{r^2-x^2}-b$, Th. $\dot{y}=\frac{-x\dot{x}}{\sqrt{r^2-x^2}}$, and $\dot{y}^2=\frac{x^2\dot{x}^2}{r^2-x^2}$; to which add $\dot{x}^2$, and we have $\dot{z}^2\ (=\dot{x}^2+\dot{y}^2)=\frac{r^2\dot{x}^2}{r^2-x^2}$ and th. $\dot{z}=\frac{r\dot{x}}{\sqrt{r^2-x^2}}$. Hence $2py\dot{z}$, the fluxion of the curve ſurface, or $\dot{C}=2p\times\sqrt{r^2-x^2}-b\times\frac{r\dot{x}}{\sqrt{r^2-x^2}}$, which is $=2pr\dot{x}-\frac{2pbr\dot{x}}{\sqrt{r^2-x^2}}$, whoſe fluent is $2prx-2pbz=$C; but when x flows =FP, the ſine of 30° $(=\frac{r}{2},)$ z is =BP. Th. the fluent then becomes pr^2-2pbz, where $b=$ coſine $30°\times r(=50,)$ and $z=$ arc of that circle whoſe rad. is $r=50$, and right ſine $x=30°$; conſequently the curve ſuperficies required is 731.2. Q. E. I.

QUESTION XXXVI. *anſwered by Mr* J. WAINE.

PUT $x=$ number of terms in the progreſſion, and let $c=$ ſemi-circumference of a circle whoſe diameter is unity; then from the theorems of *Spicilegus*, Gent. *page* 44. $\frac{xx-x}{2}=$ the diameter, and $\frac{2x^3-3x^2+x}{6}=$ circumference; whence it's evident $\frac{x^2-x}{2}\times c=\frac{2x^3-3x^2+x}{6}$. Th. $x=\mp\sqrt{\frac{9cc-6c+1}{16}}+\frac{3c+3}{4}=\frac{6c=2}{4}=2.8562$, and the ſemi-circle's area $=2.75948$.

Since we have been favoured with *Spicilegus*'s curious eſſay on the powers of arithmetical progreſſions, queſtions of this nature may now be eaſily ſolv'd, and that part of mathematics made very familiar and eaſy; as, for inſtance, this 36th queſtion depends upon the 258 queſtion in the *Ladies Diary*, which was anſwered by *Spicilegus*, Gent. in our laſt number, where it will appear that if B be put to repreſent the diameter, $\frac{1}{3}B\sqrt{8B+1}$ will be the circumference.

Th. $1 : \frac{1}{4} \times 3.14159$ &c. $:: B : \frac{1}{4}B\sqrt{8B+1}$. Th. $\frac{1}{4} \times 3.14159$ &c. $= \sqrt{8B+1}$; Th. $\frac{9}{4} \times \frac{3.14159^2}{8} - \frac{1}{8} = B$, from whence the area is eaſily obtained.

Several of our correſpondents had anſwered the above from the theorems p. 47.

QUEST. XXXVII. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

Pl. 3. Fig. 67. LET $b=s$, CB, $p=s$, AR, $x=$ſine lat. $y=$ſine twice the lat. Then 1*ſt*. In the right-angled-triangle C B D, $1 : \sqrt{1-xx} :: b : b\sqrt{1-xx} = s$, D B, or A C.

2*d*. In the right-angled-triangle C A R, $1 : b\sqrt{1-xx} :: x : p = bx\sqrt{1-xx}$; but $2x\sqrt{1-xx} = y$. Th. $by = 2p$, and $y = \frac{2p}{b}$.

The ſame anſwered by Mr W. KINGSTON.

GIVEN $a=$ ſine ſun's altitude at 6 o'clock $= 14° 32' 46''$, $n=$ coſine ſun's amplitude $= 59° 19' 55''$; let x and $y=$ ſine and coſine of the latitude, and $z=$ſine ſun's delination; then *per Shperics*, it will be $1 : z :: x : a$; Th. $a = zx$, and $z = \frac{a}{x}$. Again, $y : 1 :: z : n$; Th. $z = yn = \frac{a}{x}$, and $xy = \frac{a}{n}$ which $\times 2 = 2xy = \frac{2a}{n}$ = ſine of double the latitude $= .9848150 = 80° 00' 09''$; hence the latitude $= 40° 00' 04'' \frac{1}{2}$, and declination $= 23° 00' 01''$, which anſwers to *May 29th*, or *June 21ſt*; whence this

THEOREM.

As radius is to the ſecant of the ſun's amplitude, ſo is the ſine of twice the ſun's altitude at ſix to the ſine of double the latitude.

Mr *Robert Robinſon* gives this theorem. *Twice the ſun's altitude at ſix, drawn into the ſecant of the ſun's ampitude from north or ſouth, is the ſine of twice the latitude, or ſine of twice the co-latitude, according as the elevation of the pole is leſs or more than* 45°.

This was anſwered in the ſame manner by Mr *W. Morris*, Mr *Simpſon*, Mr *Crofsley*, Mr *Todd*, Mr *Waine*, Mr *Farrer*, &c. who all brought out the theorem as above.

The 38th queſtion being miſprinted, we could receive no ſolutions; the equation ſhould have been, $y = \sqrt{x+\sqrt{x+a}}$, &c. to find the fluent of $\dot{x} \times \sqrt{x+\sqrt{x+a}}$? I ſhall therefore firſt inſert the ingenious propoſer's anſwer, and as I was at ſome trouble in ſolving it

it myſelf, in a method quite different from his, I will for variety ſake give it.

Firſt, the Propoſer's Anſwer.

LET $v=\sqrt{x+\sqrt{x+a}}$, and we ſhall have $x=\frac{\overline{vv-a}^2}{2v}$, and $\dot{x}=\frac{1}{2}v\dot{v}-\frac{aa\dot{v}}{2v^3}$; conſequently $y\dot{x}=\frac{1}{2}v^{n+1}\dot{v}-\frac{aav^{n-3}\dot{v}}{2}$, whoſe fluent is $\frac{1}{2n+4}v^{n+2}-\frac{aa}{2n-4}v^{n-2}$, or $\frac{1}{2n+4}\times\overline{\sqrt{x+\sqrt{x+a}}}^{n+2}-\frac{aa}{2n-4}\times\overline{\sqrt{x+\sqrt{x+a}}}^{n-2}$.

N. B. This fluent is perfect, except in two Caſes, when $n=2$, and -2, and then it depends on logarithms. If the general equation had been $y=\overline{x+\sqrt{xx+aa}}^n$, its area after the ſame manner will befound to be $\frac{1}{2n+2}\times\overline{x+\sqrt{xx+aa}}^{n+1}+\frac{1}{2n-2}\times\overline{x+\sqrt{xx+aa}}^{n-1}$, which is finite, except when n is equal to 1 or -1.

By Mr F. HOLLIDAY. *Generally.*

FOR finding the fluent of $\dot{x}\times\overline{\sqrt{x+\sqrt{a+x}}}^n$, let $y^2=x$, and $v^2=a+x$; then $\dot{x}=2y\dot{y}$, and we ſhall have $2y\dot{y}\times\overline{y+v}^n$ for the fluxionary quantity; this by involving becomes $2y\dot{y}\times y^n+ny^{n-1}v+\frac{n.n-1}{2}y^{n-2}v^2$ &c. = (by reſtoring the value of v) $2y^{n+1}\dot{y}+2ny^n\dot{y}\sqrt{a+y^2}+\frac{2n.n-1}{2}y^{n-1}\dot{y}\times\overline{a+yy}+2n.\frac{n-1}{2}.\frac{n-2}{3}y^{n-3}\dot{y}\times\overline{a+yy}^{\frac{3}{2}}$, &c. Here the 1*ſt*, 3*d*, 5*th* terms will conſiſt of ſimple terms only, whoſe fluents are eaſily found. To find the fluents of the 2, 4, 6*th* terms, proceed thus:

CASE I. Let n be an odd number, then will $n-1$, $n-3$, $n-5$, &c. be even numbers; and it is evident that the fluent of $\frac{y\dot{y}}{\sqrt{a+yy}}$ is $\sqrt{a+yy}$, which call Φ; then it's plain, that Φ being given the fluents of $\frac{y^n\dot{y}}{\sqrt{a+yy}}$, $\frac{y^{n-2}\dot{y}}{\sqrt{a+yy}}$, &c. will be given by *Form* 11*th* of Mr *Emerſon*'s curious table of finding fluents; and when theſe are had, the fluents of

of $y^n\dot{y}\sqrt{a+yy}$, $y^{n-2}\dot{y}\times\overline{a+yy}^{\frac{3}{2}}$, &c. will be had by *Form* 13*th*. Therefore we ſhall have the fluent of all the terms, when n is odd.

CASE II. Let n be even; then will n, $n-2$, $n-4$, &c. be even; and it is evident that the fluent of $\frac{\dot{y}}{\sqrt{a+yy}}$ is had by *Form* 9*th*, which call Φ (which is *Ex.* 2. *page* 85. of Mr *Emerſon*'s fluxions;) then Φ being had, the fluents of $\frac{y^n\dot{y}}{\sqrt{a+yy}}$, $\frac{y^{n-2}\dot{y}}{\sqrt{a+yy}}$, &c. will be had by *Form* 11*th* as before; and thence the fluents of $y^n\dot{y}\sqrt{a+yy}$, $y^{n-2}\dot{y}\times\overline{a+yy}^{\frac{3}{2}}$, &c. by *Form* 13*th*; and when n is aſſigned in numbers, the particular fluents will be obtained as above.

QUESTION XXXIX. *anſwered by* CLEOPHILUS, *the Propoſer.*

Pl. 3. Fig. 68. LET AB$=x$, BR$=y$, and we ſhall have for the equation of the curve $\dot{y}=\frac{\dot{x}}{x-1}\sqrt{x\dot{x}-2\dot{x}}$; conſequently the fluxion of the external area will be $+\dot{y}=\dot{x}\sqrt{xx-2x}$, whoſe fluent corrected will be $\frac{1}{2}\sqrt{xx-2x}\times\overline{x-1}-1.1512\times$log. of $\overline{x-1}+\sqrt{xx-2x}$. And the fluxion of the length of the curve will be $\frac{\dot{x}}{x-1}\sqrt{2xx-4x+1}$, whoſe fluent will be found $\sqrt{2x\dot{x}-4x+1}-.01745\times$ arc whoſe radius is unity, and tangent $\sqrt{2xx-4x+1}$. The greateſt ordinate of the curve will be found 5.85, and abſciſſa 8.429. Q. E. I.

By Mr F. HOLLIDAY.

THE nature of the curve, I think, may thus be inveſtigated: Draw ED parallel to AB, EG a tangent to the circle GO, DG perpendicular to EG, and GI, DF, perpendicular to ED; and let radius DG$=r$, ſemicircle $=c$, length of the rope $=a$; then PM$=a-c$, the greateſt ordinate; let EO$=x$, OR$=y$, then EG $=\sqrt{\overline{ED}^2-\overline{DG}^2}=\sqrt{xx-2rx}$, and DI$=\frac{\overline{DG}^2}{DE}=\frac{rr}{x-r}$; let GF$=z=$ arc belonging to the ſine $\frac{rr}{x-r}$, and then EG$+$GF$+$FO$+$OB$+$BR$=a$; that is, $\sqrt{xx-2rx}+z+\frac{1}{2}c+y=a$, and $y=a-z-\frac{1}{2}c-\sqrt{xx-2rx}$, the equation of the curve, which is a mechanical curve, and when $y=0$, the greateſt abſciſſa $x=r+\sqrt{\overline{a-z-\frac{1}{2}c}^2+rr}$.

QUESTI-

QUEST. XL. *answered by* Philofluentimecanalgegeomaſtrolongo.

THIS curve touches the two lines R A, A B, perpendicular to each other in A; and the axis A D biſects the right angle A, and the two parts of the curve ACD and AED are ſimilar and equal. Pl. 3. Fig. 69.

Let A P=x, P C=y, b=1000; then $x^3+y^3=bxy$; let AS=z, S C=v; then $x\sqrt{2}+v=z$, and $v\sqrt{2}+x=y$; put $d=\frac{b}{\sqrt{2}}$, and expunging x and y, $z^3+3zv^2+dv^2-dz^2=0$; hence $vv=\frac{dz^2-z^3}{3z+d}$, and $\frac{dz^4-z^5}{3z+d}$ = a *maximum*; this in fluxions, and reduced, gives $z=\frac{d+d\sqrt{13}}{6}$, and the area of the triangle $=vz$. This curve, after its interſection at A, extends itſelf towards H and K both ways *ad infinitum*, and ſeems to be the 68th ſpecies of Sir *I. Newton*'s curves.

The ſame anſwered by Mr J. POWLE *of* Hereford.

DRAW A D, and at any point therein (S) let C I be drawn at right angles, meeting A B at I, and cutting the curve in the points C and E, let fall the perpendicular C P, and join A C.

Now, in order to find the equation within the curve, let x=A S, y=C S, z=A P, v=C P, a=1000; then, *per* queſtion, $z^3+v^3=azv$, or, aſſuming $uz=v$, $z^3+u^3z^3=auz^2$, which gives $z=\frac{au}{1+u^3}$; Th. $uz=v=\frac{au^2}{1+u^3}$, conſequently $z^2+v^2=\overline{AC}^2=\frac{a^2u^2+a^2u^4}{\overline{1+u^3}^2}$, a *maximum*; Th. in fluxions $\overline{2a^2uu+4a^2u^3\dot{u}}\times\overline{1+u^3}^2-6u^2\dot{u}\times\overline{1+u^3}\times\overline{a^2u^2+a^2u^4}=0$. By reduction $u^5+2u^3-2u^2=1$, where the only affirmative and poſſible value of u=1; whence it is manifeſt $z=v$, and A G=G D, and ſo the triangle A D G is an iſoſceles. This being diſcovered, it evidently follows, that the triangles CPI, ASI, are alſo iſoſceles, becauſe of their ſimiliarity with A D G; hence AS=S**I**, CP=PI; Th. $x\sqrt{2}=z+v$, $v\sqrt{2}=x+y$; by help of theſe and the given equation, by the rules of common algebra, we get $\sqrt{2}.x^3+3\sqrt{2}.xy^2=ax^2-ay^2$, for the equation within the curve.

Such being found to be the property of the curve, the area of the greateſt iſoſceles triangle is very eaſily inveſtigated thus: The equation $\sqrt{2}.x^3+3\sqrt{2}.xy^2=ax^2-ay^2$, gives $y=\sqrt{\frac{ax^2-\sqrt{2}.x^3}{3\sqrt{2}.x+a}}$; Th.

 $xy=$

$xy = AS \times CS = x^2\sqrt{\frac{a-\sqrt{2}.x}{a+3\sqrt{2}.x}} = A$, a *maximum*; or, ſquared and divided by x^4, $\frac{a-\sqrt{2}.x}{a+3\sqrt{2}..x} = \frac{A^2}{x^4}$; in fluxions,

$$\frac{-\sqrt{2}.\dot{x}\times\overline{a+3\sqrt{2}.x} - 3\sqrt{2}.\dot{x}\times\overline{a-\sqrt{2}.x}}{\overline{a+3\sqrt{2}.x}^2} = \frac{2A\dot{A}x^4 - 4A^2x^3\dot{x}}{x^8}; \text{ or,}$$

ſince $\dot{A} = 0$, $\frac{-\sqrt{2}.\dot{x}\times\overline{a+3\sqrt{2}.x} - 3\sqrt{2}.\dot{x}\times\overline{a-\sqrt{2}.x}}{\overline{a+3\sqrt{2}.x}^2} = \frac{-4A^2x^3\dot{x}}{x^8}$;

contracted, $\frac{a\sqrt{2}.}{\overline{a+3\sqrt{2}.x}^2} = \frac{A^2}{x^5}$; Th. $\frac{a\sqrt{2}.x}{\overline{a+3\sqrt{2}.x}^2} = \frac{A^2}{x^4} = \frac{a-\sqrt{2}.x}{a+3\sqrt{2}.x}$

from above, multiplying both ſides by $\overline{a+3\sqrt{2}.x}$, $\frac{a\sqrt{2}.x}{a+3\sqrt{2}.x} = a - \sqrt{2}.x$; reduced and ſolv'd, $x = \frac{1+\sqrt{13}}{6\sqrt{2}}a$, which ſubſtituted in the *Maximum*, gives $\sqrt{\frac{16+\sqrt{13}}{18+6\sqrt{13}}} \times \frac{a^2}{9} = 78147\frac{5}{6}$, the required area.

This was ingeniouſly anſwered by the Propoſer.

Remark. As I was at ſome pains laſt *May* in inveſtigating the quadrature and the area of this curve, I thought to have inſerted it here, as I found it Geometrically quadrable; but lately peruſing Mr *Emerſon*'s Fluxions, I find it done at *page* 195. *Ex.* 1. much neater, I think, than my own, to which I refer.

New QUESTIONS *to be anſwered in our next.*

QUESTION XLI. *By* Geminus Rhodanus Hoffman.

WHAT two numbers are thoſe, whoſe difference, ſum and product are to each other as are the numbers 2, 3, 5 reſpectively?

QUESTION XLII. *By Mr* JOHN HAMMOND.

IF a body move 4 miles the firſt minute, hour, &c. 2 miles the ſecond, one mile the third, $\frac{1}{2}$ mile the fourth, $\frac{1}{4}$ mile the fifth, &c. in a geometrical ratio, decreaſing every following ſpace of time, to the one half of what it moved the foregoing one, *in infinitum*; required the diſtance which the body cannot exceed?

QUESTI-

QUESTION XLIII. *By Mr* WILLIAM HART.

IN a right-angled plain triangle A B C, there is given the hypotenuſe =100, and the ſide of the inſcribed ſquare =34.286; required the diameter of the inſcribed circle?

QUESTION XLIV. *By Mr* THOMAS DOD.

IN the ſpring quarter in a certain place of N. lat. the ſun's declination and latitude of the place were taken in one ſum =64° 15′ 24″, and the difference between the lat. and declination =41° 28′ 36″. At one o'clock P. M. of the ſame day, I obſerved a cloud in the zenith, and its ſhadow fell at the diſtance of 3 miles from me (upon a right level.) Required the perp. height of the cloud, latitude of the place, and day of obſervation?

QUESTION XLV. *By Mr* PAUL SHARP.

THERE is a parabolical conoid, whoſe perpendicular height, or axis, =10 feet, and its ſolidity is to its ſuperficies as 1 is to .90173. Required a general theorem to determine the diameter of the baſe?

QUESTION XLVI. *By Mr* J. TODD.

IN a certain northern latitude, the ſun's altitude, when due eaſt, was obſerved to be 29°; and at 10 o'clock, the ſame forenoon, his alt. was 51° 26′; required the lat. place, and day of obſervation?

QUESTION XLVII. *By Mr* JOHN WILLIAMS, *jun.*

THERE is a ciſtern in form of a parallelopipedon, whoſe depth, breadth, and length are in arithmetical progreſſion; the common difference is $\frac{1}{7}$ of the length, and the depth is the ſhorteſt; moreover the length of the longeſt ſtrait rod, that poſſibly may be any way put entire therein, is ſo many inches as the ſaid ciſtern contains ſolid feet. What is the content in malt buſhels?

QUESTION XLVIII. *By* SPICILEGUS, *Gent.*

A. Being poſſeſſed of a conſiderable eſtate, with no other incumbrance than an annuity of 20*l.* payable to B. or his aſſigns for 60 years, offers preſent money for it, if B. will allow diſcount on each of the payments at 5*l. per cent.* ſimple intereſt; which being agreed to, they deſire the gentlemen who contribute to the *Miſcellanea Curioſa Mathematica*, to inform them how the anſwer may be obtained, without the trouble of an operation for each payment?

QUEST-

QUESTION XLIX. *By Mr* NICH. FARRER.

GIVEN the radius of a circle =50 chains; required the area of the greateſt inſcribed parabola?

QUESTION L. *By Mr* JAMES WAINE.

Quere. THE leaſt triangle that will incloſe a parabola, whoſe ordinate is double its axis?

QUESTION LI. *By Mr* ROBERT ROBINSON.

TWO ſhips depart from a port in lat. 2° 40′ N. making an angle 98° 48′ between them; the firſt runs ſouth-weſterly 595 miles, and the other ſails ſouth-eaſterly into the ſame latitude, and found ſhe had run 150 miles in ſouth latitude; required her diſtance run, and latitude arrived in?

QUESTION LII. *By Mr* W. BROWN.

GIVEN the ſlant ſide of any cone 42.75 inches, and the diameter of its baſe =16 inches, to find the area of the greateſt parabola that can poſſibly be cut from ſuch a cone?

QUESTION LIII. *By* ΦΙΛΟΣ, *Gent.*

SUppoſe r= radius of a circle.
c= chord of any arc thereof.
s= ſine
s'= co-ſine } of the ſame arc.
v= verſed-ſine

Now any two of theſe five quantities being known, the other three may be readily found; required, that the ſeveral known and unknown quantities, together with their ſolutions, may be ranged into a table fit for uſe?

QUESTION LIV. *By Mr* J. POWLE.

LET there be a curve related to the circle in the equations $x=z-m$, and $y=v$, z being any arc of a circle (whoſe diameter $=a$) m, and v, its correſponding right and verſed ſines; now it's required to ſhew in what law of velocity this ſhall be the curve of ſwifteſt deſcent?

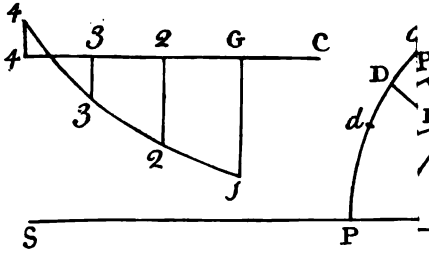
QUESTION LV. *By Mr* NIC. OATS.

Pl. 2. Fig. 36. IN the figure ADmP is given $ax^m=y^{m+n}$ expreſſing the nature of the curve; to find a general expreſſion for the value of AE, when EP is the altitude of the greateſt cylinder that can lie in a ſpace deſcribed by the rotation of ADm, upon its axis AP?

ERRATA. Page 47. line 9. *read* $F=\frac{1}{6}n^6-\frac{1}{2}n^5+\frac{5}{12}n^4*-\frac{1}{12}n^z$.
P. 62. l. 29 *for* contract *read* contact. L. 31. *for* gins *read* gims.
P. 73. l. 20. *read* $y=\sqrt{x+\sqrt{x+a}}$. L. 22. *r.* $x\times\sqrt{x+\sqrt{x+a}}$.
P. 110. l. 24. read *amplitude.*

The End of NUMBER III.

Fig. 52



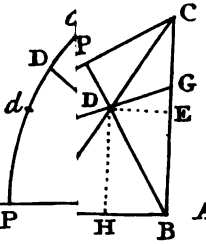


Fig. 54

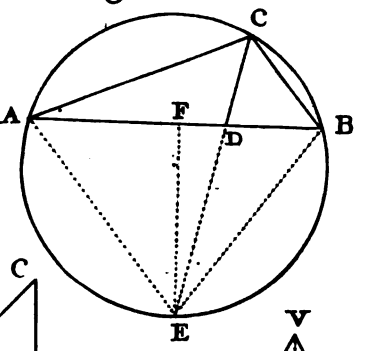


Fig 58

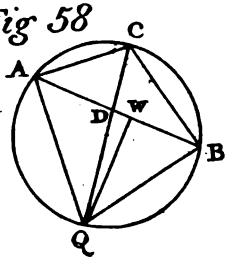


Fig 57

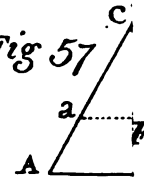


Fig. 55

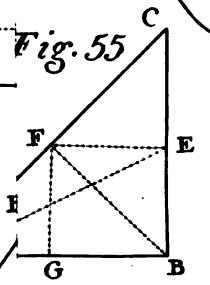


Fig. 59

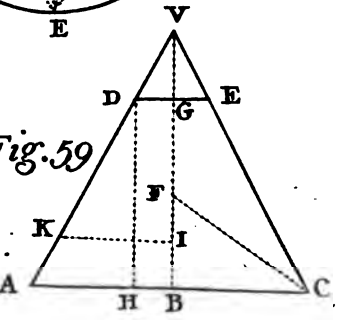


Fig. 63

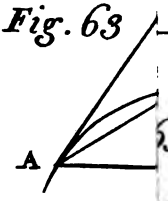
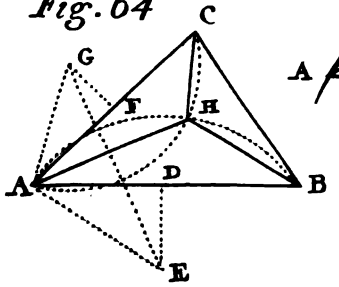


Fig. 64



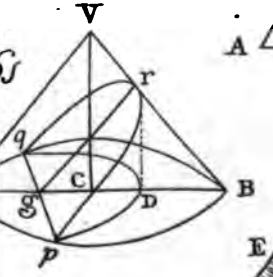


Fig. 60

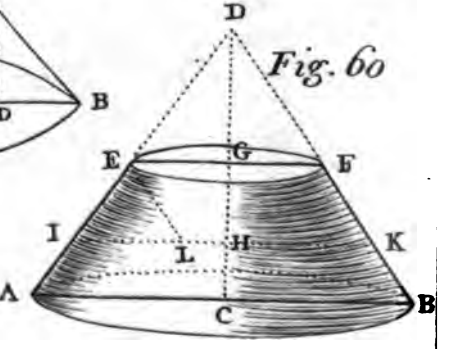


Fig. 66

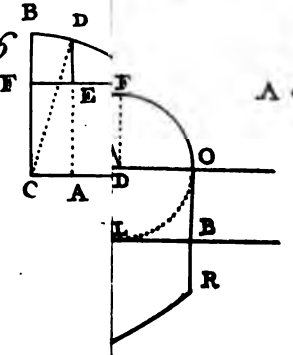


Fig. 65

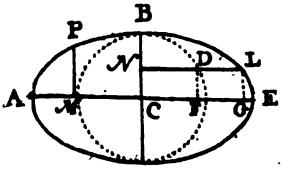
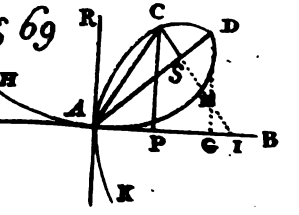
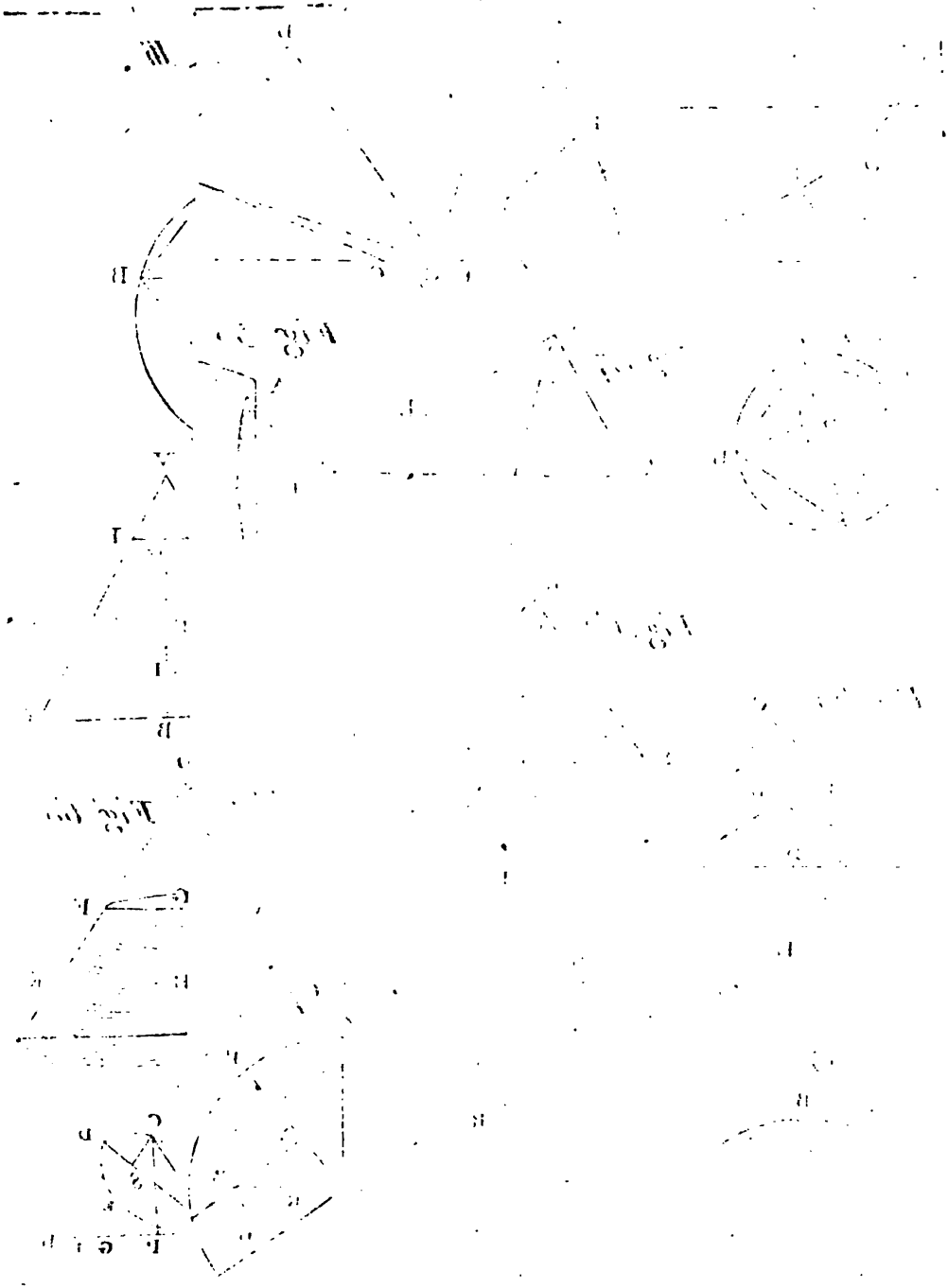


Fig 69



THE areas of all triangles, in[illegible] the products of their sides.



P. 73. l. 20. *read* $y=\sqrt{x+\sqrt{x+a}}$. L. 22. *r.* $x\times\sqrt{x+\sqrt{x+a}}$.
P. 110. l. 24. read *amplitude*.

The End of NUMBER III.

Miſcellanea Curioſa, &c.

NUMBER IV.

An ESSAY *on the properties of triangles inſcribed in, and circumſcribed about two given circles; by Mr* WILLIAM CHAPPLE.

SIR,

THE following enquiry into the properties of triangles inſcrib'd in, and circumſcrib'd about given circles, has led me to the diſcovery of ſome things relating to them, which I preſume have not been hitherto taken notice of, having not met with them in any author; though an ingenious correſpondent of mine, in the iſle of *Scilly*, to whom I communicated ſome of the propoſitions herein after demonſtrated, informs me that he had begun to conſider it ſome years ago, but did not go thro' with it; however I muſt acknowledge that a query of his to me, relating thereto, gave me the firſt hint, and induc'd me to purſue the ſubject with more attention, than perhaps otherwiſe I ſhould have done.

Tho' one or two of the following propoſitions (as the firſt and third, and perhaps the ſecond) might be laid down as *lemmata*, having been demonſtrated by others, yet, for the ſake of order, I thought it beſt to infert them among the reſt.

PROPOSITION I.

THE areas of all triangles, circumſcrib'd about the ſame circle, are as their perimeters.

This is a propoſition well known, the areas of each being equal to the perimeter multiplied into half the radius.

PROPOSITION II.

THE areas of all triangles, inſcrib'd in the ſame circle, are as the products of their ſides.

DEMONSTRATION.

Put d = the diameter of the circle, x = one of the ſides of the inſcrib'd triangle, y = another of its ſides, and p = the perpendicular let fall from the angle included by them, on the oppoſite ſide, which ſide put $=z$. Then, by *Theorem* 17, in *Ward*'s Introduction, $d : x :: y : p = \frac{xy}{d}$; but $\frac{pz}{2} = \frac{xyz}{2d}$ = the area of the triangle; conſequently the areas of all triangles are equal to the products of their ſides, divided by twice the diameter of their circumſcribing circles.

COROLLARY.

The areas of all triangles, inſcribed and circumſcribed in and about the ſame circles, are not only as their perimeters, but alſo as the products of their ſides.

PROPOSITION III.

TO inſcribe and circumſcribe a triangle in and about two concentric circles, the radius of the greater circle muſt be double the radius of the leſſer, and the triangle will always be equilateral.

DEMONSTRATION.

If the ſides of a triangle be biſected by perpendicular lines, thoſe lines will (by 5 *E.* 4.) interſect each other in the center of the circumſcribing circle; and 'tis evident that lines drawn from the center to each angle of the triangle are equal, being radii of the circumſcribing circle. Again, if perpendiculars be let fall from the center of a circle on the ſides of its circumſcribing triangle, it is evident they will fall on the point of contact, and therefore are radii of the inſcrib'd circle, and conſequently equal. Then DI, EI, FI, being equal perpendiculars of right-angled triangles, whoſe hypotenuſes AI, BI, CI are all equal, their baſes muſt therefore be equal; and being halves of the ſides of the triangle reſpectively, the ſides of the triangle are equal.

Pl. 4. Fig. 70.

And that the radius of the circumſcribing circle is equal to the diameter, or twice the radius of the inſcrib'd circle, is thus prov'd: continue the perpendicular AF to G; then, by 15 *E.* 4. GC=GI =CI; and, by 4 *E.* 1. FI=FG; therefore GI=2FI. *Q. E. D.*

PROPOSITION IV.

TO inſcribe and circumſcribe a triangle in and about two eccentric circles, the radius of the leſſer circle muſt be leſs than half the radius of the greater circle.

DE-

DEMONSTRATION.

The area of any triangle is equal to the product of the radius of its inscrib'd circle into half the perimeter; therefore, if this proposition be true, the area of any triangle doth not exceed the product of half the radius of its circumscrib'd circle into half the perimeter. Now a triangle, whose area is equal to the product of half the radius of its circumscrib'd circle into half the perimeter, is equilateral, by *Prop.* 3. So that, if this proposition be true, the greatest triangle that can be inscrib'd in any circle will be equilateral; which we come now to demonstrate.

In order to which it must be considered that the greatest inscrib'd triangle, that can stand upon any chord line to an arc of a circle, will be isosceles, because this will evidently have the greatest altitude; so that we are now to find the greatest *isosceles* triangle that can be inscrib'd in any circle.

Put d= the diameter AG, and x=AF the perpendicular (which in the present case must pass thro' the center of the circle:) Then, by the property of circles, $\overline{d-x}\times x$ $(=dx-x^2)$ = the square of half the base. Th. $\sqrt{dx-x^2}$=BF=FC, and $x\sqrt{dx-x^2}$ $(=\sqrt{dx^3-x^4})$ = the area of the triangle, and dx^3-x^4= the square of the area, which is a *maximum*, because the area is itself so. In *Fluxions*, $3dx^2\dot{x}-4x^3\dot{x}=0$; Theref. $3dx^2=4x^3$, and $3d=4x$; Th. $d=\frac{4}{3}x$, or $1\frac{1}{3}x$, and consequently the triangle is equilateral. *Q. E. D.* Pl. 4. Fig. 70.

PROPOSITION V.

AN infinite number of triangles may be drawn, which shall inscribe and circumscribe the same two circles; provided their diameters, with respect to each other, be limited, as in the two last propositions.

For put x, y and z= the sides of any triangle circumscrib'd about a circle whose radius is $=a$, and inscribed in a circle whose radius is $=b$. Then $\frac{xyz}{4b}$ = the area of the triangle, as was shewn in the demonstration of the 2d *Prop.* Also $\overline{x+y+z}\times\frac{a}{2}$ = the area. Therefore $\frac{xyz}{b}=2a\times\overline{x+y+z}$, and $xyz=2ab\times\overline{x+y+z}$. Hence it is plain, that if the sides of the triangle were required to be found by the given diameters (or the area by the same *data*, supposing x, y and z unknown quantities) the question wou'd be capable of

of innumerable anſwers; for one of the ſides, at leaſt, is unlimited, and may be put equal to any thing at pleaſure, that doth not exceed the longeſt line that can be drawn within the great circle as a tangent to the leſſer. And hence it appears, that if the diſtance of the given circles be ſo fixt, as that any one triangle may be inſcrib'd and circumſcrib'd, innumerable others may be inſcrib'd and circumſcrib'd in and about the ſame circles.

PROPOSITION VI.

THE neareſt diſtance of the peripheries of two given circles, or, which amounts to the ſame, the diſtance of their centers, in order to render it poſſible to inſcribe and circumſcribe triangles, is fixed, and will be always the ſame.

DEMONSTRATION.

Pl. 4. Fig. 71. Let two circles be ſo ſituated, as that the triangle ABC can be circumſcrib'd and inſcrib'd; then, by the laſt *Propoſition*, the triangle DEF, and innumerable others, may be inſcrib'd and circumſcrib'd; but it is evident, by inſpection, that if the leſſer circle be any how remov'd from its place, the ſides of thoſe triangles cannot be tangents to it; and therefore the ſituation of the two circles muſt be the ſame for them all. But if it be ſuſpected that their ſituation may be altered, and yet other innumerable triangles may be circumſcrib'd and inſcrib'd (as the contrary is not yet made appear) let it be conſidered, that the leſſer circle muſt, in its removal, move along on ſome one or other of the ſides of the innumerable triangles that might be inſcrib'd and circumſcrib'd at its *firſt ſituation*, and then where-ever it ſtops (unleſs it be at the ſame diſtance from the periphery of the great circle on the other ſide of the center, which is in effect the ſame ſituation) the like inconveniency will follow. For 'tis well known that triangles, circumſcrib'd about the ſame circle, and having one common baſe, will continually increaſe their altitude, the further the point of contact in the ſaid baſe is removed from the middle thereof, till at length the two lines drawn from the extremities thereof become parallel to, or diverging from each other, and the altitude of the triangle becomes infinite; conſequently (when the two circles are eccentric) the vertexes of *only two* of the circumſcribing triangles, that can be erected on the baſe AB, can be at the periphery of the circumſcribing circle; *viz.* only at C and D; and in either of theſe, the diſtance of the peripheries of the greater and leſſer circles muſt be the ſame, having the ſame baſe and alti-

Pl. 4. Fig. 72.

tude. So that the propoſition is abundantly proved; and hence we have the following

COROLLARY.

If one ſide of a triangle, inſcrib'd in and circumſcrib'd about two given circles, be given, the other two ſides are thereby limited, and may from thence be found.

[See the method of finding them in the anſwer to the Queſtion I have propoſed, which will be in your next.]

PROPOSITION VII.

OF the innumerable triangles that may be inſcrib'd and circumſcrib'd in and about two given (eccentric) circles, two muſt of courſe be iſoſcelar, the vertexes of which will be in the common diameter of thoſe circles, which will cut their baſes at right angles; now the content of that iſoſcelar triangle, which hath the leaſt baſe, and greateſt altitude, will be the greateſt, *and that of the other the* leaſt *of all the triangles that can be inſcribed and circumſcrib'd in the given circles.*

DEMONSTRATION.

In order to demonſtrate this propoſition without the help of Fluxions (which will by all the methods, I can think of, produce a very tedious equation, and very much perplex'd with ſurds) it is neceſſary to premiſe the following

LEMMATA.

1. If a line be drawn from any angular point in a triangle to the middle of the oppoſite ſide, the center of gravity of the triangle (which is that point therein from whence lines drawn to each angle will divide the triangle into three triangles, whoſe areas are equal) will be in that line, and diſtant two thirds thereof from the angular point; and, conſequently, if lines be drawn from two of the angular points to the middle of the oppoſite ſides, they will croſs each other in the center of gravity.

2. All chord lines drawn in the ſame circle are greater or leſs, according as they approach nearer to, or further from the center of the circle.

3. An angle of any triangle is greater or leſs, according as it is nearer to, or further from the center of the inſcrib'd circle.

4. In any triangle inſcrib'd in a circle, the greater any one of its angles is, the leſs will a line be that is drawn from that angle to any point in the oppoſite ſide.

The firſt three of theſe *Lemmata*, having been often demonſtrated, are known to every body; and the fourth hath almoſt the ſelf-evidence of an axiom: for if the ∠BAC be enlarged, ſo as to become equal to BAD, 'tis plain, that a line drawn from A, to any point in the line BC, muſt be greater than a line drawn from thence to any point in BD.

Pl. 4. Fig. 73.

Pl. 4. Fig. 74.

To apply this to the caſe in hand, draw the iſoſceles triangle ABC inſcrib'd in and circumſcrib'd about eccentric circles, making the extremity A of the common diameter CA (which is furtheſt from the leſſer circle) the vertex thereof; alſo draw the iſoſceles triangle $\lambda\beta\delta$, whoſe vertex let be at the other extremity δ. Now I ſay that ABC is the greateſt, and $\lambda\beta\delta$ the leaſt triangle that can be inſcrib'd and circumſcrib'd in and about the given circles; to prove which, draw AF biſecting BC in F, and CE biſecting AB in E, and croſſing AF in D. Then will AD be $\frac{2}{3}$ of AF, and CD $=\frac{2}{3}$ of CE, and the triangle CDA $=\frac{1}{3}$ of the whole triangle ABC, by *Lemma* 1.

Now innumerable lines may be drawn in the ſemicircle $A\lambda\delta$, all of which ſhall be tangents to the leſſer circle, and ſides of inſcrib'd and circumſcrib'd triangles, by *Prop.* 5. and ſuch lines muſt always be leſs than AC, by *Lemma* 2. becauſe further remov'd from O the center of the great circle, and will continually decreaſe according as their extremities are further removed from A, ſo that $\lambda\delta$ will be the leaſt of all of them. Let ac be any one of thoſe lines, and let the circumſcribing and inſcrib'd triangle abc be drawn; alſo the lines af and ce, biſecting bc and ab, and croſſing each other in d, the center of gravity of this trangle; ſo that the triangle cda $=\frac{1}{3}$ of the triangle abc, *&c.* as is above ſhewn.

Then, by *Lemma* 3. the angle at a is > than the angle at A; therefore, by *Lemma* 4. af<AF, and conſequently $\frac{2}{3}$ of af, *viz.* ad<$\frac{2}{3}$AF, *i. e.* <AD. Again, the angle at c>C, and CE>ce, by *Lemma* 3 and 4; conſequently $\frac{2}{3}$ of CE>$\frac{2}{3}$ce, *i. e.* CD>cd. Now ca being alſo <CA, as is above ſhown, the area of the triangle adc is leſs than the area of the triangle ADC (each ſide being reſpectively leſs) that is, $\frac{1}{3}$ of the triangle abc<$\frac{1}{3}$ of the triangle ABC; therefore the triangle ABC>abc.

Hence it plainly appears that the area of any circumſcribing and inſcribed triangle, which hath one angle any where between A and λ, is leſs than the area of the iſoſceles triangle ABC, and greater than the iſoſceles triangle $\lambda\beta\delta$. And that this takes in all the caſes that can poſſibly happen, will be evident to any one that conſiders that if the ∠a be between A and λ, the angle c will be ſomewhere between

between C and δ, and the angle b ſomewhere between B and β; ſo that this takes in one half of the periphery; and the arc Aβ being =Aλ, and Cλ=Bβ, it is but changing ſides with the circle, and we have the ſame ſet of triangles again; for wherever the triangle be, if it be not iſoſcelar, we may have another on the other ſide of the center of the great circle (if I may ſo expreſs it) which ſhall be equal to it, as the triangle $\gamma\Delta\mu$ is the ſame with the triangle abc. Wherefore the propoſition is every way demonſtrated; and tho' the method herein taken ſeems a little tedious and intricate, it is perhaps more conciſe and leſs troubleſome than any which Fluxions would have afforded us.

From the three foregoing propoſitions we may deduce the following

COROLLARY.

If the diſtance of the centers of the two given circles be not ſo fix'd, as that an iſoſcelar triangle can be inſcrib'd and circumſcrib'd, no triangle whatſoever can be inſcrib'd and circumſcrib'd.

And this ſuggeſts an eaſy method of finding the diſtance of the centers of the ſaid given circles, and conſequently the neareſt diſtance of their peripheries; for here we may always conſider the triangle as iſoſcelar, and then the diſtance is found as follows. Pl. 4. Fig. 74.

Put AF=x, CF=y, AO=R, and FG=r; Then δF=2R=x, and CF=$y=\sqrt{2Rx-xx}$, by a known property of circles; alſo $\sqrt{xx+yy}$=AC, by 47 *E.* 1. Put radius =1, Then as $\sqrt{x^2+y^2}$: 1 :: y : $\frac{y}{\sqrt{x^2+y^2}}$ = ſine of the $\angle$CAF; and $x-r$ (=AG) 1 :: r (=GH) : $\frac{r}{x-r}$ = ſine of the $\angle$CAF alſo. Th. $\frac{r}{x-r}=\frac{y}{\sqrt{x^2+y^2}}$; conſequently $xy-ry=\sqrt{r^2x^2+r^2y^2}$; and by involution $x^2y^2-2rxy^2+r^2y^2=r^2x^2+r^2y^2$, that is, $\left.\begin{matrix}x^2\\-2rx\end{matrix}\right\}y^2=r^2x^2$; and dividing by x, we have $\left.\begin{matrix}x\\-2r\end{matrix}\right\}y^2=r^2x$. Th. $\frac{r^2x}{x-2r}=y^2$, that is, $=2Rx-x^2$=AF$\times$CF, as above; and by reducing the fraction, and dividing by x, we have $r^2=2Rx-4Rr-x^2+2rx$; and by tranſpoſition $x^2\left.\begin{matrix}-2R\\-2r\end{matrix}\right\}x=-4Rr-r^2$ ſolv'd, $x-R-r=\sqrt{R^2-2Rr}$= the diſtance of the centers of the two circles; conſequently x (that is AF) =$R+r+\sqrt{R^2-2Rr}$, and CF (the neareſt diſtance of the peripheries)=$R-r-\sqrt{R^2-2Rr}$.

Note,

Note, That R must be = or $>2r$, or else the Root is impossible, which is another proof of the fourth *Proposition*.

Yours, &c. W. CHAPPLE.

A LETTER *from Mr* THOMAS SPARROW, *concerning the Eclipse of the Moon that happens* August 19, 1746. *with the calculation at large, for the meridian of* Mansfield *in lat.* 53° 10′ *N. and long.* 1° *west of* London, *from* Flamsted's *tables.*

SIR,

AS your intentions, in publishing a *Miscellanea Curiosa Mathematica*, is to encourage the sciences mathematical, I presume a calculation of an eclipse at large will no ways be disagreeable to many of your correspondents and readers, as it comes under the denomination of your ingenious book, which if you please to take notice of in any of your numbers, will oblige, SIR,

Mansfield, *Your very humble servant*,

April 23, 1745. THO. SPARROW.

Radix.	Sun's motion.				Motion of perihel.				
1741	9ˢ	21°	01′	58″	3ˢ	8°	13°	30′	
Years 5	11	29	47	28			4	10	
August 19	7	17.	41	04				32	
Hours 11			27	06	3	8	18	12	Perihel. place
Min. 58			2	23	5	9	00	01	Sun's mean mot.
Sec. 50				2					
Sun's mean motion	5	9	00	01	2	0	41	49	Mean anomaly.
Equat. subt.		1	40	10					
Sun's true place	♍	7	19	51					

	☽'s mean motion.				Apog.				Node retrog.			
1741	7	12	28	00	5	15	58	50	3	3	43	51
Years 5	10	00	5	52	6	23	25	54	3	6	41	46
August 19	5	13	44	50		25	44	07		12	13	58
Hours 11		6	2	21			3	04			1	27
Min. 58			31	51				16				8
Sec. 50				27					3	18	57	19
Mean motion	11	2	53	21	1	5	12	11	11	14	46	32
Sun's mean anom.	2	0	41	49	5	7	19	51	5	7	19	51
Phys. parts add			9	55	Suns place				Sun's place			
Mean ☽l. cor.	11	3	3	16	4	2	7	40	5	22	33	19
Apog. subt.		23	47	10	An. argument				Sun from node.			
Mean anom.	10	9	16	6		11	25	01	Eq. ☊ sub.		25	43
Equat. add		4	16	35	Eq. subt.				11	14	20	49
Eq. pl. in its orb.	11	7	19	51	0	23	47	10	True place of node.			
Node subt.	11	14	20	49	True place of ap.				Incl. 5		17	41
Argum. lat.	11	22	59	2	Mean ecc.						55237	
Reduct. add			1	34	True						50191	

True

True pl. in eclip.	11	7	21	25
Lat. ſouth deſc.			38	44
☽'s hourly mot.			31	35
☉'s — ſubt.			2	25
☽ *ab.* ☉			29	10
Mean time of true opp. in moon's orbit *Auguſt*	D. 19	H. 11	M. 58	Sec. 50
As 29′10″:60′::1: 34 red. in time ſub.			3	13
Ecliptic opp.	19	11	55	37
1 Equat. ſub.			6	37
2 Equat. add.			6	41
Diff. ſub.				4
App. time ecl. opp.	19	11	55	33
Diff. merid. ſub.			4	00
Ap. time at *Mansfield*	19	11	51	33
Interval of time between ecl. op. &c. add			6	26
Middle eclip.	19	11	57	59
Moon's hor. paral.			56	10
— Sun's add				10
Sum			56	20
Semi-diam. Sun ſub.			15	58
S.-diam. earth's ſhad.			40	22
Sem. diam. ☽ add			15	14
Sum			55	36
Moon's lat. ſub.			38	44
Part deficient			16	52

Diff. D. M. and L. 5046 11617

At *Mansfield.*	D.	H.	M.	Sec.
Begin *Auguſt*	19	10	36	41
Ecliptic oppoſ.		11	51	33
Middle		11	57	59
End		13	19	17
Duration		2	42	36
Digits eclips'd		6°	38′	
Time app. P. M.				

As 15′ 14″ : 6° :: 16′ 52″ : 6° 38′ digits obſcur'd.
As ſine 55′ 36″ : R :: ſine 38′ 44″ : coſ. 45° 18′ angle of incidence.
As R : ſine 45° 18′ :: 55′ 36″ : 39′ 31″ mot. of ſemi-duration.
As 29′ 10″ : 60′ :: 39′ 31″ : 1^{h} 21^{m} 18^{s} time of ſemi-duration.

Yours, &c. T. SPARROW.

SIR,

THE following queſtion was propoſed in the *Ladies Diary* for the year 1731, by Mr *Thomas Pointin*, but was never anſwered in the ſucceeding numbers; by inſerting my ſolution in your next *Miſcellanea Curioſa Mathematica*, you will, amongſt the reſt of your contributors, greatly oblige SIR, *Yours, &c.*

EDWARD HAUXLEY.

QUESTION 161. L. D. 1731. *By Mr* THO. POINTIN.

THERE is an iſland in the Atlantic *ocean, ſuppoſed perfectly round; in the center thereof a tower is erected, from which, if a*

 ſerene

ſerene ſky, and calm ſea, the inhabitants can ſee 12 miles from the ſhore. We being at anchor, a line drawn from the top of the tower to the top of the maſt was ſuppoſed to bound the utmoſt ſtretch of ſight; from thence we ſail'd to our intended port, and found the diſtance run $\frac{1}{3}$ of a degree, (allowing $69\frac{1}{2}$ miles to a degree) and with great care by the ſun's height our courſe corrected at noon was N. by E. 2° 45′ E. or 14° from N. to E. the tower's top then cutting the ſun in two halves, the maſt was 45 feet from the water, the ſhip's diſtance from the ſhore was 1.263244 leagues, its latitude 64° 20′, and the tower's latitude 63° 56′ N. Required its height, the compaſs of the iſland, and the ſquare miles it contains.

SOLUTION.

Firſt, We have given the latitude come to at noon 64° 20′ N. and becauſe, when obſerving at noon, the ſun was cut in two halves by the top of the tower, it is evident that the tower was then due ſouth from the ſhip. Now the latitude of the tower being given = 63° 56′ N. hence the difference of latitude between the tower and ſhip, which is 24 *ſexagenary* miles, repreſented in the fig. by N. S. is their diſtance aſunder; from which ſubtract ND=3.789732 *ſexagenary* miles, the ſhip's diſtance given from the ſhore, the remainder is 20.211 miles, the ſemi-diameter of the iſland. Hence the diameter DI=40=422 miles, conſequently its circumference =126.989 *ſexagenary* miles, and its content =1283.297 ſquare miles.

Pl. 4. Fig. 75.

Secondly, We have now given the latitude in at noon 64° 20′ N. and the bearing of the place ſail'd from S 14° weſterly, from which I find AB=34.9 M. the difference of latitude made northerly, and BN=8.7 M. her departure eaſterly. Therefore from the latitude, ſail'd from 64° 20′ N. ſubtract 34.9 M. the difference of latitude there remains 63° 45.1, the latitude the ſhip ſail'd firſt from. Hence we have now given 63° 45′ N. the latitude of A the ſhip ſail'd at firſt, and the latitude of S the tower =63° 56′ N. conſequently the ſhip was then 11 M = ST to the ſouthward of the tower, and =AT=BN=8.7 M. to the weſtward of the tower at firſt ſetting ſail at A. Hence the *tower* bore from the ſhip N 38° 21′ E. diſtant 14.02 miles =AS, by caſe 6th of *plain ſailing*. So that it is plain the ſhip muſt have been at A, in ſome harbour in the iſland. And ſince, from the top of the tower, an obſerver looking over the maſt head at N, whoſe height is =45 feet, perceives the limits of the horizon at 12 *ſexagenary* miles diſtance from the periphery

phery of the iſland, and conſequently at 32.211 M. diſtance from the foot of the tower, from thence it will be eaſy to find the height of the tower, as follows.

Let the circle HSO repreſent that great circle of the earth in which the ſhip S at firſt, and tower O lie in, and let HO=18 M. the ſhip's diſtance then from the limits of the horizon of the tower; and let SO=14M. the diſt. the ſhip was on the ſame great circle from the tower O when it bore N. 38 21 E. as determin'd above. Then as 25035.84 miles the circuit of the earth, is to 360°; ſo is (HO) 18 miles, to 15′ 31″=∠MCH; which ſubtracted from 90°, leaves 89° 44′ 29″=∠HMC, which taken from 180°, leaves the ∠CMT =90° 15′ 31″. Again, as 25035.84 M. is to 360°; ſo is 14 M. to 12′ 42″=∠MCT; which added to ∠CMT=90° 15′ 31″, and the ſum 90° 27′ 35″, ſubtracted from 180°, leaves the ∠MTC=89° 32′ 25″. Laſtly, we have now given in the oblique-angled plain triangle MTC, the angle MTC=89° 32′ 25″; ∠TMC=90° 15′ 31″; leg MC=23907525 feet from the maſthead to the earth's center; from whence I find, by caſe 1. of *oblique Trig.* TC=23908249.98 feet, the diſtance of the earth's center from the top of the tower; from which the earth's ſemi-tranſverſe OC being ſubtracted, leaves TO=769.9 feet, the tower's height. Pl. 4. Fig. 76.

EDW. HAUXLEY.

Solutions to queſtions in Number III.

QUESTION XLI. *anſwered by Mr* SAMUEL DUNN.

LET x= greater number, y= leſſer; then *per* queſtion $x-y : x+y :: 2 : 3$; Th. $3x-3y=2x+2y$; Th. $x=5y$. But $x+y : xy$, that is, $6y : 5y^2 :: 3 : 5$; Th. $30y=15y^2$; Th. $y=2$, and $x=10$.

The ſame anſwered by Mr THO. NAYLOR.

PUT x and y for the two numbers; Then $x+y : xy :: 3 : 5$; and $x-y : xy :: 2 : 5$. Th. $5x+5y=3xy$, and $5x-5y=2xy$; now ſubtracting the latter equation from the former, we have $10y=xy$; Th. $x=10$, and $y=2$.

The ſame anſwered by Mr W. WALLHEAD.

PUT x= diff. then $2 : 3 :: x : \frac{3x}{2}$= ſum; and *to* and *from* half their ſum $\left(=\frac{x}{4}+\frac{x}{2}\right)$ *add* and *ſubtract* half their diff. $\left(=\frac{x}{2}\right)$ and

we

we have $x+\frac{x}{4}$, and $\frac{x}{4}$ for the two numbers reſpectively, the rectangle of which is $=\frac{5xx}{16}$. But $2:5::x:\frac{5x}{2}$; Th. $\frac{5xx}{16}=\frac{5x}{2}$; Th. $x=8$; conſequently the two numbers are 2 and 10.

By VASSAL MORPETHIENSIS.

As $\left\{\begin{matrix}\text{Half the difference}\\ \text{Half the ſum}\end{matrix}\right\}$ of the ratios for their ſum and difference $\left\{\begin{matrix}.5\\ 2.5\end{matrix}\right\}$ is to the ratio for their product 5; ſo is 1, to $\left\{\begin{matrix}10=\text{the greater number required.}\\ 2=\text{the leſſer number required.}\end{matrix}\right.$

QUESTION XLII. *anſwered by* DINOCRATES.

LET a denote the firſt term, y the laſt, s the ſum of the terms, and let the ratio of the terms be that of r to 1. Then $s=\frac{ra-y}{r-1}$; but in deſcending geometrical progreſſions, whoſe number of terms are infinite, and firſt term finite, the laſt term will be $=0$. Th. $s=\frac{ra}{r-1}=8$, the miles that the body cannot exceed.

The ſame anſwered by Mr EDWARD HAUXLEY.

HERE it is evident that we have given 4 the firſt and greateſt term in an infinite decreaſing geometrical progreſſion, 2 the 2d term and common ratio, and the laſt term being leſs than any aſſignble quantity is therefore in effect $=0$; conſequently the ſum of the ſeries will be finite. Hence, *The greateſt term* 4 *being multiplied by* 2 *the common* ratio, *and that product divided by the common* ratio *leſs* 1, *the quotient is* $=8$, *the ſum of all the ſeries, which is the utmoſt limits that any moving body can go by what is given in the queſtion.*

QUESTION XLIII. *anſwered by Mr* JOHN TODD.

Pl. 4. Fig. 77. HERE is given AC$=100=a$, DE$=$EB$=$BL$=$LD$=34.286$ $=b$. Put $x=$AB$+$BC; then becauſe the ſquare of the ſum of the ſides *minus* the ſquare of the hypotenuſe is equal to twice the product of their multiplication, and the product of the ſides is equal to the product of their ſum into the ſide of the inſcribed ſquare, we have $x^2-a^2=2bx$; Th. $x=b+\sqrt{a^2+b^2}$, and $x-a=b-a+\sqrt{a^2+b^2}$ $=40=$ diameter of the inſcribed circle.

The

The ſame anſwered by Mr THOMAS COWPER.

PUT b=AC=100, c=DE=34.286, and x=FB=BH, the radius of the inſcrib'd circle. Then $\sqrt{b^2+c^2}+c$=AB+BC; Th. $b+2x$=AB+BC=$\sqrt{b^2+c^2}+c$; Th. $2x=\sqrt{b^2+c^2}+c-b$=40.

QUESTION XLIV. *anſwered by Mr* J. TURNER.

THE ſum of the lat. and declination is = 64° 15′ 24″
Their difference 41 28 36
Added, is twice the lat. of the place = 105 44 00

And 64° 15′ 24″ *minus* 41° 28′ 36″=22° 46′ 48″= twice the ſun's declination. Hence the lat. and declination was 52° 52′ north, and 11° 23′ 24″ reſpectively, anſwering to the 9th day of *April*. The ſun's altitude at one o'clock P. M. was 46° 48′; then it will be

As the ſun's co. altitude	9.835403
Is to the ſhad. of the cloud-diſt. on ground	0.477121
So is the ſun's altitude	9.862733
To perp. height cloud =3.195 miles	0.504451

The ſame by Mr R. ROBINSON.

HALF the given ſum added to half the given difference, gives the lat. = 52° 52′ N; alſo the exceſs of the half ſum above the half difference is the ſun's declin.=11° 23′ 24″N. Hence the ſun's alt. at one o'clock is 46° 48′ 28″. As rad. : length ſhade = 3 miles : : tangent ſun's alt. : cloud's height =3.1955 miles.

QUESTION XLV. *anſwered by Mr* J. TURNER.

LET AD=b=10; BD=y the ſemi-diameter of the baſe; n=.7854; t=.90173. Then by *page* 121 of the *Appendix* of *Stone*'s Fluxions, $2bny^2$= ſolidity of the paraboloid; and by *page* 135, its ſuperficial content is $=\frac{2nb}{3yy}\times 4y^2+\frac{y^4}{bb}^{\frac{3}{2}}$. But by the queſtion; as 1 : t : : $2bny^2$: $\frac{2nb}{3y^2}\times 4y^2+\frac{y^4}{bb}^{\frac{3}{2}}$. Put $z=y^2$, and $s=\frac{1}{bb}$; then, by reduction, we ſhall have $-s^3z^3-12s^2z^2-48sz+9t^2z=64$; from which equation we find, by converging ſeries, z=9.376; y=3.062, and the diameter of the baſe CB=6.124. Pl. 4. Fig. 78.

The ſame anſwered by Mr ROBERT ROBINSON.

LET $x=10$, the axis, $y=$ rad. baſe, $a=$ param. $m=3.1416$, and $1 : .90173$ as $1 : n$; then will $ax=y^2$, and $my^2\dot{x}=ma\dot{x}\dot{x}$, whoſe fluent is $\frac{1}{2}my^2x$, the ſolidity.

Again, the fluxion of the curve ſurface is $2my\sqrt{\dot{x}^2+\dot{y}^2}=\frac{2my\dot{y}}{a}\sqrt{a^2+4y^2}$; its fluent is $\frac{m}{6a}\sqrt{a^2+4y^2}^3-\frac{ma^2}{6}$; hence $1 : n :: \frac{my^2x}{2}$: $\frac{m}{6a}\sqrt{a^2+4y^2}^3-\frac{ma^2}{6}+my^2=\frac{mny^2x}{2}$, the whole ſurface. Th. $3nx^2a+a^2-6xa=\sqrt{ax\times a+4x}^3$, which reduced is $a^2+\frac{3n^2x^3-12nx^2-4x}{2nx-8}a=\frac{32xx}{3nx-12}$; or $a^2+2Aa=B$, Th. $a=\sqrt{B+A^2}-A$; hence $y=\sqrt{x\sqrt{B+A^2}-Ax}$, the radius of the baſe.

QUESTION XLVI. *anſwered by Mr* J. TURNER.

PUT $b=$ fine ſun's altitude when due eaſt, $d=$ fine of his altitude at 10 o'clock, $c=$ coſine of the hour, $\sqrt{x}=$ fine of the lat. of the place, its coſine $=\sqrt{1-x}$; Then as $1 : b :: \sqrt{x} : b\sqrt{x}=$ fine ſun's declination, its coſine $=\sqrt{1-b^2x}$. Now, by *Anderſon*'s theorem, $c\times\sqrt{1-x}\times\sqrt{1-b^2x}=d-bx$; which by involution, &c. is $\begin{matrix}+b^2 & & +c^2\\ -c^2b^2 & x^2 & +c^2b^2 \\ & & -2db\end{matrix}\ x=c^2-d^2$. Th. $x=.6689$, and $\sqrt{x}=.8180$, the ſine of 54° 53′, the latitude; and the declination is $=$ 23° 18′ N.

The ſame anſwered by Mr PAUL SHARP.

PUT the nat. ſines of the ſun's altitude, when due eaſt, and at 10 o'clock $=a$ and b, the coſine of the hour from noon $=c$, (radius $=1$) and $x=$ ſine of the latitude of the place ſought. Then will $ax=$ ſine of the ſun's declination, its coſine $=\sqrt{1-a^2x^2}$. Then in the oblique ſpherical triangle ACB, we have the coſines of the $\angle$ ABC, and ſide AC ($=c$ and b) and the ſines of the ſides AB and BC ($=\sqrt{1-a^2x^2}$, and $\sqrt{1-x^2}$.) Then, by *Theorem* 8. *p.* 99. *Thacker*'s Miſc. we have this equation; $c\times\sqrt{1-x^2}\times\sqrt{1-a^2x^2}=b-ax^2$, which ſquared and tranſpoſed gives $a^2x^4-a^2c^2x^4+c^2x^2+$ $a^2c^2x^2$

4.
Fig. 79.

$a^2c^2x^2-2abx^2=c^2-b^2$. Put $\frac{c^2+a^2c^2-2ab}{a^2-a^2c^2}=2n=2.86164$, and $\frac{c^2-b^2}{a^2-a^2c^2}=m=2.35967154$; then $x^4+2nx^2=m$. Whence $x=\sqrt{m+n^2}-n|^{\frac{1}{2}}=.81758=54^\circ\ 50'\ 37''$, the lat. of the place; or rather thus: $2x^2=2\sqrt{m+n^2}-2n=1.33689$, the verſed ſine of double the ſaid lat. Th. $ax=.39636=23^\circ\ 21'\ 4''$, the ſun's declination, anſwering the 4th or 16th day of *June*.

QUESTION XLVII. *anſwered by* LOGARITHMICUS.

Putting $d=$ the depth, $b=$ the breadth, and $l=$ the length

$$\left.\begin{array}{l} d+\frac{l}{5}=b. \quad \text{Th. } l=5b-5d \\ d+\frac{2l}{5}=l. \quad \text{Th. } l=\frac{5d}{3} \end{array}\right\} \text{whence } b=\frac{4d}{3}; \text{ but } (\sqrt{ll+bb+dd}=)$$

$\sqrt{\frac{25dd}{9}+\frac{16dd}{9}+dd}=$length of the longeſt line that can be inſerted in the ſolid; and $(l\times b\times d=)\ \frac{5d}{3}\times\frac{4d}{3}\times d=\frac{20d^3}{9}=$the ſolidity. Theref. $\sqrt{\frac{25dd}{9}+\frac{16dd}{9}+dd}=\frac{20d^3}{9}$; and $\frac{50d^2}{9}=\frac{400d^6}{81}$. Alſo $d^4=\frac{9}{1}$; from whence the malt buſhels may be deduc'd by the following logarithmical Proceſs.

From the log. of 9	0.95424
Take the log. of 8	0.90309
Divide the rem. by 4	4)0.05115
The quot. is the log. of d	0.01279
To 3 times the log. of d, = the log. of d^3	0.03837
Add the log. of 20	1.30103
And the ar. co. of the log. of 9,	$\bar{1}$.04576
The ſum is the log. of the ſolidity $\left(\frac{20d^3}{9};\right)$ in feet	0.38516
To which add the log. of 1728 (inches in a ſolid foot)	3.23754
And (ſee *Dodſon*'s anti-logarithmic canon p. 72.) the L.	4.66748
The ſum is the log. of 1.9506 (the malt buſhels)	0.29018

The

The same by Mr ROBERT ROBINSON.

LET the length $=5x$ inches, then the breadth $=4x$, depth $=3x$, and the solidity $=\frac{5x^3}{144}$ solid feet, the diagonal in inches $=5x\sqrt{2}=\frac{5x^3}{144}$; hence $x=12\sqrt[4]{2}$, and the content $=\frac{103680\sqrt{8}}{2150.4}=81.0864$ malt bushels.

QUESTION XLVIII. *answered by the Proposer.*

PUT $a=$ annuity, $n=$ number of payments thereof, $r=$ interest of 1 *l.* in 1 year, $s=$ the sum required; then

$$\frac{a}{1+r}+\frac{a}{1+2r}+\frac{a}{1+3r}+\frac{a}{1+4r}, \text{ \&c. to } \frac{a}{1+nr}=s.$$

Or putting $1+nr=m$

$$\frac{a}{m}+\frac{a}{m-r}+\frac{a}{m-2r}+\frac{a}{m-3r}, \text{ \&c. to } \frac{a}{1-n-1\times r}=s.$$

But $\frac{a}{m}=\frac{a}{m}$

$$\frac{a}{m-r}=\frac{a}{m}+\frac{ar}{m^2}+\frac{ar^2}{m^3}+\frac{ar^3}{m^4}+\frac{ar^4}{m^5}, \text{ \&c.}$$

$$\frac{a}{m-2r}=\frac{a}{m}+\frac{2ar}{m^2}+\frac{4ar^2}{m^3}+\frac{8ar^3}{m^4}+\frac{16ar^4}{m^5}, \text{ \&c.}$$

$$\frac{a}{m-3r}=\frac{a}{m}+\frac{3ar}{m^2}+\frac{9ar^2}{m^3}+\frac{27ar^3}{m^4}+\frac{81ar^4}{m^5}, \text{ \&c.}$$

Therefore putting $\frac{a}{m}=Q$, $\frac{r}{m}=q$, and A, B, C, D, &c. as in pag. 47 and 48 in number 2.

$s=Q\times\overline{A+Bq+Cq^2+Dq^3+Eq^4}$, &c.

But by the form, in pag. 47.

$A=n$

$Bq=\frac{n^2q}{2}-\frac{nq}{2}$

$Cq^2=\frac{n^3q^2}{3}-\frac{n^2q^2}{2}+\frac{nq^2}{6}$

$Dq^3=\frac{n^4q^3}{4}-\frac{n^3q^3}{2}+\frac{n^2q^3}{4}$

$Eq^4=\frac{n^5q^4}{5}=\frac{n^4q^4}{2}+\frac{n^3q^4}{3}*-\frac{nq^4}{30}$

&c. &c. There-

Therefore 1, $nq+\frac{n^2q^2}{2}+\frac{n^3q^3}{3}+\frac{n^4q^4}{4}$, &c. = *Nep.* Log. of $\frac{1}{1-nq}$.

And $\frac{1}{q}\times$ *Nep.* log. $\frac{1}{1-nq}=n+\frac{n^2q}{2}+\frac{n^3q^2}{3}+\frac{n^4q^3}{4}$, &c.

2. $1+nq+n^2q^2+n^3q^3+n^4q^4$ &c. $=\frac{1}{1-nq}$.

Th. $(\frac{1}{r}=)$ * $b\times\overline{\frac{1}{1-nq}-1}=\frac{nq}{2}+\frac{n^2q^2}{2}+\frac{n^3q^3}{2}+\frac{n^4q^4}{2}$, &c.

3. $1+2nq+3n^2q^2+4n^3q^3$, &c. $=\frac{1}{\overline{1-nq}^2}$.

Th. $(\frac{1}{1\frac{1}{2}}q=)\ \frac{1}{2}cq\times\overline{\frac{1}{\overline{1-nq}^2}-1}=\frac{nq^2}{6}+\frac{n^2q^3}{4}+\frac{n^3q^4}{3}$, &c.

4. $1+4nq+10n^2q^2+20n^3q^3$, &c. $=\frac{1}{\overline{1-nq}^4}$.

Th. $(\frac{1}{1\frac{1}{20}}q^3=)\ \frac{1}{4}dq^3\times\overline{\frac{1}{\overline{1-nq}^4}-1}=\frac{nq^4}{30}+\frac{n^2q^5}{12}+\frac{n^3q^6}{6}$, &c.

Therefore $s=Q\times\frac{1}{q}\times$ *Nep.* log. $\frac{1}{1-nq}-b\times\overline{\frac{1}{1-nq}-1}+\frac{1}{2}cq\times$

$\overline{\frac{1}{\overline{1-nq}^2}-1}-\frac{1}{4}dq^3\times\overline{\frac{1}{\overline{1-nq}^4}-1}+\frac{1}{6}eq^5\times\overline{\frac{1}{\overline{1-nq}^6}-1}$, &c.

In the present example $x=20$, $n=60$, $r=\frac{1}{10}$, $m=4$, $Q=5$, $q=\frac{1}{10}$, $nq=(\frac{60}{10}=)\ \frac{3}{4}$, $1-nq=\frac{1}{4}$,

$\frac{1}{1-nq}=4$, $b=\frac{1}{3}$,

$\frac{1}{\overline{1-nq}^2}=16$, $\frac{1}{2}cq=\frac{1}{2.6.80}$,

$\frac{1}{\overline{1-nq}^4}=256$, $\frac{1}{4}dq^3=\frac{1}{4.30.80.80.80}$.

Nep. log. $\frac{1}{1-nq}=1.38629$. Then $1.38629\times80=110.9032$.

$\frac{1}{q}=80$ $(\overline{4-1}=)\ 3\times b$ $=1.5$

$(\overline{16-1}=)15\times\frac{1}{2}cq=$ 0.015625

$(\overline{256-1}=)255\times\frac{1}{4}dq^3$ $=0.000004$

From the sum of + 110.918825 1.500004

* The values of *b*, *c*, *d*, *e*, &c. will be found in *page* 46.

Take the sum of — 1.500004
Remainder 109.418821×(Q=)5=547.0941=s.

QUESTION XLIX. *answered by Mr* J. BULMAN.

Pl. 4. Fig. 80. LET x=BF the axis, and a=100=BD, the diameter of the circle; then $a-x$=FD, and, by 13 *E.* 6. $\sqrt{ax-xx}$=GF, and $2\sqrt{ax-xx}$=GE, and $\frac{4}{3}\sqrt{ax^3-x^4}$=area of the parabola, which by the question must be a *maximum*; in *fluxions* is $\frac{4}{3}\times\frac{3ax^2\dot{x}-4x^3\dot{x}}{\sqrt{ax^3-x^4}}$ =0. Reduced, $x=\frac{3}{4}a$=75=BF, and the ordinate GE=86.602. Th. area =4330,1 square chains.

The same answered by Mr JOHN TURNER.

PUT a= diam. circle BD, x=BF, EF=y. By the property of the circle $ax-x^2=y^2$; and $x\sqrt{ax-xx}$ is as the area of the parabola, which, in *fluxions*, gives $3ax^2\dot{x}=4x^3\dot{x}$, or $x=\frac{3}{4}a$=75, and the area of the parabola =4330.1.

The same answered by Mr ROBERT ROBINSON.

LET the circle's diameter =100=a, the abscissa =x; then the semi-ordinate $=\sqrt{ax-xx}$, and the area of the parabola $=\frac{4x}{3}\sqrt{ax-x^2}$=*maximum*, or ax^3-x^4; in *fluxions* $3ax^2\dot{x}-4x^3\dot{x}=0$. Th. $x=\frac{3}{4}a$=75 chains, the abscissa; and the ordinate $=\frac{a}{2}\sqrt{3}$= 86.6025.

Again, by VASSAL MORPETHIENSIS.

PUT a=50=$\frac{1}{2}$BD, x=BF; then $2a-x$=FD, and $\overline{BF}^2\times FD=\overline{GF}\times FB^2=2ax^2-x^3$, a *maximum*; Th. its fluxion $6ax\dot{x}-4x^3\dot{x}=0$. Hence $x=\frac{3a}{2}$, and the required area $=\frac{1}{4}a^2\sqrt{3}$= 4330.125 chains.

QUESTION L. *answered by* Philosluentinecanalgegeomastrolongo.

Pl. 4. Fig. 81. SINCE DE=2BD, therefore D is the focus of the parabola, whose equation is $ax=y^2$. Th. let BD=$\frac{1}{4}a$, DE=$\frac{1}{2}a$, BC=x, CG=y; then AB=x, and AD=$\frac{1}{4}a+x=v$, and $\dot{x}=\dot{v}$. Then by

by ſimiliar triangles, $2x : y$ or $\sqrt{ax} :: v : \frac{v\sqrt{ax}}{2x} = DF$, and $AD \times DF$ = area of the triangle = a *minimum*; that is, $\frac{v^2\sqrt{ax}}{2x} = minimum$, or $\frac{v^2}{\sqrt{x}} =$ a *minimum*; conſequently $\frac{v^4}{x} =$ *minimum*. In *fluxions*, $\frac{4v^3\dot{v}}{x} - \frac{v^4\dot{x}}{xx} = 0$, and $4v^3x = v^4$, or $4x = v = \frac{1}{4}a + x$; therefore $3x = \frac{1}{4}a$, and $x = \frac{1}{12}a$, and $y = a\sqrt{\frac{1}{12}}$.

The ſame anſwered by Mr TURNER.

LET $BC = x$, $AC = 2x$, $b =$ parameter, $CM = \sqrt{bx}$, $AD = x + b$; as $2x : \sqrt{bx} :: x + b : DT = \frac{x+b}{2x}\sqrt{bx}$, and $AD \times DT = x + b \times \frac{x+b}{2x}\sqrt{bx}$ = area of the triangle = a *minimum*. Put $z^2 = x$, and $b = 1$; then the area will be $\frac{z^5 + 2z^3 + z}{2z^2} = \frac{z^4 + 2z^2 + 1}{2z}$; which put into *fluxions*, tranſpoſed, and divided, is $6z^4 + 4z^2 = 2$. For z^2 reſtore its value x, and $6x^2 + 4x = 2$, or $x^2 + \frac{2}{3}x = \frac{1}{3}$, and $x = \frac{1}{3}$. Conſequently BC is univerſally equal to $\frac{1}{3}$ of the abſciſſa.

QUESTION LI. *anſwered by Mr* J. TURNER.

PUT $AB = d$, $QC = b$, $BE = n$, s and $c =$ ſine and coſine of half the ſum of the angles at the baſe, *viz.* 40° 36′, x and y = ſine and coſine of half their difference; then is $cy + sx =$ ſine $\angle C$, and $cy - sx =$ ſine $\angle A$. As $1 : d :: cy - sx : dcy - dsx = BD$, and $1 : b :: cy + sx : bcy + bsx = QG$. Conſequently $dcy - dsx = bcy + bsx + n$. Let $dc - bc = p$, and $bs + ds = q$; then $py = qx + n$, and $p^2 - p^2x^2 = q^2x^2 + 2qnx + n^2$. In numbers, $x^2 + .4443x = .2536$, hence $x = .2170$, the ſine of 12° 32′; conſequently the eaſtermoſt ſhip's courſe was ſouth 36° 52′ eaſterly, her diſtance run = 350 miles, lat. arrived in = 2° 0′ ſouth. Pl. 4. Fig. 82.

The ſame anſwered by the Propoſer, Mr R. ROBINSON.

LET $AB = d = 595$, $BE = q = 160$, $QC = a = 150$, $\angle ABC = 98° 48'$, n and $m =$ ſine and coſine of its half, x and $y =$ ſine and coſine of half the difference $\angle$'s A and C; then $1 : a :$ ſine $\angle C = my$

$=my+nx$: QG$=amy+anx$; Th. BD$=q+amy+aux$; fine $\angle$A $=my-nx : q+amy+anx :: 1 : d$; Th. $\overline{dm-am}\times y-\overline{dn-an}\times x=q$, or $vy-wx=q$; but $v^2y^2=v^2-v^2x^2=q^2+2qwx+w^2x^2$; Th. $x^2+\frac{2qw}{v^2+w^2}x=\frac{v^2-q^2}{v^2+w^2}$; hence $x=\frac{v\sqrt{v^2+w^2-q^2}-qw}{v^2+w^2}=.2169148$, the fine of 12° 31′ 40″; hence $\angle A=28°\ 4'\ 20''$, $\angle C=53°\ 7'\ 40''$; conſequently the eaſtermoſt ſhip runs 350 miles, and arrived in 2° ſouth latitude.

QUESTION LII. *anſwered by Mr* ROBERT ROBINSON.

Pl. 4. Fig. 83. GIven AH=42.75 inches $=n$, AC=16$=m$, EC$=x$; then $m : n :: x : \frac{nx}{m}=$EG; and, by the nature of the circle, $\sqrt{mx-xx}=$DE=EF; hence the area of the parab. $=\frac{4nx}{3m}\sqrt{mx-x^2}$ $=maximum$, or mx^3-x^4, in *fluxions* $3mx^2\dot{x}-4x^3\dot{x}=0$. Th. $x=\frac{3}{4}m$; hence the *axis* GE$=\frac{3}{4}n$, *ordinate* DF$=\frac{1}{2}m\sqrt{3}$, and the area $=\frac{1}{4}mn\sqrt{3}=296.1807$ ſquare inches.

The ſame anſwered by Mr SAMUEL BAMFIELD.

PUT $d=16=$AC, $s=$ cone's ſide $=$AH$=42.75$, and $x=$EC; then $d-x=$AE; *per* ſim. $\triangle$'s as $d : s :: x : \frac{sx}{d}=$ GE, the parab. axis; by the prop. of the circle (AE$\times$EC$=\overline{EF}^2=\overline{ED}^2$) $\sqrt{dx-xx}=$EF$=$ half the parabola's baſe; Th. $\frac{2sx}{3d}\times 2\sqrt{dx-xx}=$ parab. area, $=$ *maximum*, in *fluxions* $48s^2dx^2\dot{x}-64s^2x^3\dot{x}=0$; reduc'd $\frac{3d}{4}=x=12$; from whence it appears that it is univerſal, let the ſide of the cone be what it will, whence the area $=296.18053$.

Again, by Mr WILLIAM DAVIES.

LET $a=16$ inches $=$AC, $b=42.75=$AH$=$HC, $x=$EC; then $2\sqrt{ax-xx}=$ baſe of the parabola, $a : b :: x : \frac{bx}{a}=$EG; Th. $\frac{2}{3}\times\frac{bx}{a}\times 2\sqrt{ax-xx}=$ area of the parabola $=$ *maximum*, its fluxion $\frac{6abxx-8bx^3}{3a\sqrt{ax-xx}}=0$; Th. $x=\frac{3a}{4}=12$.

The

QUESTION LIII. *answered by the Propoſer.*

Since $ss = rr - s's' = cc - vv = 2rv - vv = 2vs' + vv$, therefore

Given	Required	Solutions.	
$r, ss', v, cs' =$		$v = r - \sqrt{rr - ss}$	$c = \sqrt{2rv - vv}$
$r, s's, v, cs =$		$v = r - s'$	$c = \sqrt{r - s' \times 2r}$
$r, vs, s', cs =$		$s' = r - v$	$c = \sqrt{2rv}$
$r, cs, s', vs =$		$s' = r - \frac{cc}{2r}$	$v = \frac{cc}{2r}$
$s, s'r, v, cr =$		$v = \sqrt{ss + s's'} - s'$	$c = \sqrt{2ss + 2s's' - 2s' \times \sqrt{ss + s's'}}$
$s, vr, s', cr =$		$s' = \frac{ss - vv}{2v}$	$c = \sqrt{ss + vv}$
$s, cr, s', vr =$		$s' = \frac{2ss - cc}{2\sqrt{cc - ss}}$	$v = \sqrt{cc - ss}$
$s, vr, s, cr =$		$s = \sqrt{2s'v + vv}$	$c = \sqrt{2vv + 2s'v}$
$s', cr, s, vr =$		$s = \sqrt{s's' + \frac{1}{2}cc - \frac{1}{2}\sqrt{c^4 + 4c^2s's'}}$	$v = \sqrt{\frac{1}{2}cc + \frac{1}{2}s's' - \frac{1}{2}s'}$
$v, cr, s, s'r =$		$s = \sqrt{cc - vv}$	$s' = \frac{cc - 2vv}{2v}$

The

The ſame anſwered by Mr R. ROBINSON.

LET ſine $=s$
Coſine $=a$
Verſed ſine $=v$
Radius $=r$
Chord $=c$

Then we have $r^2=s^2+a^2$
$c^2=s^2+v^2$
$r=v+a$ or $v-a$, according as the arc is acute or obtuſe.

From which equations the following theorems are deduced, in which the upper of the double ſigns $\pm$ or $\mp$ belongs to the acute, and the under to the obtuſe arc.

	data	Theorems.
1		$r=\sqrt{s^2+a^2}$
2	s, a	$v=\sqrt{s^2+a^2}\mp a$
3		$c=\sqrt{2s^2+2a^2\mp 2a\sqrt{a^2+s^2}}$
4		$r=\frac{s^2+v^2}{2v}$
5	s, v	$a=\frac{\pm s^2\mp v^2}{2v}$
6		$c=\sqrt{s^2+v^2}$
7		$a=\mp\sqrt{r^2-s^2}$
8	s, r	$v=r\mp\sqrt{r^2-s^2}$
9		$c=\sqrt{2r^2\mp 2r\sqrt{r^2-s^2}}$
10		$a=\frac{\pm 2s^2+c^2}{2\sqrt{c^2-s^2}}$
11	s, c	$v=\sqrt{c^2-s^2}$
12		$r=\frac{c^2}{2\sqrt{c^2-s^2}}$
13		$s=\sqrt{v^2\pm 2av}$
14	a, v	$r=v\pm a$
15		$c=\sqrt{2v^2\pm 2av}$
16		$s=\sqrt{r^2-a^2}$
17	a, r	$v=r\mp a$
18		$c=\sqrt{2r^2\mp 2ar}$
19		$s=\sqrt{\frac{c^2-a^2\pm a\sqrt{2c^2+a^2}}{2}}$
20	a, c	$v=\frac{\sqrt{2c^2+a^2}\mp a}{2}$
21		$r=\frac{\sqrt{2c^2+a^2}\pm a}{2}$
22		$s=\sqrt{2rv-v^2}$
23	r, v	$a=+r\mp v$
24		$c=\sqrt{2rv}$
25		$s=\sqrt{c^2-v^2}$
26	v, c	$a=\frac{\pm c^2+2v^2}{2v}$
27		$r=\frac{c^2}{2v}$
28		$s=\frac{c}{2r}\sqrt{4r^2-c^2}$
29	r, c	$a=\frac{+2r^2\mp c^2}{2r}$
30		$v=\frac{c^2}{2r}$

Mr *Wm Jeſſon* of *Lincoln* has given a curious inveſtigation of the above, and deduced the ſame theorems, which I have omitted on purpoſe, to add ſomething concerning Fluents.

QUEST. LIV. *answered by* Philofluentimecanalgegeomaſtrolongo.

LET AE=x, EB=y, diameter of the circle CH=a, radius GD=r, CF=v, arc CD=z, ſine FD=m. Pl. 4. Fig. 84.

By the nature of the circle, $2rv-vv=mm$, $r\dot{v}-v\dot{v}=\dot{m}m$, and $\dot{v}=\frac{\dot{m}m}{r-v}$; alſo $r-v:r::\dot{m}:\dot{z}=\frac{r\dot{m}}{r-v}$. Then, by the nature of the curve AB, $y=v$, or $\dot{y}=\dot{v}=\frac{\dot{m}m}{r-v}$, and $x=z-m$, or $\dot{x}=\dot{z}-\dot{m}$ $=\frac{r\dot{m}}{r-v}-\dot{m}=\frac{v\dot{m}}{r-v}$.

But, by the nature of the curve of ſwifteſt deſcent, $\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}}$ is as the velocity in B; but $\sqrt{\dot{x}^2+\dot{y}^2}=\frac{\dot{m}\sqrt{m^2+v^2}}{r-v}$, and $\frac{\dot{x}}{\sqrt{\dot{x}^2+\dot{y}^2}}=\frac{v\dot{m}}{r-v}\times$ $\frac{r-v}{\dot{m}\sqrt{m^2+v^2}}=\frac{v}{\sqrt{m^2+v^2}}$; therefore the velocity is every where as $\frac{v}{\sqrt{m^2+v^2}}$, or as $\frac{y}{\sqrt{m^2+y^2}}$.

The ſame anſwered by CLEOPHILUS.

LET v repreſent the velocity required, s the element of the curve, that is, $\sqrt{\dot{x}^2+\dot{y}^2}$, and we ſhall have in general $2v\times\frac{\dot{s}\ddot{x}-\dot{x}\ddot{s}}{\dot{s}}=$ $\dot{v}\dot{x}$, whoſe fluent is log. $\frac{v}{a}=2$ log. $\frac{\dot{x}}{\dot{s}}$, or $v=\frac{a\dot{x}\dot{x}}{\dot{s}\dot{s}}$. In the preſent caſe, we have AP=y, PM=x=$s:\frac{a\dot{y}}{2\sqrt{ay-yy}}-\sqrt{ay-yy}$ or (which is the equation of the curve AMN) $\dot{x}=\frac{y\dot{y}}{\sqrt{ay-yy}}$, and $\dot{x}\dot{x}=$ $\frac{y^2\dot{y}^2}{ay-yy}$, and $\dot{s}\dot{s}=\frac{a\dot{y}\dot{y}}{ay-yy}$, which ſubſtituted in the general equation $v=\frac{a\dot{x}\dot{x}}{\dot{s}\dot{s}}$, we ſhall have $\frac{ay^2\dot{y}^2}{ay-yy}\div\frac{a\dot{y}\dot{y}}{ay-yy}$, or $v=y$. *Q. E. I.* Pl. 4. Fig. 85.

The

The ſame anſwered by the Propoſer.

BY the queſtion, $x=z-m$, $y=v$; Th. $\dot{x}=\dot{z}-\dot{m}$, and $\dot{y}=\dot{v}$. But, from the nature of the circle, $\dot{z}=\frac{\frac{a}{2}\dot{v}}{\sqrt{av-v^2}}$, $\dot{m}=\frac{a\dot{v}-2v\dot{v}}{\sqrt{av-v^2}}$; conſequently $\dot{x}=\frac{\frac{a}{2}\dot{v}}{\sqrt{av-v^2}}-\frac{a\dot{v}-v^2}{\sqrt{av-v^2}}=\frac{v^{\frac{1}{2}}\dot{v}}{\sqrt{a-v}}$, which ſince by hypoth. this is a curve of ſwifteſt deſcent, muſt be equal to $\frac{y^n\dot{y}}{\sqrt{a^{2n}-y^{2n}}}$ (an univerſal expreſſion for all curves of ſwifteſt deſcent, n denoting the exponent of the altitude fallen;) or, becauſe $y=v$, we have $\frac{v^{\frac{1}{2}}\dot{v}}{\sqrt{a-v}}=\frac{v^n\dot{v}}{\sqrt{a^{2n}-v^{2n}}}$, or $\frac{v^{\frac{1}{2}}}{\sqrt{a-v}}=\frac{v^n}{\sqrt{a^{2n}-v^{2n}}}$, which it's evident, can only be equal when $n=\frac{1}{2}$; conſequently the law of velocity, in this caſe, is in a *ſubduplicate* ratio of the altitude fallen.

QUESTION LIV. *anſwered by Mr* NIC. OATS, *the Propoſer.*

Pl. 2. Fig. 36. PUT $z=AE$, $c=.7854$; then $x^m : y^{m+n} :: z^m : \frac{z^m y^{m+n}}{x^m}$, and $\overline{\frac{z^m y^{m+n}}{x^m}}^{\frac{2}{m+n}} \times c \times x-z = \text{maximum}$. In *fluxions* $\frac{2}{m+n}\times \frac{y^{m+n}z^{m-1}\dot{z}}{\overline{\frac{z^m y^{m+n}}{x^m}}^{\frac{1}{m+n}}}-\frac{2}{m+n}\times\frac{y^{m+n}z^m\dot{z}}{\overline{\frac{z^m y^{m+n}}{x^m}}^{\frac{1}{m+n}}}-\frac{\overline{z\times z^m y^{m+n}}^{\frac{2}{m+n}}}{x^m}=0.\times\frac{2}{m+n}$

$y^{m+n}z^{m-1}\dot{z}-\frac{2}{m+n}y^{m+n}z^m\dot{z}-y^{m+n}z^m\dot{z}=0. \div \frac{2}{m+n}x-\frac{2}{m+n}z-z=0$; hence $\frac{m+n+2}{m+n}z=\frac{2}{m+n}x$, and $z=\frac{2}{m+n+2}x$, the value of AE required.

Now if $m=1$, $n=3$, as in *number* I. *queſtion* 15. *page* 29. then $z=\frac{2}{6}x=10$, and EP=20. If the figure be a cone, then $m=1$, $n=0$, and $z=\frac{2}{3}x$; conſequently $PE=\frac{1}{3}AP$. If the the *Apollonian* parabola, then $m=1$, $n=1$, and $z=\frac{2}{4}x$. If the ſecond parabola, then $m=2$, $n=1$, and $z=\frac{2}{5}x$, *&c. &c. &c.*

QUESTION IV. *in* N° I. *anſwered by Mr* EDWARD HAUXLEY.

GIVEN $x^{xx}=y$; required the value of x when y is a *minimum*?

SOLU-

SOLUTION.

The ſenſe I take x^{xx}, or $\overline{x}|^{\overline{x}|^{x}}$ or $\overline{x}|^{xx}$ in, is to find ſuch a number x, which involved into a power, whoſe index is expreſſed by x, thus noted $\overline{x}|^{x}$, and this power $\overline{x}|^{x}$ again involved into a power whoſe index is expreſſed by x, ſo that the laſt involution or $\overline{x}|^{\overline{x}|^{x}}$ may be the leaſt poſſible; and this I ſuppoſe is the ingenious Propoſer's meaning. Then ſince $\overline{x}|^{xx}$ is to be a *minimum*, it is evident that its hyperbolic log. which offers itſelf firſt, will alſo be a *minimum*. Therefore the fluxion of the log. of x^{xx} is $2x\dot{x}\times Lx+\frac{\dot{x}}{x}\times x^{2}=0$, or $2x\dot{x}\times Lx+x\dot{x}=0$; and divided by $x\dot{x}$, we have $2Lx+1=0$. Th. $Lx=-\frac{1}{2}$, which call y, then will the number anſwering be had by this general ſeries, *viz.* $1+y+\frac{y^{2}}{2}+\frac{y^{3}}{2.3}+\frac{y^{4}}{2.3.4}+\frac{y^{5}}{2.3.4.5}$ &c. $=.6065306$ correct to a decimal; which ſeries was found out by Sir *Iſaac Newton*, and is in his ſecond letter to Mr *Leibnitz*, as appears from *pag.* 179. of the *Commercium Epiſtolicum*, now tranſlating into *Engliſh*.

For			
$+1=$		$+$ 1.0000000	
$+y=$	$-.5$		
$+\frac{y^{2}}{2}=$		$+.125$	
$+\frac{y^{3}}{2.3}=$	$-.02083333$		
$+\frac{y^{4}}{2.3.4}=$		$+.0026041666$	$=.60653067$
$+\frac{y^{5}}{2.3.4.5}=$	$-.00026041$		
$+\frac{y^{6}}{2.3.4.5.6}=$		$+$ .0000217013	
$+\frac{y^{7}}{2.3.4.5.6.7}=$	$-.00000155$		
$+\frac{y^{8}}{2.3.4.5.6.7.8}=$		$+.0000000968$	

Th. $x=.60653067$. To find the log. of .60653067, let $1-.3934694=1-x=.6065306$; hence $x=-.3934694$, conſequently the log. of $1-x$ will, by *prob.* 2. *pag.* 130 of Mr *Emerſon's Doctrine of Fluxions*, be had by this ſeries $=x-\frac{x^{2}}{2}-\frac{x^{3}}{3}-\frac{x^{4}}{4}-\frac{x^{5}}{5}-\frac{x^{6}}{6}$ &c. $=-.49999$, or $-.5$. *Q. E. D.*

Th. x^{xx} is $=.832$ the *minimum* required.

New QUESTIONS, *to be anſwered in our next.*

QUESTION LVI. *By Mr* JOHN HAMMOND.

IF the length of a pendulum of a clock, which vibrateth ſeconds, be 39.13 inches (ſee *pag.* 71.) what ſhall be the length of that pendulum, that ſhall vibrate ten times as often in a minute as it contains inches in length?

QUESTION LVII. *By* MERCATOR SCISSOR.

A Gentleman who had *n* children, each a year older than the next younger, determined to new cloath them, and to beſtow as many yards of lace on the trimming of the youngeſt's ſuit as he was years old; as many on the ſecond as the ſum of his and the youngeſt's age; as many on the third as the ſum of their three ages, *&c.* and agreed with the taylor to pay him in pence the product of each child's age by the number of yards of lace on his cloaths, for the making of each ſuit. Now the work being done, the taylor's bill amounted to *m* pounds; required the age of each child?

QUESTION LVIII. *By Mr* NICH. FARRER.

IN a geometrical progreſſion is given the ſecond term = 1, and the firſt or leaſt term is to the number of terms as 2 to 15, and the ſum of the terms to the common ratio as 211 to 36; it's thence required to determine the terms of the ſeries.

QUESTION LIX.

ONE evening, being with ſome lovers of beer, I was deſired to tell the company, in how many days the landlord would be paid for his new ſilver tankard, reckoning its value to be 15*l.* fill'd ten times a day, at 4*d.* $\frac{1}{2}$ a time, or the value of 3 pints; and allowed 4*l. per cent. per ann.* compound intereſt, for the uſe of his money. I deſired ſome time to conſider of it, and, before the company parted, took the following meaſures in inches; *viz.* diameter at top = 4.625, height = 5.25, diagonal = 7.25 inches; and, for the reſt, deſire the aſſiſtance of thoſe gentlemen, who have leiſure and inclination to help a friend.

QUESTION LX. *By Mr* JOHN WILLIAMS, *jun.*

IF the legs of a right-angle plain triangle be $3x$ and x^3, and the line that biſects the right angle be x^2 chains, what is the content in acres?

QUE-

QUESTION LXI. *By Mr* THOMAS SIMPSON.

IN a plain triangle, ABC, are given AB+BC+AC=78; the perpendicular BD let fall from the vertex =8.4324, and the vertical angle ABC=126° 52′ 12″. *Quere* the triangle by a geometrical conſtruction?

QUESTION LXII. *By Mr* WILLIAM CHAPPLE.

LET the two diameters of the circumſcribing and inſcrib'd circles of a triangle be 94 and 40, and one of its ſides =90; *Quere* the other two ſides?

QUESTION LXIII. *By the reverend* RICH. ROBERTS.

GIVEN the area of a circular ſegment, ABD, =4472.96 (leſs than a ſemicircle:) this ſegment is divided by a right line, BC, perpendicular to its baſe, or chord AD, in ſuch ſort, that the ſegment's greater part, CBD, =4253.98, and the leſſer part, ABC, =218.98; alſo AC : CD :: 1 : 7. Required the baſe, or chord, AD, the parts of the baſe AC, CD, the perpendicular BC, the ſegment's altitude, and diameter of the circle.

QUESTION LXIV. *By Mr* JOHN BULMAN.

GIVEN the axis of a globe =20 inches; required the dimenſions and content of the leaſt cone, that is poſſible to circumſcribe it, by a ſimple equation.

QUESTION LXV. *By Mr* ROBERT ROBINSON.

REquired the dimenſions of the greateſt ſemi-ellipſis that can be inſcribed in a triangle, whoſe ſides are 21, 17, and 10?

QUESTION LXVI. *By Mr* J. POWLE.

REquired the area of the greateſt parallelogram, that can be inſcrib'd in the *Catenaria*, whoſe *abſciſſa* =50, and correſponding ordinate =100?

QUESTION LXVII. *By* CLEOPHILUS.

TO find the nature of the curve whoſe ſubtangent is $\frac{\overline{m+n}\times ax+xx}{\overline{ma+m+n}\times x}$?

QUES-

QUESTION LXVIII. *By Mr* EDWARD HAUXLEY.

DR *Clark*, in his *Latin* edition of Sir *Iſaac Newton*'s Optics, has proved, that the rays of light paſſing thro' a rarer medium into a denſer are refracted *ad perpendiculum*, conſequently thoſe rays paſſing through our atmoſphere are tranſmitted to us in the curve of the common *cycloid*, and not in ſtrait lines. Therefore, on this *theſis*, a ray of light paſſing through the atmoſphere at 5^h 50^m, in the morning, on *April* 30, is 512.87654387 *Engliſh* miles; and ſuppoſing the atmoſphere's height at that time =47.0876543 miles, and the length of a parallelogram that might be inſcribed in the cycloid (part of which curve is the ray of light) being =1404.5 miles, and breadth 400 miles; from thence it is required to find the latitude of the place, with the inveſtigation.

QUESTION LXIX. *By* SOL PRIMUS.

SUppoſe, on the 11^{th} *June* 1746, the atmoſphere to be equally free from evaporations, or exhalations, from the earth, both at *Port Royal* in lat. 18° 20′, and at *Exeter*, lat. 50° 50′. Quere the *ratio* of the ſun's rays of heat, at both thoſe places, to each other at 10^h M. at noon, and at 3^h P. M.

QUESTION LXX. *By* TOM PROBLEM.

Pl. 4. Fig. 86. TO find the law of centripetal force, by which a body (projected from A in a direction parallel to CD) will deſcribe *Tſhirnhauſe*'s *quadratrix?*

QUESTION LXXI. *By* HERMES TRISMEGISTUS.

SUppoſe the tranſverſe axis of the earth's orbit 20000, and the eccentricity to be 173 of thoſe parts; and let us imagine the law of attraction to be changed from the ſquare to the cube of the diſtance inverſely, when the earth arrives at her loweſt *Apſis*. 'Tis required to find her diſtance from the ſun, after ſhe hath moved the ſpace of two years in her new orbit; alſo the number of gyrations ſhe will have perform'd round the ſun in that time; and how many ſhe can poſſibly make, were her motion in this orbit to be continued for ever.

Of finding of Fluents, &c.

THE finding of *Fluents* being by far the moſt difficult, as well as the moſt uſeful, part of the art of *Fluxions*, for the ſake of thoſe

thoſe who are leſs ſkilled in theſe abſtruſe matters, I ſhall from time to time, as I have room, illuſtrate the method of finding *Fluents* by that excellent table in Mr *Emerſon*'s book of *Fluxions*, which I am perſuaded none of my correſpondents will be without. This method of finding *Fluents* being ſo eaſy and general, I hope the giving examples in all the principal caſes will be both uſeful and acceptable to our readers, as it ſhews the cleareſt and readieſt methods of ſolving what the noble inventer Sir *Iſaac Newton* was pleaſed to call *Moleſtiſſimum et omnium difficillimum problema*, "the moſt difficult and troubleſome of all problems."

Note, *In comparing a given Fluxion with the table, the* radical *quantity muſt be compared with the* radical *quantity; the* index *of the variable quantity within the* radical, *with the* index *within the* radical; *and the* index *without, with the* index *without the* radical, *&c. ſo that by making equations between like quantities, the particular values of each may be diſcovered, in order to find the* Fluent *required.*

EXAMPLE I.

Let $ax^3\dot{x} - \frac{13b\dot{x}}{x} = \dot{z}$, be propoſed.

1. To find the *Fluent* of $ax^3\dot{x}$; comparing this with *Form* 1. you will have $z = x$, $n - 1 = 3$, or $n = 4$, and therefore the *Fluent* of $x^3\dot{x}$ is $= \left(\frac{x^n}{n} = \right) \frac{x^4}{4}$, and conſequently the *Fluent* of $ax^3\dot{x} = \frac{ax^4}{4}$.

2. For the *Fluent* of $-\frac{13b\dot{x}}{x}$; comparing this with *Form* 2. $z = x$, and the *Fluent* $x^{-1}\dot{x}$ or $\frac{\dot{x}}{x}$ is $= \mathrm{L} \times \log. x$, conſequently the *Fluent* of $-\frac{13b\dot{x}}{x}$ is $= -13b\mathrm{L} \times \log.$ of x, that is, $= -13b \times 2.302585 \log. x$

Hence therefore the whole *Fluent* is $\frac{ax^4}{4} - 13b \times 2.302585 \log. x = z$. Where *note* that the logarithms here uſed are the common tabular logarithms; but if you have a table of hyperbolic logarithms, it will be better to uſe that, and then you muſt leave out the number L, or 2.302585, which only ſerves to reduce the one ſort to the other.

EXAMPLE II.

To find the *Fluent* of $d\sqrt{aa-bby^3}\times y^2\dot{y}-\overline{aa+bby^2}\,|^{\frac{1}{2}}y\dot{y}+\frac{a\dot{x}}{\sqrt{a-x}}-\frac{2bx\dot{x}}{\sqrt{aa-xx}}$.

1. For the *Fluent* of $d\sqrt{aa-bby^3}\times y^2\dot{y}$; comparing this with *Form* 3. we have $z=y$, $n=3$, $\mu=\frac{1}{2}$, $\alpha=aa$, $\beta=-bb$; whence the *Fluent* $\overline{aa-bby^3}\,|^{\frac{1}{2}}y^2\dot{y}=\left(\frac{\overline{\alpha+\beta z^n}\,|^{\mu+1}}{\mu+1.n\beta}=\right)\frac{\overline{aa-bby^3}\,|^{\frac{3}{2}}}{\frac{3}{2}\times-3bb}=-\frac{2}{9bb}\times\overline{aa-bby^3}\,|^{\frac{3}{2}}$; and the *Fluent* of $d\sqrt{aa-bby^3}\times y^2\dot{y}=-\frac{2d}{9bb}\times\overline{aa-bby^3}\,|^{\frac{3}{2}}$.

2. For $\overline{aa+bbyy}\,|^{\frac{1}{2}}\times y\dot{y}$; by *Form* 3. $z=y$, $n=2$, $\mu=\frac{1}{2}$, $\alpha=aa$, $\beta=bb$, and *Fl.* $\overline{aa+bby^2}\,|^{\frac{1}{2}}y\dot{y}=\frac{\overline{aa+bbyy}\,|^{\frac{3}{2}}}{\frac{3}{2}\times 2bb}$, and *Fluent* $-\overline{aa+bbyy}\,|^{\frac{1}{2}}y\dot{y}=-\frac{3}{8}\times\frac{\overline{aa+bbyy}\,|^{\frac{3}{2}}}{bb}$.

3. For $\frac{a\dot{x}}{\sqrt{a-x}}$, or $\overline{a-x}^{-\frac{1}{2}}\times a\dot{x}$; by *Form* 3. $z=x$, $n=1$, $\mu=-\frac{1}{2}$ $\alpha=a$, $\beta=-1$, and *Fluent* $\overline{a-x}^{-\frac{1}{2}}\dot{x}=\frac{\overline{a-x}^{\frac{1}{2}}}{\frac{1}{2}\times-1}=-2\times\sqrt{a-x}$, and *Fluent* $\frac{a\dot{x}}{\sqrt{a-x}}=-2a\sqrt{a-x}$.

4. For $\frac{-2bx\dot{x}}{\sqrt{aa-xx}}$, by *Form* 3. $z=x$, $n=2$, $\mu=-\frac{1}{2}$, $\alpha=aa$, $\beta=-1$, and *Fluent* $\frac{x\dot{x}}{\sqrt{aa-xx}}=\frac{\overline{aa-xx}^{\frac{1}{2}}}{\frac{1}{2}\times-2}=-\sqrt{aa-xx}$; and multiplying by $-2b$, *Fluent* $\frac{-2bx\dot{x}}{\sqrt{aa-xx}}=2b\sqrt{aa-xx}$; whence the whole *Fluent* required is $-\frac{2d}{9bb}\times\overline{aa-bby^3}\,|^{\frac{3}{2}}-\frac{3}{8}\times\frac{\overline{aa+bbyy}\,|^{\frac{3}{2}}}{bb}-2a\sqrt{a-x}+2b\sqrt{aa-xx}$.

EXAMPLE III.

Let $\frac{4aax\dot{x}}{a^3b+bbxx}-\frac{8\dot{y}}{b-ay}$ be propoſed.

1. For

1. For $\frac{4aax\dot{x}}{a^3b+bbxx}$; compare this with *Form* 4. and $z=x$, $n=2$, $\alpha=a^3b$, $\beta=bb$, and *Fluent* $\frac{x\dot{x}}{a^3b+bbxx}=\left(\frac{L}{n\beta}\times\text{log. of }\overline{\alpha+\beta z^n}=\right)$ $\frac{L}{2bb}\times\text{log. }\overline{a^3b+bbxx}$; and *Fluent* $4aa\times\frac{x\dot{x}}{a^3b+bbxx}=\frac{2aa\times 2.30258}{bb}$ $\times\text{log. }\overline{a^3b+bbxx}$.

2. For $\frac{-8\dot{y}}{b-ay}$; by *Form* 4. $z=y$, $n=1$, $\alpha=b$, $\beta=-a$, and *Fluent* $\frac{\dot{y}}{b-ay}=\frac{L}{-a}\times\text{log. }\overline{b-ay}$, and *Fluent* $\frac{-8\dot{y}}{b-ay}=2.3025\times\frac{8}{a}\times\text{log. }\overline{b-ay}$. Th. the *Fluent* required is $2.30258\times\frac{2aa}{bb}\times\text{log. }\overline{a^3b+bbxx}+2.3025\times$ $\frac{8}{a}\times\text{log. }\overline{b-ay}$; or, if you please, the *Fluent* required is $=\frac{2aa}{bb}\times$ hyperbolic logarithm of $\overline{a^3b+bbxx}+\frac{8}{a}\times\text{hyp. log. }\overline{b-ay}$.

EXAMPLE IV.

To find the *Fluent* of $\frac{6\dot{x}}{aa+xx}$; compare this with *Form* 5. and $n=2$, $\alpha=aa$, $\beta=1$; and *Fluent* of $\frac{\dot{x}}{aa+xx}=\left(\frac{2N}{n\sqrt{\alpha\beta}}\times\text{degrees whose}\right.$ natural tangent is $\sqrt{\frac{\beta z^n}{\alpha}}=\left.\right)\frac{2N}{2\sqrt{aa}}\times$ degrees in the arc whose tangent is $\sqrt{\frac{xx}{aa}}$ or $\frac{x}{a}$; therefore the *Fluent* of $\frac{6\dot{x}}{aa+xx}=\frac{6N}{a}\times$ degrees whose tangent is $\frac{x}{a}$. Now if you desire to work it into numbers, suppose, for example, that $a=20$, $x=43$, then $\frac{x}{a}=2.15$ a tangent; look for it in the table of *natural tangents*, and you will find it belong to 65 deg. and 3 min. nearly, that is, to something more than $65\frac{3}{60}$ degrees. And to find the degrees exactly, say

As the difference of the two nearest tangents in the table (2.1510378—2.1494020=)	.0016358
To the difference between the lesser and given tangent (2.15—2.1494020=)	.0005980

So

So is 1 minute 1.0000000
To a proportional part to be added .3657

Therefore the number of degrees belonging to the tangent 2.15 is $65\frac{3.3657}{60}$, or 65.05609; Th. the Fluent of $\frac{6\dot{x}}{aa+xx}$ is $=\frac{6N}{a}\times$. $65.05609=\frac{6\times.0174533\times65.05609}{20}=.34063$.

But if you will ſuppoſe x to be infinite, then $\frac{x}{a}$ or $\frac{x}{20}$, will be infinite, and the degrees belonging to the infinite tangent will be 90°, and therefore in this caſe the *Fluent* of $\frac{6\dot{x}}{aa+xx}$ will be $\frac{6\times.01745\times90}{20}=.47124$.

EXAMPLE V.

Let the Fluxion propoſed be $\frac{a\dot{x}}{ax^{\frac{1}{2}}+6x^{\frac{1}{2}}}$, or $\frac{ax^{-\frac{1}{2}}\dot{x}}{a+6x}$.

Compare this with *Form* 5. and $z=x$, $n=1$, $\alpha=a$, $\beta=6$, and *Fluent* $\frac{x^{-\frac{1}{2}}\dot{x}}{a+6x}=\left(\frac{2N}{n\sqrt{\alpha\beta}}\times\right.$ degrees whoſe tangent is $\left.\sqrt{\frac{\beta z}{\alpha}}=\right)\frac{2N}{\sqrt{6a}}\times$ degrees whoſe tangent is $\sqrt{\frac{6a}{a}}$, and *Fluent* $\frac{ax^{-\frac{1}{2}}\dot{x}}{a+6x}=\frac{2Na}{\sqrt{6a}}\times$ degrees whoſe tangent is $\sqrt{\frac{6a}{a}}=.017453\sqrt{\frac{a}{1}}a\times$ degrees in the arc whoſe tangent is $\sqrt{\frac{6x}{a}}$.

EXAMPLE VI.

To find the *Fluent* of $\frac{7bx\dot{x}}{d^4+x^4}$.

By *Form* 5. $z=x$, $n=4$, $\alpha=d^4$, $\beta=1$; and *Fluent* $\frac{x\dot{x}}{d^4+x^4}=\frac{2N}{2\sqrt{d^4}}\times$ degrees whoſe tangent is $\sqrt{\frac{x^4}{d^4}}=\frac{N}{dd}\times$ degrees whoſe tangent is $\frac{xx}{dd}$; and *Fluent* $\frac{7bx\dot{x}}{d^4+x^4}=\frac{7bN}{dd}\times$ degrees in the arc whoſe tangent is $\frac{xx}{dd}$

Two

Two new meaſures ſubſervient to mechanical practice; by M. C. Polhem. *Done from the* Latin *of the* Acta Literaria Sueciæ.

IT is allowed by all, who are ſkilled in theſe matters, that the uſe of ſpeculative mathematics is twofold; namely one, that it is excellently ſubſervient to various arts, which either relieve the neceſſities of human life, or adorn it, and promote its conveniences; the other uſe, and that none of the meaneſt, is, that its ſtudy endues the underſtanding with a certain ſenſe, and, as it were, with an habit of evidence, whereby it may proceed with greater eaſe and ſucceſs in underſtanding and cultivating the other ſciences. As to the firſt, I take it that both the learned and unlearned are agreed, but not ſo as to the laſt; for there are ſome who imagine that all application to the higher parts of mathematics is uſeleſs, on this ſcore, namely, as they cannot ſee what theſe barren ſpeculations, as they call them, and only recommendable from their difficulty, can conduce towards the increaſe of arts. And, tho' I am of opinion that theſe men may be ſufficiently convinced of their error by various reaſonings, and manifold experiments, yet I think there can ſcarce a ſtronger argument be made uſe of to oblige ſuch people to give the mathematical ſciences their due value, than by ſhowing that even theſe more abſtracted truths may ſeveral ways be made ſubſervient to the purpoſes of common life. Tho' this has been done very diſtinctly and learnedly by many, yet I thought it worth the while to adduce ſome things from the higher parts of the mathematics, whereby mechanical practice may at leaſt be facilitated, if not improved. At preſent it ſhall ſuffice to deſcribe two meaſures, or feet, which I have deviſed, together with their uſe.

The firſt of theſe meaſures ſeems, from its uſe, not to be improperly called a *minute-foot*; for it has that ratio to the length of a ſimple pendulum, each of whoſe vibrations is perform'd in a ſecond of time, which 5 has to 18; conſequently, ſince, by *Huygens*'s experiments, the length of ſuch a pendulum is 3 foot $8\frac{1}{2}$ lines *Paris* meaſure, the minute-foot will be 10 inches $2\frac{1}{3}$ lines, which, in reſpect of our *Swediſh* foot, ſuppoſing the ratio of 1000 to 1099, which is that of the *Swediſh* and *Pariſian* foot, makes 9 inches, 3 lines, $2\frac{3}{4}$ points; for this meaſure, divided in the uſual manner into its decimal, centeſimal and milleſimal parts, is of that nature, that if there be made a pendulum equal in length to that foot, the num-

ber of thirds, wherein that pendulum performs each vibration, will be $=\sqrt{1000}$: whence it follows, that if the length of any other pendulum be had in the milleſimal parts of the minute-foot, and the ſquare root be thence extracted, I ſay, it follows, that that ſquare root expreſſes the number of thirds, in which that other pendulum vibrates.

The uſe of this meaſure is as follows; if the time be required in which any effect is produced, as for inſtance, the time in which the wheel of any machine performs one revolution, take a pendulum of any length, which let be given in the milleſimal parts of the minute-foot, and reckon the vibrations anſwering to one revolution; and then from the length of the pendulum, thus expreſs'd, extract the ſquare root; this multiplied by the number of vibrations, ſhews the number of thirds anſwering to one revolution.

Suppoſe it be a great water-wheel, which revolves once, while a pendulum (whoſe length let be two minute-feet and 2 inches, or 4400 points) performs four vibrations, the ſquare root of 4400 is 65 very nearly, which multiplied by 4, the number of vibrations, gives 260 thirds, or 4 minutes, 20 ſeconds, for the time of one revolution.

It appears, without my hinting it, that the ſame method may likewiſe be commodiouſly applied for diſcovering the quickneſs of the pulſe in ſick people, for finding the velocities in found, and the like.

The other meaſure, which I have here propoſed to deſcribe, I, for diſtinction ſake, uſually call the *mechanical foot*; it is equal to the height which a body, falling freely from a ſtate of reſt, runs over in the fourth part of a ſecond, that is, 1 foot, 3 lines, $7\frac{1}{2}$ points *Swediſh* meaſure.

By help of this meaſure, the height of the fall, from the time given, as alſo the time, from the height given, may be eaſily found; for, if the time ſpent in falling be t ſeconds, the ſpace run over will be $=16tt$ of the mechanical foot; and, reciprocally, if the height, which a heavy body in its fall paſſes over, be put d foot, the time will be $=\frac{1}{4}\sqrt{d}$ ſeconds. The ſame method may be applied for computing the times of the ranges of bombs, as alſo for finding the quantities of water burſting out of flood-gates, water-mills, wears, *&c.*

If the amplitude of the range of a bomb, at the elevation of 45 degrees, be a feet, the time of the range will be $=\frac{1}{4}\sqrt{a}$ ſeconds; if the ſine of the angle of elevation of the mortar be put $=m$, the radius

dius $=r$, and the amplitude of the range of 40 degrees $=a$ feet, the time will be $=\frac{m\sqrt{a}}{r\sqrt{2}}$.

If the horizontal diſtance of the mark from the mortar be put $=b$ feet, the height of the mark above the level of the mortar $=\pm b$ feet, and the range of 45 degrees, the time will be $\frac{b}{4\sqrt{a}}$ ſeconds, ſuppoſing the horizontal amplitude is a feet.

Suppoſing ſtill the ſame things, if the angle of the elevation be not half a right angle, but its ſine given $=b$, and the radius $=r$, then the time of the range will be $=\frac{m\sqrt{a}}{2r\sqrt{2}}+\frac{\frac{1}{4}\sqrt{2m^2a}}{r^2}+b$.

The quantity of water, or any other liquor, in one ſecond of time, running out at an aperture, whoſe area is aa feet, is $8aa\sqrt{t}$ cubic feet, ſuppoſing the height of the preſſure $=t$. If the wheel, whoſe mean periphery is p, be turn'd by water, and the altitude of the preſſion of the water be $=t$, and the height of its fall m, each revolution will be perform'd in the ſpace of $\frac{p}{8\sqrt{m+t}}$ ſeconds.

As ſome particular caſes of the 55th *Queſtion*, *page* 116, have engaged the attention of the curious in ſeveral of their mathematical performances, 'tis hoped a *general ſolution* thereof may be acceptable to our readers; and as all curves of that nature have the ſame inclination towards their axes, the general expreſſion here found can admit of no ambiguity, being applicable to any particular caſe whatſoever. This did not come to hand till the printing of the laſt ſheet, which is the reaſon for inſerting of it here.

GENERAL SOLUTION; *by Mr* NICH. OATS.

GIVEN $ax^m=y^{m+n}$ (the equation of the curve.) Let $z=AE$; Pl. 2. Fig. 36. and put $\frac{m+n}{m}=v$. Then $x:y^v::z:\frac{zy^v}{x}$; and if $c=3.1416$, then, by the queſtion, $\overline{\frac{zy^v}{x}\Big|^{\frac{z}{v}}}\times\overline{x-z}\times c=$ *maximum*, which put into *Fluxions*, and exterminating y^v and c, we have $\frac{2}{v}\frac{\dot{z}}{\overline{\frac{z}{x}\Big|^{\frac{v-2}{v}}}}-\frac{2}{v}\frac{z\dot{z}}{x\times\overline{\frac{z}{x}\Big|^{\frac{v-2}{v}}}}-\dot{z}$

$-\dot{z}\times\frac{z}{x}\Big|^{\frac{2}{v}}=0$; multiplying by $\frac{z}{x}\Big|^{\frac{v-2}{v}}$, and expunging $\dot{x}$, gives $\frac{2}{v}-\frac{2}{v}\times\frac{z}{x}-\frac{z}{x}=0$; hence $z=\frac{2}{2+v}x$, and by reſtoring $\frac{m+n}{m}$, $z=\frac{2m}{3m+n}x$. $=AE$. *Q. E. I.*

Or $\frac{m+n}{3m+n}x=EP$, the altitude of the greateſt cylinder. Now if m be ſuppoſed infinite in reſpect to n, then the whole generated ſolid will be a cone, and $\frac{m+n}{3m+n}x=\frac{x}{3}$.

In *Queſtion* 15 *Miſc. Curioſa*, Numb. I. where $m=1$, $n=3$, then $\frac{m+n}{3m+n}x=\frac{4}{6}x=20$.

See Mr *Robinſon*'s Anſwer, *page* 63. Numb. 2.

By a like proceſs $\frac{m+n}{2m+n}x$ is the altitude of the greateſt parallelogram in ſuch curves. NIC. OATS.

We here alſo take notice that, beſides the above, Mr Oats *anſwered queſt.* 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, *and propoſed a new problem, which came too late.*

ERRATA.

Page 72. queſtion 30. read $y^2+x^4-x=2yx^2$.
106. line 3. read $9=dd$, and not q.

We here obſerve that the ſolution to queſtion 5. by Mr *William Chapple*, was communicated by him to the Author of the *Syntagma Matheſios* long before the publication of Numb. I. See *page* 55.

The End of Number IV.

Plate

Fig

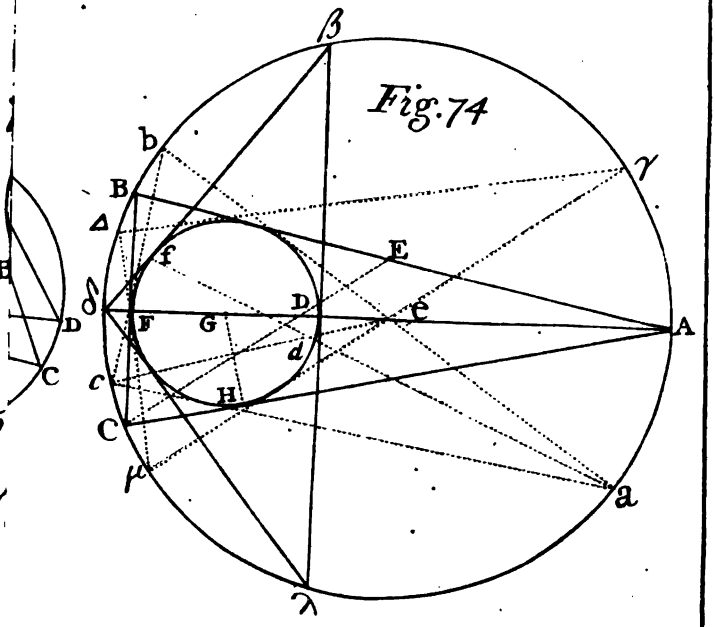


Fig.74

Fig 75

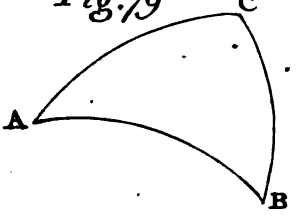


Fig.79

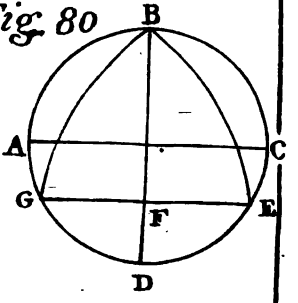
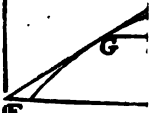


Fig. 80

Fig 81



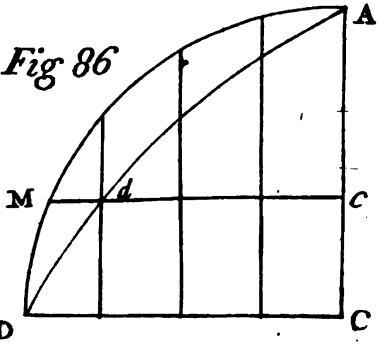


Fig 86

Fig.83

Miſcellanea Curioſa, &c.

NUMBER V.

A LETTER *from Mr* Edward Hauxley, *teacher of the mathematics in the royal navy, concerning a very curious uſeful aſtronomical problem for finding the place of a comet*, &c.

SIR,

I HERE ſend you a calculation at large for finding the place of a comet, exemplified in that which appeared in the year 174$\frac{1}{4}$; the obſervations were made in the *Mediterranean*. I flatter myſelf it will meet with a place in your next *Miſcellanea Curioſa Mathematica*, which will oblige SIR,

May 20, 1746. *Your very humble ſervant*, EDW. HAUXLEY.

A PROBLEM.

BY having the latitude and longitude of four fixed ſtars, not lying in the arc of the ſame great circle, or apparent right line, nor in the form of an apparent ſquare or oblong; to find the latitude and longitude of that point of the heavens, where two apparent right lines, each paſſing through two ſtars, interſect each other.

In this uſeful problem are two caſes.

1ſt. When the apparent right lines paſs laterally; and

2d. When they paſs diagonally.

CASE I. *Exemplified in the late Comet.*

On *February* 2, 174$\frac{3}{4}$, the *nucleus* of the comet appear'd, at *Hieres Bay* in the *Mediterranean* near *Toulon*, in a right line with *Lucida Pleiadum*, and the bright ſtar in the head of *Aries*; alſo in another right line with *Andromeda*'s head, and the bright ſtar in the right foot; from whence the latitude and longitude of the comet may be determined, as follows.

PREPARATION.

Pl. 5. Fig. 87. Let A repreſent the north, and C the ſouth poles of the ecliptic BR; and the primitive circle ABRC the ſolſtitial colure, or circle of longitude paſſing through *Cancer* and *Capricorn*.

			Latitude		Longit.
And let D repreſent	*Lucida Pleiadum*	4° 01′	N.	♉	26 25
And let E repreſent	The bright * in the *head* of ♈	9 57	N.	♉	4 05
And let F repreſent	*Andromeda*'s right foot	35 21	N.	♉	8 51
And let G repreſent	*Andromeda*'s head	25 41	N.	♈	10 44

The ſtars D, E, F and G, being laid down in the projection by their latitudes and longitudes, let great circles be drawn through D, E, F and G; and where they interſect each other, as at the point H, repreſents the place of the comet in the projection; from whence the reaſon of the following calculation will appear manifeſt.

ILLUSTRATION *and* CALCULATION.

I.

In the oblique-angled ſpherical triangle, ADF, are given

The { Leg AD = 85° 59′, the co-lat. of *Lucida Pleiadum*.
Leg AF = 54 39, the co-lat. of *Andromeda*'s right foot.
∠DAF = 17 34, their diff. of longitude from each other.

From whence is eaſily found the angle ADF = 25° 13′, the ∠AFD = 148° 37′, and ſide DF = 35° 18′.

II.

In the oblique-angled ſpherical triangle ADE, we have

Given { AD = 85° 59′, the co-lat. of *Lucida Pleiadum*.
AE = 80 03, the co-lat. of the bright * in he head of ♈.
∠EAD = 22 20, their diff. of longitude from each other.

From whence I find ∠ADE = 73° 49′, the ∠AED = 103° 25′, and ſide DE = 22° 56′, the diſtance of the ſtars D and E.

III.

In the oblique-angled triangle AFE, we have

Given { AE = 80° 03′, the co-lat. of the bright ſtar of ♈.
AF = 54 39, the co-lat. of *Andromeda*'s right foot.
∠FAE = 4 46, their diff. longitude.

From whence I find the ∠AFE = 169° 10′, the ∠AEF = 8° 58′, and FE = 25° 47′.

IV.

In the oblique-angled ſpheric triangle DEF, we have now

Given

Given { DF = 35° 18′, DE = 22 56, FE = 25 47 } Hence I find { ∠DFE = 42° 14′, ∠DEF = 94 32, ∠FDE = 48 37 }

V.

In the oblique-angled triangle AEG, we have

Given { AE = 80° 03′, the co-lat. of the bright ſtar of ♈. AG = 64 19, the co-lat. of *Andromeda*'s head. ∠EAG = 23 21, their diff. of longitude. }

From whence I find ∠AEG = 51° 30′, ∠EGA = 121° 12′, and the ſide EG = 27° 21′.

VI.

In the oblique ſpherical triangle AFG, we have

Given { AF = 54° 39′, the co-lat. of *Andromeda*'s right foot. AG = 64 19, the co-lat. of *Andromeda*'s head. ∠GAF = 28 07, their diff. of longitude. }

From whence I find ∠AGF = 67° 06′, ∠AFG = 97° 46′, and the ſide FG = 24° 40′.

VII.

In the oblique-angled ſpherical triangle EFG, we have

Given { EF = 25° 47′, FG = 24 40, GE = 27 21 } Hence I find { ∠EFG = 67° 30′, ∠EGF = 61 00, ∠FEG = 57 04 }

VIII.

From	360° 00′; and from 180° 00′
Subt. ∠FGE + 180° =	241 00. Subt. ∠DEF + ∠FEG = 151 36
Remains the ∠EGH =	119 00. Remains ∠GEH = 28 24

IX.

We have now given in the oblique-angled ſpherical triangle EGH.

The { ∠GEH = 28° 24′, ∠EGH = 119 00, Side EG = 27 21 } Hence I find { GH = 20° 54′, the diſt. of the comet from *Andromeda*'s head. }

X.

Laſtly, we have now given in the oblique-angled ſpherical triangle AGH,

The { Leg AG = 64° 19′, the co-lat. of *Andromeda*'s head. Leg GH = 20 54, the diſt. of the comet from *Androm.* head ∠HGA = 112 54, the included angle. }

From whence I find the ∠HAG = 20° 12′, the comet's difference of longitude from *Andromeda*'s head; and the leg AH = 73° 59′, the co-latitude of the ſame northerly. Conſequently the comet's latitude was 16° 01′ N. and its longitude from ♈, according

to the ſucceſſion of the ſigns, is 350° 32′, or 20° 32′ in the ſign *Piſces*.

CASE II.

Let it be required to find the latitude and longitude of that point in the heavens, at the interſection of a great circle paſſing through *Lucida Pleiadum* (D) and *Andromeda*'s head (G), with another great circle paſſing through *Andromeda*'s right foot (F) and the bright ſtar in the head of *Aries* (E).

Let the ſtars D, G, F and E be laid down by their latitudes and longitudes, as in the figure, with the other neceſſary circles as there drawn; alſo let a great circle be drawn through D and G, and another great circle be drawn through F and E; and where they they interſect, put K, which will be the place in the projection whoſe latitude and longitude is required; to find which, obſerve what follows. Firſt, having, through A and K, drawn the circle of longitude AKC, then

I.

By the operation in *Caſe* I. I find DF=35° 18′, FG=24° 40′, GE=27° 21′, DE=22° 56′, FA=54° 39′, ∠EFG=67° 30′, ∠EGF=61° 00′, ∠FEG=57° 04′, ∠GEH=28° 24′, and the ∠EFA=169° 10′.

II.

Then 180°—28° 24′ (∠GEH)=151° 36′=∠DEG.

III.

In the oblique-angled ſpherical triangle DEG, we have given the ſide DE=22° 56′, the ſide EG=27° 21′, and the included angle DEG=151° 36′; from whence I find DG=47° 34′, ∠DGE=14° 17′, and ∠GDE=16° 45′.

IV.

The ∠FGE—∠DGE=46° 43′=∠DGF.

V.

In the oblique ſpherical triangle FKG, are now given GF=24° 40′, the ∠KGF=∠DGF=46° 40′, the ∠KFG=∠EFG=67° 30′; from whence I find FK=42° 15′.

VI.

And, laſtly, we have now given in the oblique ſpherical triangle FKA, the ſide FK=42° 15′, the ſide FA=54° 39′, and the angle KFA=(∠EFA=) 169° 10′, from whence I find AK=82° 53′, the

53′, the co-latitude of the point K; and the angle FAK=7° 19′, the difference of longitude of K from (F) *Andromeda*'s right foot; then 38° 51′ *minus* 7° 19′=31° 32′=∠KA ♈. Hence the place of K, referred to the ecliptic, is ♉1° 31′. *Q. E. I.*

Yours, &c. EDW. HAUXLEY.

A LETTER *from a gentleman at* Cambridge, *concerning the obſervations made there on the late Comet.*

SIR,

I Herewith ſend you (according to promiſe ſome time ago) all the obſervations made at *Cambridge* on the late comet, whoſe exactneſs may be depended upon. The places of the ſtars were taken from *Flamſteed*'s *Hiſtoria Cæleſtis*, and the allowance that was made for the annual promotion of the ſtars in longitude was 50″ 30‴, which is a medium between *Flamſteed* and *Caſſini*. The logarithmic tables that were uſed, were thoſe of Mr *Gardiner*, vaſtly ſuperior to any other yet extant, for their correctneſs, as well as many other excellencies.

February 13. The comet ſet exactly at 6ʰ 52′, at which time the tail was about 21 degrees long, very much bent in the middle in the direction of its path, but the *nucleus* did not ſeem to repreſent the flame of a candle with a ſharp *apex*, upwards in the tail, ſo much as on the 9th, at which time it was obſerv'd (by 4 perſons) that a ſtrait ſtream iſſued out from the ſouth ſide of the tail, about 4 degrees above the *nucleus*. But, what is remarkable, this morning, *viz.* *February* 13. at 5ʰ 30′, the tail did not ſeem above half as long as in the evening, and quite ſtrait, though the horizon ſeemed very clear: but I ſuppoſe this was owing to the great quantity of vapours then aſcending. The tail made about half a right angle when it roſe, but was perpendicular to the horizon when it ſet.

I am, *Yours*, &c. S. B.

See the Table *the back of* Plate V.

Diſtance of STARS from one another, taken at different times,

α *Andr.* and γ *Peg.*	13	57	00	α *And.* and *Markab.*	20	11	30
	13	56	35		20	11	10
β *Ari.* and γ *Peg.*	24	42	50	β *Ari.* and *Markab.*	40	38	00
	24	42	40		40	38	00
γ *Peg.* and *Markab.*	16	30	40	α *And.* and η *Piſc.*	23	35	00
	16	31	10	γ *Peg.* and *Scheat*	20	37	40
α *And.* and *Scheat*	14	12	30	η *Piſc.* and γ *Peg.*	18	52	30
Scheat and *Markab.*	12	51	10	β *Ari.* and α *And.*	25	24	20
β *Ari.* and η *Piſc.*	7	45	30				

The deductions I have been able to make from the above obſervations are as follows, *viz.*

Inclination of the orbit	32°	36′
Aſcending node ☊	18	32
Perihelion ♑	14	32

Time of Perihelion *Feb.* 19. 6^h 43′ P. M.

Perihelion diſtance from the ſun = 117.382 parts of the radius of the *orbis magnus* which I aſſumed = 490.

Yours, S. B.

A LETTER *from* Philo-Whimſicolos, *concerning an octave on the monochord, divided by geometrical proportionals.*

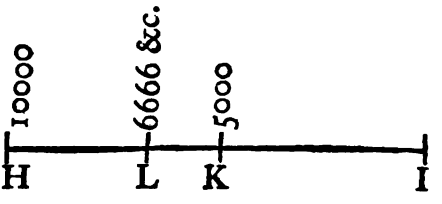
SIR,

I Don't know whether the following ſubject has yet been conſider'd in the ſame light or not; you are at liberty to publiſh it in your *Miſcellanea Curioſa Mathematica*, if you think it deſerves a place there, or that it may induce better judges to communicate their thoughts upon it. *I am, Yours,*

July 22, 1746. PHILO-WHIMSICOLOS.

An OCTAVE *on the monochord, divided by geometrical proportionals.*

SUppoſe a wire, or ſtring, as HI, to be divided into 10000 equal parts, and ſtrain'd to ſuch a pitch that the whole length ſhould be the note C, or



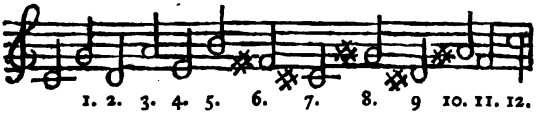
Biſect it, with a moveable bridge, into equal parts, at K, and each ſegment will be an octave above the whole.

It is a receiv'd opinion that $\frac{2}{3}$ of a ſtring will be a 5th (or 7 half tones) above the whole; but I ſhall endeavour to prove that it is more acute. To this perhaps may be owing the difficulty of tuning a key'd inſtrument in ſuch a manner, that a piece of muſic ſhall be equally agreeable to the ear, when tranſpoſed to, or play'd in any other keys, ſuppoſing the performer to play equally well in them all.

Let the bridge be moved to L (IL being $\frac{2}{3}$ds) then by the common rule, IL ſhould be or 6666 parts, and HL (or 3333

its

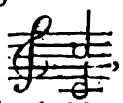
its half) will be the octave to it above, or ; but neither of thoſe ſegments are a 5th or 12th to the whole line, notwithſtanding they are an octave one from the other. For a ſtring may be biſected that the parts ſhall be diſtant from each other, any number of whole or half tones; but then neither of thoſe parts will have a true harmonical ratio to the whole.

If $\frac{2}{3}$ of a ſtring are a 5th from thewhole, then $\frac{2}{3}$ of thoſe $\frac{2}{3}$ muſt be another 5th above the firſt 5th, and ſo on through the whole octave to as follows

where every aſcending 5thcontains 7 half tones, and (which is the ſame in effect) every deſcending 4th 5 half tones; which 4ths are ſo placed, that one ſtave, or cliff, may contain all the half tones in an octave, without the addition of ledger lines. Now there being 12 ſuch 5ths and 4ths from the lower to the upper C, 10000 multiplied by 2, and the product divided by 3 (is 6666), that quotient multiplied by 2, the product divided by 3, and ſo on 12 ſeveral times, the laſt quotient, if this ratio were exact, would be 5000; but on tryal it will be found to be no more than 4931, as by the following table in colum P, which will make the upper C be .0069 parts too acute.

M		P	R
12th	C	4931	5000
5	B	5266	5297
10	B b or A ♯	5548	5612
3	A	5925	5946
8	A b or G ♯	6241	6299
1	G	6666	6674
6	G b or F ♯	7022	7071
11	F	7396	7491
4	E	7900	7937
9	E b or D ♯	8322	8409
2	D	8888	8909
7	D b or C ♯	9362	9439
	C	10000	10000

N. B. The figures in column M point out the aſcending 5ths as they come in courſe.

To rectify this defect, ſuppoſe the ſtring to be biſected at K as before;

fore; then each ſegment will contain 5000 equal parts. Find as many geometrical means between 5000 and 10000, as there are half tones between the two extreams  , *viz.* 11, which means are contained in the column R. The bridge being moved to thoſe ſeveral means, the longeſt part of the ſtring will be the true pitch of every half tone throughout the octave, the firſt (9439) having the ſame ratio to the whole (10000), as the ſecond (8909) has to the firſt (9439); the ſame by all the others: conſequently every interval, or half tone, muſt be equal.

'Tis to be obſerv'd that the numbers in the column R increaſe in geometrical proportion, and that thoſe in the column P are very irregular. The cauſe of the error may be that $\frac{2}{3}$ds (or 6666 parts) come ſo near the true 5th (differing only .0008 parts, which is not $\frac{1}{100}$ of a half tone) that it cannot be diſtinguiſh'd by the ear, and that $\frac{2}{3}$ds, or 2 and 3, were pitch'd upon for the ſake of whole numbers; which of theſe 5ths is moſt agreeable to the ear, I leave to better judges.

This difference, though ſmall in the firſt 5th, is very conſiderable in ſeveral others, and is the cauſe of the great irregularity in the column P. *Two thirds* of a ſtring cannot therefore conſtitute a true *fifth* to the whole. The ratio $\frac{2.00240014}{3.00000000}$, &c. is exact; for multiplying 10000 by the numerator, and dividing the product by 3, twelve ſeveral times, the laſt quotient, allowing for defective decimals, will be 5000, or half the monochord, and an octave above the whole.

N. B. When after multiplying and dividing any of the quotients are leſs then 5000, they are to be doubled; and then as many octaves, as are thought fit, may be added to this, by halving the numbers in the column R.

Yours, &c. PHILO-WHIMSICOLOS

Perhaps it won't be improper to obſerve to our readers, that the antients made uſe of the monochord to determine the proportion of ſounds to one another, when the chord was divided into two equal parts; ſo that when the terms were as 1 and 1, they called them *uniſons*; but if they were as 2 to 1, they call'd them *octaves* or *diapaſons*; when they were 3 to 2, they called them *fifths*, or *diapentes*; when they were as 4 to 3 they called them *fourths*, or *diateſſarons*, &c. the monochord being thus divided, was properly what they called a *ſyſtem*, of which there were many kinds.

A TABLE

A TABLE *ſhewing ſome of the properties of regular polygons*; *by* POLYGONUS.

PUT S for the ſide, R radius of the circumſcribing circle, r radius of the inſcrib'd circle, and A area; alſo let $n=$ the numbers of ſides, $s=$ the ſine of $\frac{360}{2n}$ degrees, $s'=$ the coſine, and $t=$ tangent thereof; then,

Given	R=	r=	A=	S
S	$\frac{S}{2s}$	$\frac{S}{2t}$	$\frac{nSS}{4t}$	
R		Rs'	$nRRss'$	$2Rs$
r	$\frac{rt}{s}$		$nrrt$	$2rt$
A	$\sqrt{\frac{A}{nss'}}$	$\sqrt{\frac{A}{nt}}$		$2\sqrt{\frac{At}{n}}$

Hence by aſſuming S, R, r, and A ſeverally equal to unity, the following tables were calculated.

When S is 1.

n	R	r	A
3	0,5773503	0,2886751	0,4330127
4	0,7071068	0,5000000	1,0000000
5	0,8506508	0,6881910	1,7204774
6	1,0000000	0,8660254	2,5980762
7	1,1523825	1,0382617	3,6339124
8	1,3065630	1,2071068	4,8284271
9	1,4619022	1,3737387	6,1818242
10	1,6180340	1,5388418	7,6942088
11	1,7747329	1,7028437	9,3656404
12	1,9318516	1,8660254	11,1961524

When R is 1.

S	r	A	n
1,7320508	0,5000000	1,2990381	3
1,4142136	0,7071068	2,0000000	4
1,1755705	0,8090170	2,3776412	5
1,0000000	0,8660254	2,5980762	6
0,8677674	0,9009689	2,7364102	7
0,7653668	0,9238795	2,8284271	8
0,6840403	0,9396926	2,8925437	9
0,6180340	0,9510565	2,9389263	10
0,5634651	0,9594931	2,9735250	11
0,5176381	0,9659259	3,0000000	12

When r is 1.

n	S	R	A
3	3,4641016	2,0000000	5,1961524
4	2,0000000	1,4142136	4,0000000
5	1,4530851	1,2360680	3,6327128
6	1,1547005	1,1547005	3,4641016
7	0,9631491	1,1099160	3,3710222
8	0,8284271	1,0823919	3,3137084
9	0,7279405	1,0641776	3,2757315
10	0,6498394	1,0514622	3,2491970
11	0,5872521	1,0422172	3,2298913
12	0,5358984	1,0352760	3,2153904

When A is 1.

S	R	r	n
1,5196716	0,8773827	0,4386912	3
1,0000000	0,7071068	0,5000000	4
0,7623870	0,6485251	0,5246678	5
0,6204033	0,6204033	0,5372849	6
0,5245813	0,6045183	0,5446520	7
0,4550899	0,5946034	0,5493420	8
0,4021996	0,5879764	0,5525172	9
0,3605106	0,5833184	0,5547687	10
0,3267617	0,5799148	0,5564242	11
0,2988585	0,5773503	0,5576775	12

The quadrature of figures of tangents and secants by the hyp. logarithms.

I.

Pl. 5. Fig. 88. Suppose the quadrantal arc AMD extended into the right line $\alpha\mu d$, describe the figure of secants $\delta d\alpha\varphi\varepsilon$, and the figure of tangents $\delta d\alpha\pi\varkappa$. Then 1st, the arc of any portion $\alpha\pi\mu$ of the figure of tangents is = *Naper*'s log. of CT, the secant of the correspondent arc, supposing the radius CA=1.

DEMONSTRATION.

Draw AB perpendicular and equal to CA, and with the asymptotes CG, CH, and vertex B, describe the equilateral hyperbola BNK; put ED=x, arc AM=$\alpha\mu=a$. Then R$m=\dot{x}$, M$m=\dot{a}$, from similar triangles CTM, MmR, we have CT : MT :: Mm ($\dot{a}$) : MR (fluxion of AP); whence MT$\times\dot{a}$ (fluxion of the figure of tangents) is =MR$\times$CT=MR$\times$PN (because PN=CT) = fluxion of the hyperbolic space ABNP; consequently the spaces themselves, *i. e.* the hyperbolic space ABNP, and figure of tangents $\alpha\pi\mu$, are equal. But, from the nature of the hyperbola, the hyperbolic space ABNP is the logarithm of $\frac{CA}{CP}=\frac{CT}{CA}$=CT (CA being =1.)

II.

The area of any part, $\alpha\varkappa\varphi\mu$, of the figure of secants, is = *Naper*'s log. of the tangent of $\frac{90^\circ+a}{2}$.

DEMONSTRATION.

Bisect the arc MD in V; then AV=$\frac{90^\circ+a}{2}$, and AS, its tangent, will be =AQ+QC=$\frac{\overline{2-x}^{\frac{1}{2}}}{x^{\frac{1}{2}}}$. Then *per* sim. triangles CME, MRm, we have EM : CM :: Rm : Mm, that is, $\overline{2x-xx}^{\frac{1}{2}}$: 1 :: $\dot{x}$: $\frac{\dot{x}}{\overline{2x-xx}^{\frac{1}{2}}}=\dot{a}$; this $\times$CT$\left(=\frac{1}{\overline{2x-xx}^{\frac{1}{2}}}\right)$ is $\frac{\dot{x}}{2x-xx}$= fluxion of the figure of secants $\alpha\varkappa\varphi\mu$. Draw CX perpendicular and equal to CD, and with the asymptotes DC, DY, and vertex X, describe the equilateral hyperbola LXF; and making CO=CE, draw OL, EF parallel to DY. Then DO=2$-x$, and from the hyperbola we have LO+EF=$\frac{2}{2x-xx}$, whence $\frac{2\dot{x}}{2x-xx}$= fluxion of the hyperbolic

bolic ſpace EFLO = twice the fluxion of the figure of the ſecants. Therefore the hyperbolic ſpace is equal to twice the figure of the ſecants; and making DZ=CP, the hyperbolic ſpace EFZZ will be equal to the figure of ſecants (as is evident from the quadratures of the external hyperbolic areas). But this ſpace is equal to *Naper's* log. of $\frac{DZ}{DE}=\frac{\sqrt{2x-xx}}{x}=\frac{\overline{2-x}^{\frac{1}{2}}}{x^{\frac{1}{2}}}$ = tangent AS. *Q. E. D.*

The ſecond propoſition may be otherwiſe demonſtrated thus: Pl. 5. Fig. 89. The ſame things being ſuppoſed as in demonſtration ſecond, draw C*t* infinitely near CT, and from the center C, with the radius CT, deſcribe the infinitely ſmall arc T*a*; call AT, *t*; CT, *s*; then T*t*=*ṫ*, and *at*=*ṡ*. Then from ſimilar triangles T*ta*, TCA we have CT : AT :: T*t* : *ta*, that is, $s : t :: \dot{t} : \dot{s}$; and, by compoſition, $s+t$ (AB) : $s :: \dot{s}+\dot{t} : \dot{t}$, whence $\frac{\dot{s}+\dot{t}}{s+t}=\frac{\dot{t}}{s}$. But $\frac{\dot{s}+\dot{t}}{s+t}$ is = the fluxion of the log. AB, and $\frac{\dot{t}}{s}$ is the fluxion of the figure of ſecants to AM (put radius = 1). Conſequently their fluents, that is, the log. of AB, and the figure of ſecants to AM, are equal. *Q.E.D.*

Solutions to Queſtions in Number IV.

QUESTION LVI. *anſwered by Mr* W. CHAPPLE.

PUT 39.13=*a*, 10=*b*, 60=*c*, and *x*= the length of the pendulum required; then *bx*= the number of its vibrations in a minute; and, becauſe pendulums are reciprocally to each other as the ſquares of their vibrations, $b^2x^2 : c^2 :: a : x$. Th. $b^2x^3=ac^2$; hence $x=\sqrt[3]{\frac{ac^2}{b^2}}=11.2096$.

The ſame anſwered by Mr J. TURNER.

THE lengths of pendulums being to one another inverſely as the ſquares of their vibrations in the ſame time, if we put *x*= length of the pendulum required, it will be as $39.13 : \overline{60}^2 :: x : \overline{10x}^2$, or $x^3=1408.68$; conſequently *x*=11.21 inches, which ſaid pendulum will vibrate 112.1 times in a minute.

QUESTION LVII. *anſwered by Mr* ROBERT ROBINSON.

LET *x*= the youngeſt's years, then their reſpective ages will be, *x*, *x*+1, *x*+2, *x*+3, &c.... to *x*+*n*−1, and the yards of lace

lace for each ſon $=x$, $2x+1$, $3x+3$, $4x+6$, &c. to $nx+\frac{n\times\overline{n-1}}{2}$; hence the taylor's bill for

The 1ſt ſon $=x^2$

2 $=2x^2+3x+1$

3 $=3x^2+9x+6$

4 $=4x^2+18x+18$.

And the n^{th} $=nx^2+\frac{3n^2-3n}{2}x+\frac{n}{2}\times\overline{n-1}^2$. Hence the ſum of the homologous terms will be thus expreſſed:

$x^2:\times 1+2+3+4$, &c. $\ldots\ldots+n\ldots\ldots=\frac{n\cdot\overline{n+1}}{2}x^2$.

$x:\times 3+9+18$, &c. $\ldots\ldots+\frac{3n}{2}\times\overline{n-1}\ldots\ldots=\frac{\overline{n^2-n}\cdot\overline{n+1}}{2}x$.

and $1+6+18$, &c. $\ldots\ldots+\frac{n}{2}\times\overline{n-1}^2=\frac{n-1}{6}\times n^3-2n^2+6n-6$,

whoſe ſum $=240m$; hence $x^2+n-1\times x=\frac{1440m+\overline{6-6n+2n^2-n^3}\times\overline{n-1}}{3n\cdot\overline{n+1}}$,

which ſolv'd gives $x=\frac{1-n}{2}+\sqrt{\frac{5760m:+24-27n+8n^2-n^3\times n-1}{12n\cdot\overline{n+1}}}$

The ſame anſwered by MERCATOR SCISSOR, *the Propoſer.*

LET $x=$ age of the youngeſt.

$x+1=$ 2d.

$x+2=$ 3d.

$x+3=$ 4th.

&c.

Then $x=$ yards of lace on the youngeſt's cloths.

$2x+1=$ 2d.

$3x+3=$ 3d.

$4x+6=$ 4th.

&c.

And xx $=$ price of making the youngeſt's ſuit,

$2xx+3x+1$ $=$ the 2d's.

$3xx+9x+6$ $=$ the 3d's.

$4xx+18x+18$ $=$ the 4th's.

&c.

Th. $\frac{n\cdot\overline{n+1}}{1\cdot 2}xx+\frac{\overline{n-1}\cdot n\cdot\overline{n+1}}{1\cdot 2\cdot 3}3x+\overline{n-1}+\frac{\overline{n-1}\cdot\overline{n-2}}{1\cdot 2}5+$

$\frac{\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}}{1\cdot 2\cdot 3}7+\frac{\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}\cdot\overline{n-4}}{1\cdot 2\cdot 3\cdot 4}3=m.$ Th.

Th. $xx+\frac{n-1}{1}x+\frac{2\cdot\overline{n-1}}{n\cdot n+1}+\frac{\overline{n-1}\cdot\overline{n-2}}{n\cdot n+1}5+\frac{\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}}{3\cdot n\cdot n+1}7+$

$\frac{\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}\cdot\overline{n-4}}{3\cdot 4\cdot n\cdot n+1}3=\frac{2m}{n\cdot n+1}$; and (putting

$\frac{2\cdot 3\cdot 4\cdot m-2\cdot 3\cdot 4\cdot\overline{n-1}-3\cdot 4\cdot 5\cdot\overline{n-1}\cdot\overline{n-2}-4\cdot 7\cdot\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}-3\cdot\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}\cdot\overline{n-4}}{3\cdot 4\cdot n\cdot\overline{n+1}}$

$=r$) $xx+\overline{n-1}\times x=r$. Th. $x=\sqrt{r+\frac{\overline{n-1}^2}{4}}-\frac{n-1}{2}$.

If $n=7$, and $m=7l.$ 10$s.$ 6$d.$ $=1806d.$

Then $2\cdot 3\cdot 4\cdot 1806=43344$

$2\cdot 3\cdot 4\cdot 6 = 144$

$3\cdot 4\cdot 5\cdot 6\cdot 5 = 1800$

$4\cdot 7\cdot 6\cdot 5\cdot 4 = 3360$

$3\cdot 6\cdot 5\cdot 4\cdot 3 = 1080$

6384

Remainder 36960

$3.4.7.8=672)36960(55=r$

$x=(\sqrt{55+\frac{6\times 6}{4}}=\sqrt{64}-\frac{6}{2}$

$=8-3=)$ 5. *Q. E. I.*

The same answered by Mr PAUL SHARP.

PUT $x=$ youngest child's age; then, by the question, x, $x+1$, $x+2$, $x+3$, &c. .... $x+n-1$ will be their ages; and x, $2x+1$, $3x+3$, $4x+6$ .... &c. to n terms $=$ the yards of lace; Th. $x^2+\overline{2x+1\times x+1}$, $+\overline{3x+3\times x+2}$, $+\overline{4x+6\times x+3}$ ..... &c. to n terms $=240m$, *per* question; now put $2c=$ sum of all the co-efficients of the second terms of each product, and the sum of the last terms of the said products $=S$, then $nx^2+2cx=240m-S$; Th. $x=\sqrt{\frac{240m-S}{n}+\frac{c^2}{n^2}}-\frac{c}{n}=$ the age of the youngest; whence the rest is known.

QUESTION LVIII. *answered by Mr* ROBERT ROBINSON.

LET $x=$ common ratio, then the first term $=\frac{1}{x}$, and $2:15::\frac{1}{x}:$ $\frac{15}{2x}=$ number of terms; again $36:211::x:\frac{211x}{36}=\frac{x^{\frac{15}{2x}}-1}{xx-x}$ $=$ sum of the terms. Assume $1.4+v=x$, and the equation will become $\overline{1.4+v}^{\frac{15}{2.8+2v}}=\frac{\overline{211}^2\times\overline{1.4+v}^2\times\overline{.4+v}}{36}$; which turned

 into

into a ſeries and ſolv'd, $v=.09998$, hence $x=1.5$, and the terms are $\frac{2}{3}$. 1 . $1\frac{1}{2}$ and $2\frac{1}{4}$.

The ſame anſwered by Mr J. WAINE.

PUT $n=$ number of terms in the progreſſion, $x=$ common ratio, and from what is given, we get $\frac{1}{x}=$ the firſt term, and thence $\frac{x^n-1}{xx-x}=$ ſum of all the ſeries; alſo from the given proportions, $\frac{1}{x} : n :: 2 : 15$; Th. $n=\frac{15}{2x}$, and $\frac{x^n-1}{xx-x} : x :: 211 : 36$; Theref. $211x=\frac{36x^n-36}{xx-x}$. For n in this equation ſubſtitute its value from the foregoing, and it becomes $211x^3-211x^2+36=36x^{\frac{15}{2x}}$, which being ſolv'd by the method of exponentials, will give $x=\frac{3}{2}$, and thence $n=5$, and the terms as above.

Again, by Mr JOHN TURNER.

LET $x=$ common ratio, then is the firſt term $=\frac{1}{x}$; and as $2 : 15 :: \frac{1}{x} : \frac{15}{2x}=$ number of terms; as $36 : 211 :: x : \frac{211x}{36}=$ ſum of all the ſeries. Now, by the nature of geometrical progreſſion, $\frac{211x}{36}=\frac{\frac{1}{x}\times x^{\frac{15}{2x}}-\frac{1}{x}}{x-1}$, or $x^{\frac{15}{2x}}=\frac{211x^3-211x^2}{36}+1$. 'Tis alſo evident that $2x$ muſt be a ſubmultiple of 15, and x muſt be greater than unity; but if ſo, x can only be equal to $1\frac{1}{2}$, $2\frac{1}{2}$, or $7\frac{1}{2}$; and, from the other conditions of the problem, x muſt be $=\frac{3}{2}$, the common ratio, and the number of terms 5, *viz.* $\frac{2}{3}$, 1, $\frac{3}{2}$, $\frac{9}{4}$, $\frac{27}{8}$, and the ſum of all the ſeries $=\frac{211}{24}$.

Thus by Mr N. FARRER, *the Propoſer.*

LET $2 : 15 :: d : m$, and $211 : 36 :: n : q$; put $x=$ common ratio, then the firſt term is $\frac{1}{x}$, the number of terms $=\frac{m}{dx}$, ſum of the terms $=\frac{nx}{q}$, and the laſt term $=\frac{nx-n}{q}+\frac{1}{xx}$; hence we have $x^{\frac{m}{dx}}=\frac{nx^3-nx^2}{q}+1$, whence $x=1.5$, and the terms of the ſeries $\frac{2}{3}$. 1; 1.5; 2.25; and 3.375.

QUESTION

QUESTION LIX. *answered by* GLIPHORI.

IN the frustum ABED, of a right cone, Pl. 5. Fig. 90.

Given $AB=d=4.625$, $AC=b=5.25$, $BD=\delta=7.25$; required the solidity.

Draw DF parallel and $=AC$, let $x=AF$; then $FB=d+x$. Now $\overline{BD}^2=\overline{BF}^2+\overline{FD}^2$, or $\delta\delta=bb+dd+2dx+xx$. Hence $x=(\sqrt{\delta\delta-bb}-d=)\ 0.375$, and $DE=D=(d+2x=)\ 5.375$; then $(\overline{DD+dd+Dd}\times b\times 0.2618=)\ 103.276821$ is the solidity in cubic inches.

From the cubic inches in three pints	$=105.75$
Take the cubic inches in the tankard	$=103.276821$
Leaves the cubic inches gain'd on each filling	$=2.473179$

Therefore the gain *per* day is 24.73179 cubic inches, and is worth 0.00438506 9*l*.

Now here is given, the daily pension $=a=0.00438506 9$

The amount $=m=15$

Rate of interest $=r=0.00010746012\ (=\sqrt[365]{1.04}-1)$

Required the time $=n$.

By well known rules, $n=\dfrac{\log.\ \overline{mr+a}-\log.\ a}{\log.\ \overline{r+1}}$.

Hence the tankard will be cleared in 7 years, 358 days.

The same answered by Mr THOMAS COWPER.

LET $b=BA=4.625$, $DF=c=5.25$, $d=BD=7.25$ inches, and $x=AF$, half the difference of the diameters; then $x+b=BF$, and *per* 47 *E.* 1. $x+b=\sqrt{dd-cc}$, *i. e.* $x=\sqrt{d^2-c^2}-b=.375$; whence $ED=5.375$ inches, and the content of the tankard 103.2770320312 5 solid inches, which is 2.4729679687 5 less than the solid inches in three pints; the value of which, at 1*d.* $\frac{1}{2}$ *per* pint, amounts to .0004384694 98*l*. If this deficiency be consider'd as an annuity, and 1.0000109589 the rate of interest for $\frac{1}{10}$ part of a day, I find that it will amount to 15*l*. in 2906.9 days, or 7 years, 351.9 days, the time in which the landlord will be paid for his tankard.

QUESTION LX. *answered by Mr* WILLIAM DAVIES.

PUT $3x=AB$, $x^3=BC$, $x^2=DB$. Then $xx\sqrt{\frac{1}{2}}=DE=DF$, $x^3-xx\sqrt{\frac{1}{2}}=BC-EB=EC$. And $AB:CB::DE:EC$, that Pl. 5. Fig. 91.

that is, $3x : x^3 :: xx\sqrt{\frac{1}{2}} : x^3 - xx\sqrt{\frac{1}{2}}$; hence $3x \times \overline{x^3 - xx\sqrt{\frac{1}{2}}} = x^3 \times xx\sqrt{\frac{1}{2}}$. Th. $x = \frac{3+\sqrt{3}}{\sqrt{2}}$, and $\frac{3x^4}{2} = 188.028$ chains $= 18.028$ acres.

The ſame anſwered by Mr ROBERT ROBINSON.

HERE is given $AB = x^3$, $BC = 3x$, and $BD = x^2$; then $DE = DF = \frac{xx}{\sqrt{2}} = ax^2$, $AF = x^3 - ax^2 : ax^2 :: ax^2 : EC = 3x - ax^2$;

Th. $\overline{x-a} \times \overline{6-2ax} = x$; hence $x = \frac{3}{2a} \pm \sqrt{\frac{9}{4aa} - 3} = 1.5\sqrt{2} \pm \sqrt{1.5}$ $= 3.3460644$, or 0.8965746; hence the content is

A.	R.	P.
18	3	8.489
or 0	0	15.508.

The ſame anſwered by Mr CH. SMITH.

HERE $AB = x^3$, $BC = 3x$, and $BD = x^2$, biſecting the right angle; put $z =$ ſine of the angle $ADB = BDC$, $s =$ ſine of $45°$. Then $z : x^3 :: s : \frac{sx^3}{z} = AD$, and $z : 3x :: s : \frac{3sx}{z} = DC$;

but $AB \times BC - AD \times DC = \overline{BD}^2$, *i. e.* $3x^4 - \frac{3s^2x^4}{z^2} = x^4$; Th. $3 - \frac{3s^2}{z^2} = 1$; Th. $z = s\sqrt{\frac{3}{2}} = .866025$, the nat. ſine of $60°$. Put p and q for the ſine and coſine of $75° = \angle C$, then $p : x^3 :: q : 3x$, Th. $x = \sqrt{\frac{3p}{q}} = 3.346$; hence $x^2 = 11.196154$, $3x = 10.038$, $x^3 = 37.4623$, and the content is $18^{A.}\ 3^{R.}\ 8^{P.}$

Again, by Mr WILLIAM JEPSON.

LET ABC be the triangle; draw the perpendicular DF, and it will be the ſide of the inſcrib'd ſquare, becauſe BD biſects the right angle. Put $s =$ ſine of $45°$, and we ſhall have $1 : x^2 :: s : sx^2 = DF$; then $AB \times BC = DF \times \overline{AB + BC}$, *i. e.* $3x^4 = sx^5 + 3sx^3$; Th. $3x = sx^2 + 3s$, by tranſpoſition, &c. $x = n \pm \sqrt{n^2 - 3} = \begin{cases} 3.34612 \\ .89656 \end{cases}$

Now becauſe either of the legs may be $3x$, or x^3, indifferently, the queſtion will admit of the two following anſwers, *viz.*

If

If $x=\begin{cases} 3.34612 \begin{cases} AB=x^1=37.464 \\ BC=3x=10.038 \end{cases} \} = 18\ 3\ 8.688 \text{ area.} \\ .89656 \begin{cases} AB=3x=2.689 \\ BC=x^1=.7206 \end{cases} \} = 15\frac{1}{2} \text{ sq. perches area.} \end{cases}$

QUESTION LXI. *answered by Mr* JOHN TURNER.

CONSTRUCTION.

MAKE EF= sum of the 3 sides =78, and upon it describe a segment of a circle capable of containing an angle (of 126° 52′ 12″ *plus* half its complement to 180°)=153° 26′ 6″, as the angle EBF. Draw B*n* parallel to EF, at the distance of the given perpendicular BD, and where it cuts the circle, as at B, will be the vertex of the triangle; draw BE, and BF; lastly, make the angle EBA=∠BEA, and the angle FBC=∠BFC, which will compleat the triangle. Pl. 5. Fig. 92.

DEMONSTRATION.

Since the angle EBA=∠BEA, the side AB=EA, and for the same reason BC=CF; and because the angle BAC=twice the angle BEF, and the angle BCA= twice the angle BFE; therefore the angle ABC=∠EBF *minus* half its complement to 180°.

CALCULATION.

In the triangle ES*m*, there is given E*m*, and the angle ES*m*, to find ES and S*m*; in the triangle BS*n*, given SB, and S*n*, to find B*n*; in the triangle EBD, given ED, and BD, to find the angle BED, the double of which is the angle BAC, and hence BCA is known, and consequently all the sides of the triangle ABC separately.

This was answered by the Proposer, the same way.

The same answered by Mr W. CHAPPLE.

PUT x= the base
r= radius of the inscrib'd circle
a= perpendicular =8.4324
b= perimeter =78. Pl. 5. Fig. 93.

Then ax= twice the area, and br also = twice the area; Th. $ax=br$, and $x=\frac{br}{a}=9.25r$, consequently the ratio of the base to the rad. of the inscrib'd circle is as 9.25 to 1.

Therefore assuming 1 for the radius of the inscrib'd circle, the

baſe will be 9.25, from which *data* a triangle ſimilar to that in the queſtion may be projected thus.

By 26 *E.* 3. the angle made by radii drawn from the center of the circumſcribing circle to the extremities of the baſe will be double the angle at the circumference, *viz.* double the given vertical angle; but as this exceeds 180°, being 253° 44′ 24″, its ſupplement to 360° muſt be taken, *viz.* 106° 15′ 36″. With this as a vertical angle, and the aſſum'd baſe, form an Iſoſcelar triangle GAC; then will the ſides GA, GC (which will each be =5.7813) be radii of the circumſcribing circle, of which G is the center; draw LM parallel to the baſe AC, at a diſtance equal the radius of the inſcrib'd circle before aſſum'd, then will the center of the inſcrib'd circle be ſomewhere in LM, and the radii of both circles being known, the diſtance of their centers may be thus had: Continue the perpendicular GS to F, and make FE (=TS) = the radius of the inſcrib'd circle, and on GE as an hypotenuſe form a right-angled triangle GNE; then will $GP=\sqrt{R^2-2Rr}$ (putting R= radius of the greater circle, and r= radius of the leſſer.) But this is equal the diſtance of the centers of the two circles by *Theorem* in *page* 123 of the *Miſcellanea Curioſa Mathematica*; with this therefore as a radius, and on G as a center, deſcribe an arc, which will croſs LM in δ and P, either of which is the center of the inſcrib'd circle, which circle being deſcribed, draw the tangents AB and BC (which will meet at B in the periphery of the circumſcribing circle) and the triangle is formed; ſo that now we have projected a triangle ſimilar to that in the queſtion.

Now the perpendicular GS in the iſoſcelar triangle AGC being found =3.4688, and TS (=the rad. of the inſcrib'd circle, =δY) being =1 according to the above aſſumption, as alſo the diſtance of the centers found as above, =4.6755, the ∠GδT=∠AXδ is found by *plane trig.* =72° 54′, and hence δX, YX, XS, AY, and conſequently the ∠δAY (=17° 6′ 10″), and the ∠BAD (=34° 12′ 20″) are found; and (the perpendicular BD being let fall) ABD is found =55° 47′ 40″; alſo DBC=71° 4′ 32″, and BCD=18° 55′ 28″; from which all the required parts of the triangle are found, *viz.* AB=3.75, BD=2.1081, and BC=6.5. Hence, by ſimilarity of triangles, we have, in the triangle required in the queſtion, AB=15, BC=26, and AC=37. *Q. E. I.*

But if we diſregard a geometrical conſtruction, the above queſtion may be more eaſily and expeditiouſly ſolv'd thus.

Having found the ratio of the baſe to the radius of the inſcrib'd circle,

circle, and assum'd 9.25, and 1 for them in a similar triangle as above, and having the angle AGC, *viz.* 106° 15′ 36″, we have AG, or GC, (the rad. of the circumscrib'd circle) by *plane trig. viz.* 5.7813, also GS=3.4688, and consequently GT=4.4688. Put R=5.7813, then (the other radius being =1) we have by the theorem above referr'd to (*viz. p.* 123. *Misc. Curiosa*) $\sqrt{R^2-2Rr}=\sqrt{R^2-2R}$= the distance of the centers =4.6755=Gδ. Then *per* 47 *E.* 1. δT=YS is found =1.375, which put =*e*; then *per* theorem in my solution of *Quest.* 62. (putting B= the base =9.25) $\sqrt{2R+e^2+\frac{4R^2}{B^2}}+\frac{2R}{B}\pm e=\frac{BC}{AB}$. In numbers $5.125\pm 1.375=\frac{BC=6.5}{AB=3.75}$. Hence the perpendicular is found =2.1081, and consequently in the Δ required in the question, which is similar to this, and wherein the perpendicular is 8.4324 (or 4 times this) AB is = 15, BC=26, and AC=37.

QUESTION LXII. *answered by Mr* CHAPPLE, *the Proposer.*

PUT *a*= the radius of the lesser circle =20, and *b*= radius of the greater =47; then *per Corollary* to *Prop.* 5, 6, and 7 in the Essay, the distance of their centers $=\sqrt{b^2-2ab}=18.1383=C\mu$. Hence it appears that the center of the greater circle is within the lesser, and consequently within the triangle. Let fall the perpendicular μH, then HD=AH=45, (being half the given side AD) which put =*n*, and draw Kμ parallel to AD. Then *per* 47 *E.* 1. $\sqrt{b^2-n^2}=\mu H=KE=13.564$, and CE−KE, *i. e.* 20−13.564 =6.436=CK; then $\sqrt{C\mu^2-CK^2}=16.958=K\mu=EH$, which put =*e*. Put *m*= AD the given side, *x* = the greater required side=BD, and *z*=AB the lesser; then $\frac{am+ax+az}{2}$= the area of the triangle $=\frac{mxz}{4b}$, as is shewn in the demonstration of *Prop.* 2[d] of of my Essay *p.* 118. Th. $z=\frac{2abm+2abx}{mx-2ab}$, consequently the area of the triangle $=\frac{am^2x+amx^2}{2mx+4ab}$, it being $=\frac{mxz}{4b}$ as above. And because *am*= the area of AFCGD, $\frac{am^2x+amx^2}{2mx+4ab}-am$ = area of BFCG,

Pl. 5. Fig. 94.

BFCG, which divided by a gives $\frac{m^2x+mx^2}{2mx+4ab}-m=FB=BG$. Again, $\frac{m}{2}+e=ED=GD$, and $\frac{m}{2}-e=AE=AF$; Then $\frac{m^2x+mx^2}{2mx+4ab}-m+\frac{m}{2}+e=BG+GD=BD=x$, reduc'd $mx^2\ \frac{-2me}{-4ab}x=2abm-4abe$, solv'd $x=\frac{me+2ab}{m}\pm\sqrt{2ab+e^2+\frac{4a^2b^2}{m^2}}$, the negative value of which laſt expreſſion will be always impoſſible, (as ſhall be ſhown by and by) the other gives $x=88.8756$.

If $AE=\frac{m}{2}-e$ had been taken inſtead of $ED=\frac{m}{2}+e$ (as it muſt have been, had x been put $=AB$) x would have come out $=\frac{2ab-me}{m}+\sqrt{2ab+e^2+\frac{4a^2b^2}{m^2}}$, which in the above ſolution was put $=z)=54.9596$, which might otherwiſe have been found by conſidering that z is $=\frac{2abm+2abx}{mx-2ab}$, as above; or that the difference of the required ſides is always $=2e$; for it muſt be always $=$ the difference of AF $\left(\frac{m}{2}-e\right)$, and GD $\left(\frac{m}{2}+e\right)$, BF and BG being equal. And this ſuggeſts another eaſy method of finding the required ſides; thus, $z=\frac{2abm+2abx}{mx-2ab}=x-2e$; Th. $2abm+2abx=mx^2-2abx-2mex+4aeb$, and by tranſpoſition $2abm-4aeb=mx^2-4abx-2emx$, as before.

Now that the negative value of $\sqrt{2ab+e^2+\frac{4a^2b^2}{m^2}}$ (before mentioned) will be always impoſſible, may be thus prov'd. If it be poſſible in the value of x the greater ſide, it will likewiſe be ſo in that of z the leſſer, their difference being always $=2e$; which difference ariſes from the other part of the expreſſion in each, *viz.* $\frac{2ab+me}{m}$, and $\frac{2ab-me}{m}$. And it cannot be poſſible in the value of z, unleſs $2ab<me$; but 'tis evident m cannot be greater than $2b$ (the diameter of the greater circle), and that e cannot be greater than a; therefore, &c.

From the whole, then, we may deduce, for finding the two unknown ſides, this general

THEO-

THEOREM.

$$\sqrt{2ab+e^2+\frac{4a^2b^2}{m^2}}+\frac{2ab}{m}\pm e.$$

Where the affirmative value of e muſt be taken for the greater, required ſide, and the negative for the leſſer.

The ſame anſwered by Mr JOHN TURNER.

FROM Aμ, AH being given, may be found the angle AμH= ∠ABD 73° 13′½, which put =m, n=AD=90, s and c= ſine and coſine of half the ſum of the angles at the baſe, being 53° 23′½, x and y= the ſine and coſine of half their differences, p= 20=FC.

Then as $m:n::sy+cx:\text{BD}=\frac{n}{m}\times\overline{sy+cx}$.

as $m:n::sy-cx:\text{AB}=\frac{n}{m}\times\overline{sy-cx}$.

as $1:n::sy-cx:\text{AN}=n\times\overline{sy-cx}$. Conſequently $\frac{n}{m}\times$ $\overline{sy+cx\times n\times sy-cx}=p\times n$, $+\frac{n}{m}\times sy+cx$, $+\frac{n}{m}\times sy-cx$, which reduc'd is $y^2-\frac{2sp}{n}y=c^2+\frac{mp}{n}$. Hence y=.9532, the coſine of 17° 36′, and the angle BAD=70° 59′¼, BDA=35° 47′¼, BD= 88.87, AB=54.97. *Q. E. I.*

QUEST. LXII. *anſwered by* Philoſluentimecanalgegeomaſtrolongo.

LET the inſcrib'd triangle be QWR, the diameter of the circumſcribing circle BD perpendicular to QW; let S be the center of the inſcrib'd circle, and draw ASV parallel to QW; then there will be known AP, CP, BP, BA, DA, DP, and conſequently BW, BV. Pl. 5. Fig. 95.

Let BW=d, BV=b, BA=s, BP=t, BR=x, BD=a=94, QW=p=90, PA the radius of the inſcrib'd circle =f=20. Then by ſimilar triangles $x:a::s:\frac{sa}{x}=\text{BS}=\frac{bb}{x}$, and $x:a::t:\frac{ta}{x}=\text{BT}$ $=\frac{dd}{x}$; whence RS=$x-\frac{bb}{x}$, and TS=$\frac{bb-dd}{x}$; and ſince ∠TQS =∠SQR, TS : SR :: QT : QR :: by the ſimilar triangles QTR,

QTR, BWR) BW : BR, that is, $\frac{bb-dd}{x} : \frac{xx-bb}{x} :: d : x$, therefore $dxx-dbb=bbx-ddx$, or $dxx+ddx=bbx+bbd$, and dividing both ſides by $dx+dd$, then $x=\frac{bb}{d}$, whence PT will be known, and then BW (d) : BR (x) :: QT : QR :: and WT : WR.

QUESTION LXIII. *anſwered by Mr* NIC. FARRER.

Pl. 5. Fig. 96. LET OB=a, CF=x, and AC=z; then 1 : 7 :: z : $7z$=CD. Th. $7z-z=2x$, and AC=$\frac{x}{3}$, AF=$\frac{4x}{3}$, OF=$\frac{1}{3}\sqrt{9a^2-16x^2}$, and the fluent of $x\sqrt{a^2-x^2}=ax-\frac{x^3}{6a}-\frac{x^5}{40a^3}-\frac{x^7}{112a^5}-\frac{5x^9}{1152a^7}$, &c. Hence the area of the part BCFE=$\frac{.7222x^3}{a}+\frac{.37x^5}{a^3}+\frac{.342x^7}{a^5}+\frac{.386x^9}{a^7}$ &c. =2017.5; therefore this equation is $.7222a^6x^3+.37a^4x^5+.342a^2x^7+.386x^9=2017.5a^7=q\times.72$, hence $x=\overline{qa}|^{\frac{1}{3}}-.1708qa^{-1}-.0413q^{\frac{5}{3}}a^{-\frac{7}{3}}-.01629q^{\frac{7}{3}}a^{-1\frac{1}{3}}$, &c.

Again $\frac{1.712x^3}{a}+\frac{1.559x^5}{a^3}+\frac{2.568x^7}{a^5}+\frac{5.138x^9}{a^7}$, &c. = 2236.48, the area EDF; then by reduction we have $x^3+911x^5a^{-2}+1.5003x^7a^{-4}+3.002x^9a^{-6}$ &c. =1306.4a=da, hence $x=\overline{da}|^{\frac{1}{3}}-\frac{.304d}{a}-\frac{.131d^{\frac{5}{3}}}{a^{\frac{2}{3}}}-\frac{.088d^{\frac{7}{3}}}{a^{1\frac{1}{3}}}$, &c. and by equating the two values of x we have an infiniteſimal equation, where a the radius of the circle will be found.

QUESTION LXIV. *anſwered by Mr* NIC. FARRER.

Pl. 5. Fig. 97. LET CD=DF=DE=10=a, $2y$=AB, and x=CV; then DV=$x-a$, GV=$x-2a$, and, *per* 47 *E.* 1. $\sqrt{x^2+y^2}$=AV; hence *per* 4 *E.* 6. $\sqrt{x^2+y^2} : y :: x-a : a$, hence $d\sqrt{x^2+y^2}=yx-ya$, conſequently $y^2=\frac{a^2x}{x-2a}$, and area of the baſe=$\frac{3.1416a^2x}{x-2a}$; hence the ſolidity =$\frac{1.0472a^2x^2}{x-2a}$, its fluxion

$$\frac{2.0944a^2x\dot{x}\times\overline{x-2a}:-1.0472a^2x^2\dot{x}}{\overline{x-2a}|^2}=0.$$

Th. $2\dot{x}-4\dot{a}-\dot{x}=0$; hence

hence $x=4a=40$, the cone's altitude, the diameter of its base 28.28497, and its content 8377.6 inches.

The same answered by Mr JOHN TURNER.

LET FD=GD=DC=10=b, $n=.7854$, VG=x; then is FV=$\sqrt{2bx+xx}$. As $\sqrt{2bx+xx} : b :: 2b+x : AC=\frac{2b^2+bx}{\sqrt{2bx+xx}}$. Hence $\frac{b^2x^2+4b^3x+4b^4}{2bx+xx}\times 4n=$ area of the base, which multiplied by $\frac{2b+x}{3}$ gives the solidity, that is, it is as $\frac{x^2+4bx+4b^2}{x}$ which in fluxions is $x^2\dot{x}-4b^2\dot{x}=0$, and $x=2b$, consequently VC =40, AB=28.284, and the solid content $=32b^3\times.2618=8377.6$ cubic inches.

The same answered by Mr W. JEPSON.

IF the cone be cut by a plane at right angles to its base thro' its axis, it will, with its inscribed sphere, be divided into two equal parts, and AFVEBCA will be the figure of the section, and consequently GFCG a great circle of the sphere. From the center D, draw DE to the point of contact, and let DC=DG=DF=10 =a, GV=x; then *per* 47 *E.* 1. $\sqrt{x^2+2ax}$=VE, and *per* sim. triangles $\sqrt{x^2+2ax} : a :: x+2a : \frac{ax+2a^2}{\sqrt{x^2+2ax}}$=BC; Th. AB=$\frac{2ax+4a^2}{\sqrt{x^2+2ax}}$, and $\frac{4a^2x^2+16a^3x+16a^4}{x^2+2ax}\times .7854=$ $\frac{x^2+4ax+4a^2\times 3.1416a^2}{x^2+2ax}=$ the area of the cone's base; Th. $\frac{x^2+4ax+4a^2\times 3.1416a^2}{x^2+2ax}\times\frac{x+2a}{3}=\frac{x^2+4ax+4a^2\times 3.1416a^2}{3x}=$ the cone's solidity, a *minimum*; in *fluxions* is $\frac{3x^2\dot{x}-12a^2\dot{x}}{9x^2}=0$. Th. $x^2=4a^2$, and $x=2a$; hence the cone's altitude =40, diameter of its base =28.284, slant side =42.426, and its solidity =8377.6 cubic inches.

Again, by Mr ROBERT ROBINSON.

LET the radius of the sphere FD=n=20, and x=AC, y=CV, then VD=$y-n$, and $n^2 : \overline{y-n}^2 :: x^2 : x^2+y^2$, and multiplying extreams and means, we have $n^2y=yx^2-2nx^2$, Th. $x^2=$ n^2y

$\frac{x^2y}{y-2n}$; hence the expreſſion $\frac{y^2}{y-2n}$ is in a conſtant ratio to the ſolidity of the cone, and therefore a *minimum*, whoſe *fluxion* is $2y\dot{y}-4n\dot{y}y-y^2\dot{y}=0$. Th. $y=4n=40$ inches, the cone's altitude, and $x=n\sqrt{2}=14.1421$, the radius of its baſe. *Q. E. I.*

QUESTION LXV. *anſwered by Mr* R. ROBINSON, *the Propoſer.*

Pl. 5. Fig. 98. HAVING the three ſides of the triangle, find the ſegments AD $=6=n$, DH$=15=d$, perp. KD$=8=m$; put $w=d+n$, and $a=d-n$; alſo let $x=$EG, $y=$EF, $z=$EC. Then EF : EG :: EG : EH$=\frac{x^2}{y}$; in like manner EA$=\frac{x^2}{z}$; Th. $\frac{x^2}{z}+\frac{x^2}{y}=w$, and $x^2=\frac{wzy}{y+z}$. Again HD : DK :: HF : FM$=\frac{m}{dy}\times x^2-y^2$, AD : DK :: AC : CI$=\frac{m}{nz}\times x^2-z^2$, BC$\times$CG : $\overline{\text{CI}}^2$ :: BE$\times$EG : $\overline{\text{EL}}^2=\frac{m^2x^2}{n^2z^2}\times x^2-z^2$, and :: BF$\times$FG : $\text{FM}^2=\frac{m^2\times x^2-z^2\times x^2-y^2}{n^2z^2}=\frac{m^2\times\overline{x^2-y^2}^2}{d^2y^2}$; Th. $x^2=\frac{wa z^2y^2}{d^2y^2-n^2z^2}=\frac{wzy}{y+z}$. This reduc'd is $azy^2+ayz^2+n^2z^2-d^2y^2=0$; write vy for z, and it is $avy+av^2y+n^2v^2-d^2=0$; Th. $y=\frac{d^2-n^2v^2}{av+av^2}$, $z=\frac{d^2-n^2v^2}{a+av}$, $x^2=\frac{w}{a}\times\frac{d^2-n^2v^2}{\overline{1+v}^2}$, and $\text{EL}^2=\frac{m^2w}{n^2a}\times\frac{aw-d^2+n^2v^2}{\overline{1+v}^2}$; hence $\overline{\text{EL}}^2\times\overline{\text{EG}}^2=\frac{m^2w^2}{a^2n^2}\times\frac{d^2-n^2v^2\times aw-d^2+n^2v^2}{\overline{1+v}^4}=$ *maximum*, which put into *fluxions*, and reduc'd, gives $v^3+\frac{2d^2-aw}{2n^2}v^2+\frac{aw-2d^2}{2n^2}v+\frac{awd^2-d^4}{n^4}=0$. Theref. $8v^3+21v^2-29v-50=0$, which divided by $1+v$ gives $8v^2+21v-50=0$; Th. $v=\frac{5\sqrt{20.41}-10.5}{8}=1.5110892496$, Th. $x=7.269209844$, the ſemi-tranſverſe, and EL$=5.513097565$, the ſemi-conjugate.

The ſame anſwered by TOM PROBLEM.

Pl. 5. Fig. 99. SINCE the three ſides of the triangle ABC are given, there will be given the line BF that biſects the baſe.

Let

Let CF, or FA, $=s$, BF$=d$, TF, or FG, $=x$; draw TN, GH, parallel to CA, and it will be, by similar triangles, $d : s :: d - x : \frac{d-x}{d}s =$ NT, and $d : s :: d+x : \frac{d+x}{d}s =$ GH; but by the *Conic Sections* NT$\times$HG$=$OF$^2=\frac{dd-xx}{dd}ss$, whence TF$\times$FO$=$ *maximum*, that is, $\frac{sx}{d}\sqrt{dd-xx} =$ *maximum*, or $ddxx-x^4$ a *max.* and $2ddx\dot{x}-4x^3\dot{x}=0$, reduc'd, $x=\frac{1}{2}d\sqrt{2}$.

QUESTION LXVI. *answered by Mr* R. ROBINSON.

LET $m=$CE$=50$, $n=$EB$=100$, $a=$ the tenſion, or force drawing in the direction of the curve in the point C, $x=$CI, $y=$IH. Then by nature of the *catenaria* $n=2.30258a\times\log.\frac{a+m+\sqrt{m^2+2am}}{a}$, where a is found $=107.4583$. Pl. 5. Fig. 100.

Again, $2my-2yx=$ area $\square$HF, its fluxion $m\dot{y}-y\dot{x}-x\dot{y}=0$; Th. $\dot{y}=\frac{y\dot{x}}{m-x}=\frac{a\dot{x}}{\sqrt{x^2+2ax}}$ by the nature of the curve, hence $y=\frac{ma-ax}{\sqrt{x^2+2ax}}$; then we have $\frac{m-x}{\sqrt{x^2+2ax}}=2.30258\times\log.\frac{a+x+\sqrt{x^2+2ax}}{a}$, ſolv'd $x=16.397$. Th. $y=58.632$, and the area of the greateſt inſcrib'd parallelogram $=3940.42$.

QUESTION LXVII. *answered by Mr* NIC. FARRER.

LET AC repreſent the required curve, where $x=$AB, and $y=$ BC; then $\dot{x}=$bc, $\dot{y}=$Cb; and *per* ſim. triangles, $\dot{y} : \dot{x} :: y : \frac{y\dot{x}}{\dot{y}} =$ BD$=\frac{\overline{m+n}\times ax+x^2}{ma\pm\overline{m+n}\times x}$, which divided by $\dot{x}$ gives $\frac{y}{\dot{y}}=\frac{\overline{m+n}\times ax+x^2}{ma\dot{x}\pm\overline{m+n}\times x\dot{x}}$. Pl. 5. Fig. 101.

Then it is evident that $\frac{y}{\dot{y}}=\frac{ma\dot{x}+\overline{m+n}\times x\dot{x}}{\overline{m+n}\times ax\pm x^2}$, whoſe fluent is hyp. log. $y-=\frac{m}{m+n}\times$ hyp. log. $\frac{\overline{m+n}\times a}{x}\pm 1 : \pm\overline{m+n}\times$ hyp. log. $\overline{\overline{m+n}\times a\pm x}$, the equation or property of the curve.

The ſame anſwered by CLEOPHILUS, *the Propoſer.*

IN the firſt place we have $\frac{\overline{m+n}\times ax+xx}{ma\pm\overline{m+n}\times x}=\frac{yx}{y}$, or $\frac{\dot{y}}{y}=\frac{\overline{max}+\overline{\overline{m+n}\times xx}\cdot\dot{x}}{\overline{m+n}\times ax\pm xx}$. To find the *fluent*, multiply the numerator and denominator of the ſecond member by $\frac{n}{m}x^{\frac{m-n}{n}}$, and the equation will be $\frac{\dot{y}}{y}=\frac{n}{m+n}\times\frac{\frac{m}{n}ax^{\frac{m-n}{n}}\dot{x}\pm\frac{m+n}{n}\times x^{\frac{m}{n}}\dot{x}}{ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}}$. Let $z=ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}$, and $\dot{z}=\frac{m}{n}ax^{\frac{m-n}{n}}\dot{x}\pm\frac{m+n}{n}x^{\frac{m}{n}}\dot{x}$; ſubſtitute $\dot{z}$ and z inſtead of their values, and we ſhall have $\frac{\dot{y}}{y}=\frac{n}{m+n}\times\frac{\dot{z}}{z}$, whoſe *fluent* is log. $y=\frac{n}{m+n}\times$ log. z, or log. $y=\frac{n}{m+n}\times$ log. $ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}$, by reſtoring the value of z. But $\frac{n}{m+n}\times$ log. $ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}$ is $=\overline{\text{log. }ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}}^{\frac{n}{m+n}}$. Theref. log. $y=\overline{\text{log. }ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}}^{\frac{n}{m+n}}$, or $y=\overline{ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}}^{\frac{n}{m+n}}$; conſequently $y^{m+n}=\overline{ax^{\frac{m}{n}}\pm x^{\frac{m+n}{n}}}^{n}=x^m\times\overline{a\pm x^n}$, which is an equation for all ſorts of hyperbolas with reſpect to the diameter when $\overline{a+x^n}$, and of all ſorts of ellipſes when $\overline{a-x^n}$. *Q. E. I.*

QUESTION LXVIII. *anſwered by Mr* N. FARRER.

Pl. 5. Fig. 102. LET NP$=a=400$, GP$=702.25=d$, BD$=512.87654387=2m$, and $n=47.0876543$. Put $x=\mu$M, then KM$=\sqrt{2ax-x^2}$, and then is $\sqrt{2ax-x^2}+.0174$ B$x=d$, where B $=$ degrees in that arc whoſe ſine is $\sqrt{2ax-a^2}$, and $x=312.532$.

Again, let $x=$LQ, $q\sqrt{2x}$, LH$=y+w^2$, LI$=\overline{y-w^2}$; then will

will $2qw=m$, and $w=5.1284$, and $z=\sqrt{n^2-C^2}+\sqrt{y-w^2}=\sqrt{p^2-D^2}+\sqrt{y+w^2}$, hence $\sqrt{n^2-C^2}-\sqrt{p^2-D^2}=$

$4wy$ $\left\{\begin{array}{l}\text{whence } C=y-w\sqrt{2x-y-w^2}+.0174Qx\\ \text{and } D=y+w\sqrt{2x-y+w^2}+.0174Zz.\end{array}\right\}$ where Q and Z = the degrees in the arches, whose verſed ſines are $\sqrt{y-w^2}$, and $\sqrt{y+w^2}$, from whence the value of y may be found; and thence the inclination LF to QD, which is the ſun's true altitude, together with the declination and hour, the latitude may be found by ſpherical trigonometry.

The ſame anſwered by Mr HAUXLEY, *the Propoſer.*

FIRST let ACIDB repreſent the curve of the cycloid, part of whoſe arc is IR, the ray of light paſſing thro' the atmoſphere on *April* 30, AM=512.876, at 5^h 50^m. Let $2b$=CD, EF=1404.5 the length, and b=CE=DF=GH=400, the breadth of the inſcrib'd parallelogram, and let x=IG, and all the other lines drawn as by the figure; then $b+x$=IH the diameter of the generating circle; put $p=3.1416$, then $pb+px$= periphery of the ſame circle, and therefore = the tract or bounding ordinate AB of the cycloid. Now by the property of the circle $\sqrt{IG\times GH}=\sqrt{bx}$=KG=GL, then $l-\sqrt{bx}$=CK. In the right-angled triangle IKG, $\sqrt{KG^2+GI^2}=\sqrt{bx+x^2}$=KI, by *E.* 1. 47. and by M. *Huygens*'s theorem $\frac{8\sqrt{bx+x^2}-2\sqrt{bx}}{6}$= arc KI of the circle, and by the property of the cycloid =CK. Now CK+KG=CG, that is, $\frac{8\sqrt{bx+x^2}-2\sqrt{bx}}{6}+\sqrt{bx}=b$, from whence the value of x may be found. We have now given $x+b$=IH=d, the diameter of the generating circle. Now the eye being ſuppoſed at I, the vertex of the cycloid, we have alſo given the arc IR, the ray of light paſſing thro' the atmoſphere, =512.876, to find the chord IR of the cycloid; in order to which thro' R draw RM parallel to AB, and alſo the correſpondent chord of the circle IN, which by the property of the cycloid is $=\frac{IR}{2}=256.438$; then $\sqrt{HI^2-IN^2}$=NH, from whence the angle IHN=∠INP is eaſily found by *plane trig.* as alſo IP and PN. Then as 360° is to the circle's circumference, ſo is the ∠IQN=2∠IHN, to the arc of the circle IN, and =NR by

Pl. 5. Fig. 103.

by property of the cycloid, Then *per Fig.* PN+NR=PR, and *per E.* 1. 47. $\sqrt{PR^2+PI^2}=IR$, the chord of the cycloid correspondent to the ray IR.

Pl. 5. Fig. 104. Second. We have now given IR the chord of the cycloid correspondent to the ray of light (which with S the sun is in one and the same right line, as AIRS), as also IZ the perp. height of the atmosphere $=47.08=b$, also ID=IN+ND, the sum of the earth's diameter and height of the atmosphere $=b$, to find the $\angle$ZIR, the co-lat. of the sun at the given time, at the place I, where the eye is supposed to be; in order to which, let $x=AI$, then $ZI\times ID=AI\times IR$, by the property of the circle, that is, $xc=bb$, Th. $x=\frac{bb}{c}$. Again, let $y=ZC$, then $b-y=IC$, and let $m=ZD$; then $m-y=CD$, and $\sqrt{my-y^2}=RC$ by the property of the circle. Theref. $my-y^2=c^2-b^2+2by-y^2$, Th. $y=\frac{c^2-b^2}{m-2b}=CZ$, hence $b-y=CI$; then in the right-angled triangle ICR are now given IC and IR, from whence, by *plane trig.* we may easily find $\angle$CIR, the co-altitude of the sun required.

Lastly, we find now given the hour of the day, the declination obtain'd from the day of the month, and the co-altitude found as above, to find the latitude of the place.

QUESTION LXIX. *answered.*

SINCE the latitude of the place $=b$, cosine $=c$, sine of the sun's distance from the pole $=p$, its cosine $=d$, sine of the hour from noon $=x$, its arc $=m$; I suppose it granted that the number of rays falling, in any given moment of tme, on a given horizontal plane, is as the sine of the sun's altitude *directly*, and the square of his distance from that plane *reciprocally*. The sine of the sun's altitude $\pm bd+cp\sqrt{1-xx}$, being multiplied by $\frac{\dot{x}}{\sqrt{1-xx}}=$ the fluxion of m, (or the time), will be as $\pm\frac{bd\dot{x}}{\sqrt{1-xx}}+cp\dot{x}$, the fluxion of heat, whose fluent $cpx\pm bdm$ is the heat required.

QUESTION LXX. *answered by* Tom Problem.

Pl. 5. Fig. 86. LET $b=\frac{2}{3.1416}$, CA$=r$, A$c=x$, AM$=z$, M$c=s$, $dc=y$; then the equation of the curve is $y=bz$, whence $\dot{y}+b\dot{z}$; but by the nature

nature of the circle $\dot{z}=\frac{r\dot{x}}{s}$, Th. $\dot{y}=\frac{br\dot{x}}{s}$, or $\dot{x}=\frac{s\dot{y}}{br}$. Now ſince $\dot{y}$ repreſents the progreſſion or horizontal velocity of the body, Th. $\frac{y}{br}$ is given, therefore the deſcending velocity of the body, which is as $\dot{x}$, will alſo be as s, or Mc, the correſponding ordinate in the circle.

Laſtly, ſince $ss=2rx-xx$, and $\dot{s}=\frac{r-x}{s}\dot{x}$, and the centripetal force at d is as $\dot{s}$, the fluxion of the velocity; Th. this force is as $\frac{r-x}{s}\dot{x}$, or as $\frac{r-x}{s}\times\frac{s\dot{y}}{br}$, that is, (becauſe $\frac{\dot{y}}{br}$ is given), the force is as $r-x$, or Cc, the diſtance from CA. *Q. E. I.*

Again, by Mr R. ROBINSON.

LET the rad. AB$=1$, DC$=x$, GC$=y$, arc DF$=z$, and the quadrantal arc AFD$=m$; then, by the nature of the curve, $1:m::x:z=mx$, and by the nature of the circle $Ba=\sqrt{1-yy}$; Th. $Da=1-\sqrt{1-y^2}$, and $\dot{z}=m\dot{x}=\frac{\dot{y}}{\sqrt{1-y^2}}$; but $\dot{y}:\dot{x}::y:CE=\frac{y\dot{x}}{\dot{y}}=\frac{y}{m\sqrt{1-yy}}$, hence $GE=\frac{y}{m}\sqrt{\frac{m^2-m^2y^2+1}{1-y^2}}$, and, by ſim. triangles, GE : GC :: EB : BH$=\frac{y+\overline{m-mx}\times\sqrt{1-y^2}}{\sqrt{m^2-m^2y^2+1}}$; then is $\frac{-1}{\overline{BH}^2}\frac{m^2y^2-m^2q}{y+\overline{1-x}\times m\sqrt{1-y^2}}$, where $q=1+\frac{1}{m^2}$, its fluxion is $\frac{2m^3y\dot{y}-2m^3x\dot{y}+2m^3y^2\dot{y}-2m^3q\dot{y}-2m^3q\dot{x}+2m^3q\dot{x}y\dot{y}+2m^2q\dot{y}\sqrt{1-y^2}}{y\sqrt{1-y^2}+m-my^2-mx+mxy^2}$ ſubſtitute $\frac{\dot{y}}{m\sqrt{1-y^2}}$ for $\dot{x}$, and the laſt expreſſion becomes $\frac{2m^3y\dot{y}\sqrt{1-y^2}\times\overline{1+y-x-q+qx}:-2m^2qy^2\dot{y}}{\sqrt{1-y^2}\times m+mxy^2-mx-my^2+y\sqrt{1-y^2}}$, hence the law of centripetal force is as $\frac{y\sqrt{1-y^2}\times m^2y+x-1:-mqy^2}{\sqrt{1-y^2}\times m+mxy^2-mx-my^2+y\sqrt{1-y^2}}$

Pl. 5. Fig. 105.

QUEST. LXXI. *answered by* Philofluentimeeanalgegeomaſtrolongo.

THIS queſtion may be reſolved by *page* 194. of Mr *Muller*'s fluxions, or by *page* 272. of *Emerſon*'s fluxions; by the latter we have $vv=bb+\frac{ppa}{n+1.q}-\frac{ppx^{n+1}}{n+1.qa^n}$. Now let the velocity be meaſured by the ſpace uniformly deſcrib'd in one year, then $g=1$, $b=b$, alſo $p=2q$; and ſince here $n=-3$, therefore we ſhall have $vv=bb-pa+\frac{pa^3}{xx}$; and ſince $s=1$, therefore $\dot{t}=\frac{\dot{x}x}{\sqrt{v^2x^2-a^2b^2}}=$

$$\frac{\dot{x}x}{\sqrt{bbxx-paxx+pa^3-a^2b^2}}=\frac{\dot{x}x}{\sqrt{bb-pa}\times\sqrt{x^2-a^2}}.$$

Pl. 5. Fig. 106. 1ſt. To find b and p, let $VO=r=10000$, $CO=d=173$, $VC=r-d=a$, $c=3.1416$.

Then as the time 1 year is to the area of the ellipſis, or $\sqrt{rr-dd}$, ſo is t', a particle of time, to $t'cr\sqrt{rr-dd}$ the area deſcrib'd in that time, and $\frac{2rct'\sqrt{rr-dd}}{a}=$ length deſcrib'd in the ſame time by the velocity at V; and t' : 1 year :: ſo that length : to $\frac{2cr\sqrt{rr-dd}}{a}$, the length deſcribed in a year; Th. $\frac{2cr\sqrt{rr-dd}}{a}=b$.

2d. Again let $e=$ velocity of a body deſcribing a circle at the diſtance CV, then by the laws of centripetal force $r+d:r::bb:ee=\frac{bbr}{r+d}$, and $e=2cr\sqrt{\frac{r}{a}}=$ arc deſcribed in that circle in a year. Then by *Cor.* 9. *Prop.* IV. *Princip.* L. I. $2cr\sqrt{\frac{r}{a}}=\sqrt{2aq}=\sqrt{ap}$, and $ap=\frac{4ccr^3}{a}$, whence $bb-ap=\frac{4ccrr}{aa}\times\overline{rr-dd}-\frac{4ccr^3}{a}=\frac{4ccrr}{aa}\times$ $\overline{rr-dd-ar}$; but $rr-dd-ar=dr-dd=ad$, whence $bb-ap=\frac{4ccrrd}{a}$, and $\sqrt{bb-ap}=2cr\sqrt{\frac{d}{a}}$, whence $\dot{t}=\frac{\dot{x}x}{2cr\sqrt{\frac{d}{a}}\times\sqrt{xx-aa}}$, and the fluent $t=\frac{\sqrt{xx-aa}}{2cr\sqrt{\frac{d}{a}}}=z$ by the queſtion; let $m=2cr\sqrt{\frac{d}{a}}$, then

$tm=$

$tm = \sqrt{xx-aa}$, and $x = \sqrt{aa+t^4m^2} = 1.9354$, the diſtance ſought, CI.

3d. Alſo $\dot{z} = \frac{aab\dot{x}}{x\sqrt{bb-pa}\times\sqrt{xx-aa}} = \frac{aab\dot{x}}{mx\sqrt{xx-aa}}$, and the fluent $z = \frac{-b}{a}\times$ are whoſe ſine is $\frac{aa}{x}$, radius $= a$; and corrected, $z = \frac{b}{m}\times$ arc whoſe coſine is $\frac{aa}{x}$, radius a; that is, $z = \sqrt{\frac{r+d}{d}}\times$ arc whoſe coſine is $\frac{aa}{x}$, and radius a; therefore $z =$ an arc of 456.01 degrees, or 1.267 revolutions.

4th. Laſtly, when t is infinite, $\sqrt{xx-aa}$ is infinite, and x infinite; then coſine $\frac{aa}{x} = 0$, and the arc $= 90° = \frac{1}{4}$ a revolution; therefore $z = \frac{1}{4}\sqrt{\frac{r+d}{d}}$ revolutions $= 1.9171$ revolutions.

QUESTION LXXI. *anſwered.*

LET ABB be the earth's elliptic orbit, R the center, S the focus in which the ſun is ſituate, and A the neareſt *Apſis*; alſo let ACCC be the new curve deſcribed by the earth after the law of attraction is changed in A, and AOO a circle whoſe center is S, and radius SA. Put $SA = a$, and $RA = b$, then $\overline{2ab-a^2}^{\frac{1}{2}}$ is $=$ ſemi-conjugate axis, $2a\times\frac{2b-a}{b} =$ *latus rectum*, and the area of the ellipſe will be $= pb\times\overline{2ab=a^2}^{\frac{1}{2}}$ (ſuppoſing $p =$ area of a circle whoſe radius $= 1$); alſo ſuppoſing $1 =$ velocity wherewith a projectile would deſcribe the circle AOO, $\overline{\frac{2b-a}{b}}^{\frac{1}{2}}$ will expreſs the velocity in A required to deſcribe the ellipſe ABB. Now if we put $SC = x$, an arc of the circle AOO which meaſures the projectile's angular motion $=$ A, and the ſpiral ſpace $SAC = S$; becauſe SA is perpendicular to the path of the moving body, and the law of attraction as the cube of the diſtance inverſely, we ſhall have $\dot{S} = \frac{na}{\overline{2\times n^2-1}^{\frac{1}{2}}}\times\frac{x\dot{x}}{\sqrt{x^2-a^2}}$, and $\dot{A} = \frac{n}{\sqrt{n^2-1}}\times\frac{a^2\dot{x}}{x\sqrt{x^2-a^2}}$, n being $= \overline{\frac{2b-a}{b}}^{\frac{1}{2}}$, and finding the fluents according to *Emerſon*'s rules, and ſubſtituting

Pl. 5. Fig. 107.

$2b-a$

$\frac{2b-a}{b}\Big|^{\frac{1}{2}}$ for n, it will be $S=\frac{a}{2}\times\frac{\overline{2b-a}}{b-a}\Big|^{\frac{1}{2}}\times\overline{x^2-a^2}|^{\frac{1}{2}}$, and $A=\sqrt{\frac{2b-a}{b-a}}$ $\times$ arc Am (of the circle AQO) whose secant is $=x$.

1. Then, because the area described by a body moving in the spiral is equal to the area described in the same time by a body moving in the ellipse, we shall have (from the conditions of the problem) this equation $\frac{a}{2}\times\frac{\overline{2b-a}}{b-a}\Big|^{\frac{1}{2}}\times\overline{x^2-a^2}|^{\frac{1}{2}}=2pb\times\overline{2ab-a^2}|^{\frac{1}{2}}$, whence $x=$ 19354, $=$ the distance sought; *i. e.* the distance of the earth from the sun would be (on this supposition) at the end of two years, almost double its present mean distance.

2. SA : SD (=SC) : : R : secant of the arc Am, whence the arc itself will be found $=59°\ 09'$; consequently A is $=59°\ 09'\times\left(\frac{2b-a}{b-a}\Big|^{\frac{1}{2}}=\right)$ $7.66834=456°\ 08'=$ one gyration $+96°\ 08'$, the angular motion perform'd in two years.

3. When the arc Am is $=90°$, then SD=SC becomes infinite, and the body can no longer revolve about its center S, but will converge towards it asymptote; Theref. $90°\times7.66834=690°\ 09'=$ one gyration $+330°\ 09'$ is the whole angular motion of a body moving in the Curve ACCC. *Q. E. I.*

Note. If the angle A Sb be taken $=29°\ 51'$, and ST be drawn perpendicular to Sb, and $=\sqrt{\frac{SN}{SR}}\times SA$, a line drawn thro' the point T parallel to Sb will be the asymptote of the curve.

New QUESTIONS *to be answered in our next.*

QUESTION LXXII. *By* SIMPLEX.

'TIS required to divide the number z into two such parts x (the greater) and y (the lesser) that zx may be equal to (p) 672, and $\overline{x-y}\times z$ may be (d) 320?

QUESTION LXXII. *By* GEORGE INNES ubique Domo.

BEING one day measuring some timber, I found a square piece under the following conditions, *viz.*

If it were $\left\{\begin{matrix}\text{Inch}\\2\\3\\4\end{matrix}\right\}$ broader $\left\{\begin{matrix}\text{Inch}\\3\\4\\2\end{matrix}\right\}$ thicker $\left\{\begin{matrix}\text{Inch}\\4\\2\\3\end{matrix}\right\}$ longer it would be $\left\{\begin{matrix}10944\\14724\\12729\end{matrix}\right\}$ Inches

Inches more than at firſt; I demand the dimenſions and content?

QUESTION LXXIV. *By Mr* WILLIAM SUTTON.

REquired to divide the cube b^3 into two other cubes, whoſe ſolidities ſhall have any aſſigned relation to each other, as 1 to t; and alſo to divide the ſaid cube into two other cubes, whoſe ſides ſhall have any aſſigned relation to each other, as 1 to s; whereby a method is found to divide the ſaid cube into innumerable pairs, and varieties of cubes, two of which taken together ſhall be always equal to the whole?

QUESTION LXXV. *By Mr* W. CHAPPLE.

GIVEN the hypotenuſe of a right-angled triangle = 100, and the neareſt diſtance of the rightangle from the periphery of the inſcrib'd circle = 8; required the legs and area of the triangle by a ſimple equation, with a general *theorem* for queſtions of this nature?

QUESTION LXXVI. *By Mr* CH. SMITH.

REquired the baſe and hypotenuſe of a right-angled triangle, whoſe perpendicular is = 14.4, and difference betwixt the area of its inſcrib'd ſquare and that of its greateſt inſcrib'd parallelogram = 5.4?

QUESTION LXXVII. *By Mr* EDWARD HAUXLEY.

IN latitude 60° N. being at ſea diſtant 900 leagues from an iſland bearing due weſt, we ſet ſail to the iſland; and tho' we ſteer'd exactly weſt 900 leagues by the log line, and had made our courſe good, yet we ſurpriſingly miſs'd the iſland. Quere the reaſons, and how the place ſail'd from bears, and how I may diſcover my courſe to the iſland?

QUESTION LXXVIII. *By Mr* J. WAINE.

TWO Ships ſet ſail both from the ſame place in a certain north latitude, the one ſteering between ſouth and weſt includes an angle with the meridian ſail'd from of 22° 30′, and makes her way the neareſt ſhe poſſibly can to a port under the equator, the other going between ſouth and eaſt makes an angle with the former meridian of 11° 15′, and after ſailing upon a great circle the ſpace of $203\frac{7}{11}$ leagues, finds ſhe is in latitude 20°, then alters her courſe, and makes her neareſt way to fall in with the other upon the equinoctial. Required the diſtance run by each ſhip, and the latitude of the port ſail'd from?

QUESTION LXXIX. *by Mr* R. HALL.

GIVEN two meridian ſhades of the ſame ſteeple, equal to 287.9 yards, and 56.3 yards; it was alſo obſerved that the ſun was 41° nearer to the elevated pole, when the ſteeple gave the leaſt ſhade. Required the altitude of the ſteeple?

QUESTION LXXX. *by Mr* ROB. ROBINSON.

SOmetime in the ſummer 1746, at 6 o'clock in the morning, I obſerved the ſun's altitude =15°, and alſo when due eaſt his altitude was 22° 58′; 'tis required to find the latitude, and day of obſervation?

QUESTION LXXXI. *by Mr* NICH. FARRER.

IN what north latitude may a ſouth erect dial, declining 75° 54′ 17″ eaſterly, be made, to have the ſtile's height, and ſubſtile's diſtance from the meridian, in the *ratio* of 42.1 to 63.15?

QUESTION LXXXII. *by Mr* JOHN BULMAN.

THERE is a caſk of the firſt variety, whoſe length is 80 inches, the bung diameter is a geometrical mean betwixt the head and length, and the content is =267.3668 ale gallons. Required the bung and head diameters?

QUESTION LXXXIII. *by Mr* SAM. DUNN.

IN an ellipſis, whoſe tranſverſe is 60 chains, and conjugate 40, a right line drawn to the periphery from a point in the tranſverſe (which point is 5 chains diſtant from the ellipſis's center) makes an angle of 70° with the leſſer ſegment of the tranſverſe. Quere the diſtance of the point in the periphery from the neareſt focus?

QUESTION LXXXIV. *by Mr* JOHN TURNER.

Pl. 5. Fig. 108. LET the triangle DEF be given in ſpecie, *viz.* the angle DEF =50 degrees, DFE=60°, and EDF=70°; it is required to inſcribe therein the triangle ABC, whoſe ſides AB=40, BC=48, and AC=50, ſo that the triangle DEF ſhall be a *maximum?*

QUESTION LXXXV. *by Mr* PAUL SHARP.

THERE is a tree 22.539 feet high, ſtanding perpendicular to the plane of the horizon, at a certain place in north latitude, on

on a certain day of the year 1746, which with the ſhadow of its ſummit, traces out an ellipſis, whoſe tranſverſe axis is =10, and periphery =28.36334 poles; by which the latitude of the place, and day of the year may be found, and are here required, with the inveſtigation of the ſame?

QUESTION LXXXVI. *by Mr* WILLIAM BROWN.

REquired two general theorems for finding the ſolidity and ſuperficial convexity of a ſemi-cubical paraboloid, the abſciſſa and ſemi-ordinate of whoſe generating parabola are x and y reſpectively; and when your hand is in, ſuppoſing $x=8$, and $y=12$, tell me what is the ſolidity, and convex ſurface of ſuch a particular paraboloid?

QUESTION LXXXVII. *by Mr* NICH. OATS.

TWO perſons *A* and *B* purchaſe a piece of land, to be bounded by an abſciſs, ſemi-ordinate and curve, whoſe equation is $ax=y^3$, and its area a *minimum*, *A* being to have the triangle VBP, and *B* the remainder. Now there are given AP=90 poles, and AB=40 (perpendicular thereto,) 'tis required to find the ſhare of each, and to determine the points V and M, in PM prolong'd, and PM drawn parallel to AB, by a geometrical conſtruction? Pl. 5. Fig. 109.

QUESTION LXXXVIII. *by Mr* HOLLIDAY.

IF a globe of iron whoſe diameter is 3 inches, be projected perpendicularly upwards with a velocity ſufficient to carry it 1000 feet high in *vacuo*, to find the height to which it will aſcend in air, the time of its aſcent and deſcent, and the velocity it falls again to the earth with, ſuppoſing the body acted on uniformly by gravity, and the air of uniform denſity, and 7000 times rarer than iron, and the ſpace a heavy body deſcends in a ſecond $=15\frac{1}{12}$ feet *Paris*, in which all the other meaſures are ſuppoſed to be taken.

Note, 1 foot *Paris* is =1.0654 feet *Engliſh* =12.785 inches *Engliſh*, by page 188. N° 465 of the *Philoſophical Tranſactions*.

The End of Number V.

ERRATA.

The reader is desired to correct these, and to excuse any other that possibly may have escaped notice.

PAGE 120, line 29 and 30, for *the same circle*, read *equal circles*; in the same page, line the last, read *greater and lesser circles must be the same, and the triangles the same, having the same base*, &c.

Page 123, line 20, *for* $\delta F = 2R = x$ *read* $\delta F = 2R - x$.

157, l. 39, *read* (against α *And.* and *Scheal.*) 12 12 30.

162, l. 5, *read* $\delta d\alpha\pi n$ *for* $\delta dznx$; also in the same l. *for are read area.*

166, l. 9, *read* $211x^3 - 211x^2 +$ &c.

171, l. 10, *for* $\frac{BC}{AB}$ *read* $\frac{BC}{AB}$

176, l. 19, *read* $8v^3 + 29v^2$, &c.

177, last line but one, *read* $y = -$ &c.

178, last line, *for* $q\sqrt{2x}$ *read* $q = \sqrt{2x}$.

179, l. 13, *for* $2b = CD$ *read* $2l = CD$.

180, line the last, *read* $y = bz$.

180, in the margin, *for* pl. 5. fig. 86 *read* pl. 4. fig. 86.

181, line 15, *read* $\frac{-1}{BH^2} =$

N. B. *All persons who are pleased to encourage this undertaking, by answering or sending new problems, essays, dissertations, or experiments in natural philosophy, &c. are desired to direct their letters to* E. Cave, *at* St John's Gate, London, *before the second of* February *next. To prevent mistakes, whoever please to send new problems are desired to send their solutions along with them.*

*** *We observe in the letters from our contributors, how well the method for finding Fluents is received; we shall in our next go on with a continuation from page* 148, *according to their request, part of which would have been continued in this number, had room permitted.*

Fig. 91

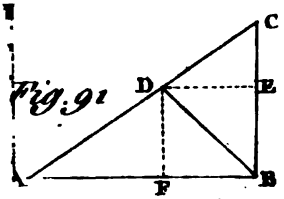
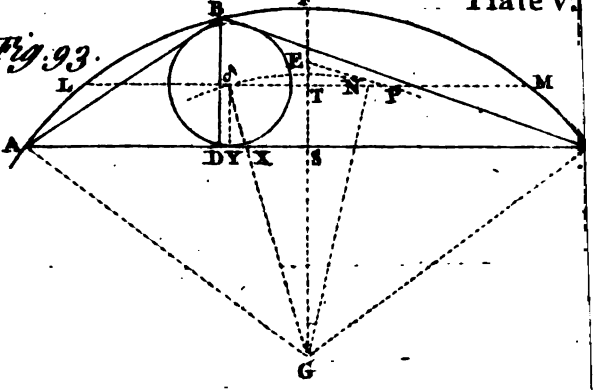


Fig. 93.



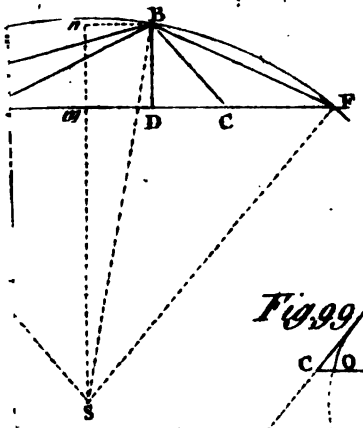


Fig. 99

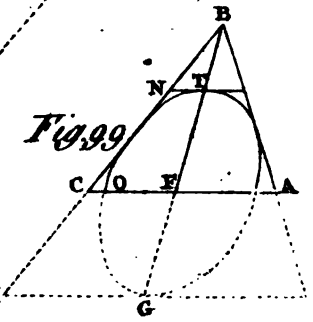
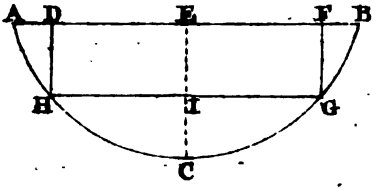


Fig. 100.



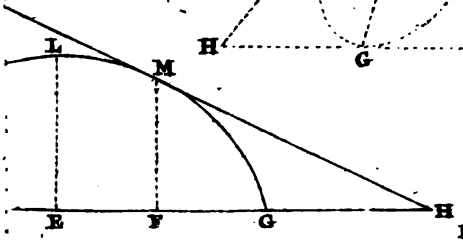


Fig. 101.

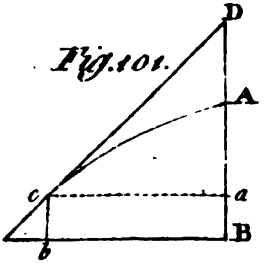


Fig. 107.

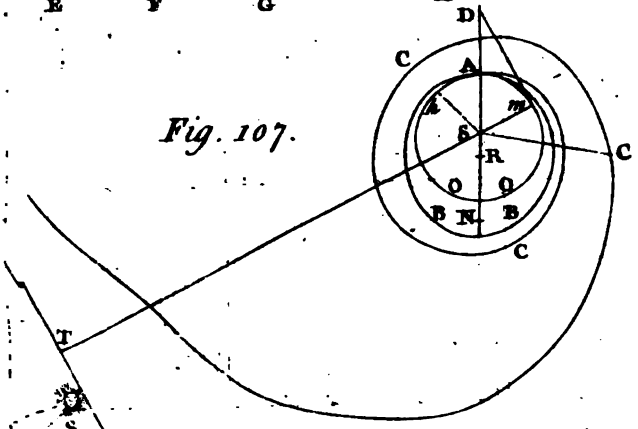


Fig. 108

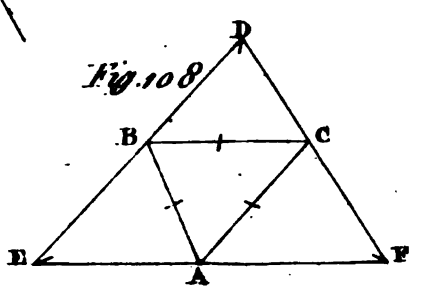
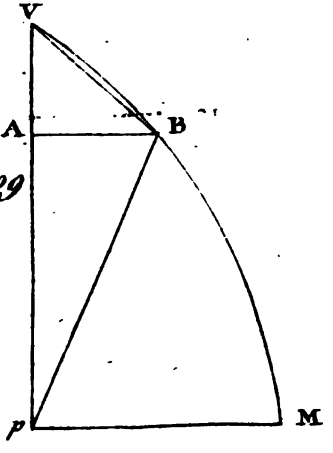


Fig. 109



S. B. (See p. 157.)

Cam	*Jan.* 12. 6h o′.	*Jan.* 13. 6h o′.	*Jan.* 14. 8h 15′.
Sun	♒ 3 12 28	♒ 4 12 27	♒ 5 19 06
		♈ 4 34 11	
Lon	♈ 4 55 40		♈ 4 09 23
		19 03 32	
Lat	18 59 40		19 07 59
C	Markab 14 10 10	Markab 13 47 30	Markab 13 21 30
	Peg. 6 26 25	α And. 8 44 40	Scheat 14 08 35
		γ Peg. 6 30 50	γ Peg. 6 40 35
		β Ar. 27 11 10	β Ar. 27 36 50
		η Pisc. 22 49 00	

	Feb. 13. 6h o′. A.M.	*Feb.* 13. 6h 35′ P.M.	*Feb.* 18. 6h 22′ A.M.
Sun	♓ 5 02 40	♓ 5 26 56	♓ 10 02 37
Lon	♓ 12 23 03	♓ 11 31 18	♓ 4 00 00
Lat	16 15 10	15 44 38	8 48 00
C	Aquil. 42 32 30	α Andr. 28 47 25	α Aquil. 39 12 10
	Caſſio. 52 19 00	at 6h 30′,	at 6h 10′.
		Lu.Cyg. 44 40 40	Venus 36 46 48
		at 6h 41′.	at 6h 33′.

Miſcellanea Curioſa, &c.

NUMBER VI.

An ESSAY *on Fluxions, wherein the firſt Principles are clearly demonſtrated in a very eaſy and conciſe Method, adapted to Beginners.*

SIR,

AS there has been no way of conceiving the nature of mathematical figures more eaſily than by ſuppoſing them generated by the motion of points, lines and ſurfaces (which is no new invention) ſo there is no calculation more eaſy and more certain than, upon that ſuppoſition, of finding and comparing the velocities of that motion. This was an invention reſerved for the illuſtrious Sir *Iſaac Newton*; whence it happens that the operations by fluxions, tho' certain in the concluſion, becauſe evidently agreeing with thoſe drawn from principles that are unconteſted, and of another kind, has been long the object of cavils, as uncertain and not *ſcientific*, or not agreeing with that ſtrictneſs of demonſtration uſed by the ancients. And hence it is, that ſo many who utterly condemn the method of *increments*, as not ſtrictly demonſtrable, yet always have recourſe to that very method, if I miſtake them not, in finding the fluxion of the rectangle of two variable quantities. For what elſe can they mean by their infinitely ſmall parallelogram, which they reject as inſignificant or vaneſcent in reſpect of the others? To me it is evident that if $\dot{x}\dot{y}$ be incomparably ſmaller than $x\dot{y}$ or $y\dot{x}$, then is $x\dot{y}$ or $y\dot{x}$ incomparably ſmaller than xy, for which reaſon theſe quantities are made to vaniſh. For a further illuſtration of this, the reader would do well to conſult Mr *Emerſon*'s excellent Preface to his *Treatiſe of Fluxions*.

If therefore I can demonſtrate the firſt principles of Fluxions, without uſing any infinitely ſmall quantities, or vaneſcent parallelograms, I doubt not but it will be acceptable to beginners, and alſo put an end to all cavils, for the future, from any one that underſtands *Euclid*'s Elements of Geometry, and thoſe of Algebra:

Pl. 6. Fig. 110. I. If, therefore, two right lines AB and CD are ſuppoſed to be generated, in the ſame time, by the uniform motion of two points A and C, then the velocities of the generating points will be to one another in the ſame proportion as the lines themſelves; and the proportion of theſe two lines will be directly, as the velocities drawn into unity, or as any two ſpaces, very ſmall, or of any length whatever, deſcrib'd in the ſame time, by the generating points. Therefore as the method of ſubſtituting algebraic characters for lines is *arbitrary*, as, for inſtance, we may call AB, y, and CD, x, ſo we may expreſs the velocities of the points, whereby theſe two lines are uniformly generated, by what characters we pleaſe, for *example*, by $\dot{y}$ and $\dot{x}$, the former letters, with a point over them; or theſe velocities may be repreſented by two lines A*a*, and C*c*, paſſed over in the ſame time by the points A and C. The velocities of the points A and C are the *Fluxions* of the lines AB and CD, which are properly expreſs'd by $\dot{y}$ and $\dot{x}$, or $1\dot{y}$ and $1\dot{x}$, and repreſented by A*a* and C*c*, any two lines bearing the ſame proportion to one another that the velocities do.

Again, the line AB may be conceived to be traced out gradually by a point moving on from A to B, either with an equable motion, or with a velocity in any manner varied. And the velocity or degree of ſwiftneſs, with which this point moves in any part of the line AB, is called the fluxion of this line at that place.

Fluxions then in general are the velocities with which magnitudes, varying by a continued motion, increaſe or diminiſh; and the magnitudes themſelves are reciprocally called the *Fluents* of thoſe *Fluxions*.

And as different *Fluents* may be underſtood to be deſcribed together in ſuch manner, as conſtantly to preſerve ſome one known relation to each other, the doctrine of Fluxions teaches how to aſſign, at all times, the proportion between the velocities, wherewith homogenous magnitudes, varying thus together, augment or diminiſh.

This doctrine alſo teaches, on the other hand, how from the relation known between the Fluxions, to diſcover what relation the *Fluents* themſelves bear to each other. It is by means of this proportion only, that *Fluxions* are applied to geometrical uſes; for this doctrine never requires any determinate degree of velocity to be aſſigned for the Fluxion of any one Fluent; and the proportion between the Fluxions of magnitudes is aſſignable from the relation known between the magnitudes themſelves. I now proceed to article the ſecond.

II. If

II. If the two rectangles AD and EH are ſuppoſed to be generated in the ſame time by the uniform motion of the lines AB and EF, always parallel to themſelves, along the lines BD and FH, then the rectangle AD will be to the rectangle EH, as AB drawn into the velocity of its motion is to EF drawn into the velocity of its motion; or AB into the velocity of the point B is to EF into the velocity of the point F, as the rectangle AD to the rectangle EH. Pl. 6. Fig. 111, 112.

That is, if we call AB, a; EF, b; BD, y; FH, x; and the Fluxions of BD, $\dot{y}$, of FH, $\dot{x}$, as before; or repreſent them by the ſpaces Bb, and Ff, or bn and fr, paſſed over in the ſame time, by the lines AB and EF; then will $a\dot{y}$ be to $b\dot{x}$, as the rectangle AD to the rectangle EH: And the velocity wherewith the line AB is moved along, drawn into that line, is the *Fluxion* of the rectangle AD; and the velocity, wherewith the line EF is moved, drawn into that line, the Fluxion of the rectangle EH; which are properly expreſs'd by $a\dot{y}$ and $b\dot{x}$, and may be repreſented by the rectangles Ab and Ef, or an and er, deſcrib'd in the ſame time, by the uniform motion of the lines AB and EF.

Pl. 6. Fig. 113, 114.

III. But if the two lines AB and EF ſhould become variable, and flow uniformly; *i. e.* at the ſame time that the points B and F move uniformly along BD and FH, the lines DC and GH ſhould increaſe uniformly from o, or the points B and F, till they become DC and HG; then 'tis evident that by ſuch a motion, inſtead of the rectangles AD and EH, there will only be deſcrib'd the triangles BDC and FHG. And the triangle BDC will be to the triangle FHG, as the rectangle of the generating lines (taken at the ſame time) into the velocities wherewith they are moved along; that is, ſuppoſing DC$=u$, and HG$=z$, at any time, the Fluxions of the triangles may be expreſs'd by $u\dot{y}$ and $z\dot{x}$ (ſuppoſing BD and FH repreſented as above) and may be repreſented by the rectangles bb and ff, or cn and gr, deſcribed in the ſame time by the lines bc and fg, taken at the length they are in any one moment of time. Hence, in general,

IV. The Fluxion of any quantity whatever is equal to the rectangle, or product, of the generating quantity drawn into the velocity of its motion; and may be repreſented by the rectangle of the generating quantity, at any point of time, drawn into a line proportional to the velocity. This definition is plain and general, being eaſily underſtood, and applicable to all Fluxions; as I ſhall afterwards make appear.

V. If the Fluxion of the rectangle of two variable quantities be required, or by what motion of two flowing quantities the rectangled

angled parallelogram EFHG may be deſcribed, I anſwer thus:

Pl. 6. Fig. 115. Let the point F move uniformly along the line FH, while at the ſame time the line HG uniformly increaſes from *o*, at the point F, till it becomes equal to GH; then 'tis evident that by that motion there will be deſcribed the triangle FHG. And at the ſame time ſuppoſe a point from F moves uniformly to D, while the line DG increaſes uniformly from *o*, at the point F, till it becomes =DG; then, for the ſame reaſon, by that motion will be deſcribed the triangle FDC. But theſe two compleat the rectangles; therefore if we ſuppoſe FH=DG=x, and FD=GH=y, and expreſs their Fluxions by $\dot{x}$ and by $\dot{y}$, or repreſent them by the ſpaces fb and dc, paſſed over in the ſame time, the Fluxion of the triangle FHG will be properly expreſs'd by $y\dot{x}$, and repreſented by the rectangle bh; and the Fluxion of the triangle FDG expreſſed by $x\dot{y}$, and repreſented by cb, by *Art.* III. Hence the Fluxion of the whole rectangle DFHG, is properly expreſs'd by $y\dot{x}+x\dot{y}$, and repreſented by the two rectangles cb and bh; I ſay *repreſented* by theſe two rectangles, which are proportional to the Fluxions, but are not the Fluxions themſelves. For the Fluxions are rectangles under the generating quantities and velocities, and ſo far as velocity, which is a property of motion, may be repreſented by a right line, ſo far, and no farther, may the Fluxion of the rectangle be repreſented by two rectangles cb, and bh, provided, the lines cd and fh bear the ſame proportion to one another that the velocities do, wherewith the lines FD and FH are generated.

Pl. 6. Fig. 116. VI. Hence it will be eaſy to find the Fluxion of the parallelopipedon AH; for put AB=a, AD=b, AF=x, and its Fluxion =$\dot{x}$, the Fluxion of the ſolid will be $ab\dot{x}$, and may be repreſented by the little ſolid ABCD $abcd$, being ſuppoſed to be generated by the uniform motion of the plane ABCD along the line AF with an uniform velocity.

Pl. 6. Fig. 117. VII. In like manner if the lines AB=x, AD=y, and AF=z, be all variable, and their Fluxions be $\dot{x}$, $\dot{y}$, and $\dot{z}$, then while x and y move uniformly along AF, they will deſcribe the pyramid AFGHE, and the Fluxion will be $xy\dot{z}$. Suppoſing them to increaſe from *o* till they become FE and FG; z and y by an uniform motion from A to BE and BC by, and uniformly increaſing, will deſcribe the pyramid AEHCB, and their fluxion will be $zy\dot{x}$. And, laſtly, while z and x move uniformly from the point A along the line AD, and uniformly increaſe till they become DG and DC, they will by ſuch a motion deſcribe the pyramid ACDGH, the Fluxion

Fluxion of which will be $zx\dot{y}$. But these three pyramids compose the parallelopipedon, and therefore the sum of their Fluxions, *viz.* $zx\dot{y}+zy\dot{x}+xy\dot{z}$, will be the Fluxions of that solid. *Q. E. I.*

Hence the Fluxion of xy is $x\dot{y}+y\dot{x}$, and the Fluxion of xyz is $xy\dot{z}+xz\dot{y}+zy\dot{x}$ $(=xy\dot{z}+x\dot{y}z+\dot{x}yz)$; but if $x=y=z$, then the Fluxion of xx, or x^2, is $=x\dot{x}+\dot{x}x=2x\dot{x}$, and the Fluxion of xxx or x^3 is $=xx\dot{x}+x\dot{x}x+\dot{x}xx=3xx\dot{x}=3x^2\dot{x}$.

VIII. But now let us suppose that, the line HG should not increase uniformly, but that its square should always bear a certain proportion to the Space passed over by the point F, then the line FG, instead of being a right one, will become a curve, and the area FGH not equal to the area of the triangle in *Art.* III. Yet still, supposing that space generated, in the same time, by the uniform motion of the line HG from the point F to GH, the Fluxion may be properly expres's'd by $y\dot{x}$ (calling GH, y; FH, x; and its Fluxion $\dot{x}$) and represented by the rectangle ff or gr, the difference here, and in *Art.* III. consisting only in y, or the line gf, or the proportion that GH bears to FH. Hence also it will be easy to find the Fluxion of a solid generated by the rotation of any curve around its axis, or by the uniform motion of a circle along a right line, whose radius increases in any given proportion. I have been the fuller in the explanation of these first principles, because they have been (but with prejudice) contested; and if this essay meet with encouragement amongst your mathematical readers (as I can't see why it should not) you shall have some further essays on the same subject. Pl. 6. Fig. 118.

August 6, 1746. *From yours*, &c.

ARCHIMEDES.

A LETTER *from Mr* Edward Hauxley, *shewing how to solve all manner of simple Fluxionary exponential problems.*

SIR,

I HAVE here sent you a general method for solving all manner of simple fluxionary exponential problems, how high soever they be; the rule being so extreamly easy, I hope it will meet with your approbation.

The General RULE.

DIVIDE minus 1 *by a number less* 1 *that expresses the number of* x*'s in the given exponential, and that quotient represented vulgar fraction-wise, will be the hyperbolic logarithm of* x, *when the given exponential is a* minimum; *from whence the number answering is had*

by this ſeries, $1+x+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+\frac{x^4}{1.2.3.4}+\frac{x^5}{1.2.3.4.5}+$ &c. (continued by the ſame viſible law) *by ſubſtituting the ſaid hyperbolic logarithm, and its ſeveral powers, divided as in the ſeries, and added to unity.*

Hence when $\left\{\begin{matrix} x^x \\ \overline{x}|^{xx} \\ \overline{x}|^{x^3} \\ \overline{x}|^{x^4} \end{matrix}\right\}$ is a *minimum*, the hyp. log. of x is $\left\{\begin{matrix} \frac{-1}{1} \text{ or } -1. \\ -\frac{1}{2} \\ -\frac{1}{3} \\ -\frac{1}{4} \end{matrix}\right.$

When $\overline{x}|^{x^x}$ is noted thus, $\overline{x}|^{xx}$, which is the ſame, then -1 divided the numerical index will be the hyperbolic logarithm of x, when the given expreſſion is a *minimum*; and in this caſe, is, as above, $=-\frac{1}{2}$; and generally for exponentials thus noted, -1, being divided by the numerical index, will be the hyperbolic logarithm of x, when any ſuch expreſſion is a *minimum*.

Nov. 22, 1746. *Yours*, &c. E. HAUXLEY.

Solutions to Queſtions in Number V.

QUESTION LXXII. *anſwered by Mr* CH. TATE.

$zx=p=672$; $\overline{x+y}\times z=d=320$; Th. $x+y=z$, and $x\times\overline{x+y}=p$, *per Queſt.* Alſo $\overline{x+y}\times\overline{x-y}=d$; Th. $y^2+xy=p-d$, and $\overline{x+y}|^2=2p-d$; Th. $x+y=z=\sqrt{2p-d}=32$, whence $x=\frac{p}{\sqrt{2p-d}}=21$, and $y=\frac{p-d}{\sqrt{2p-d}}=11$.

Again, by VASSAL MORPETHIENSIS.

BY the *Queſt.* $2p-d$ $(=zx+zy)=zz$; Th. $z=\sqrt{2p-d}=32$, $x=21$, and $y=11$.

QUESTION LXXIII. *anſwered by* Sodalis Mathematicus.

PUT $x=$ ſide of the ſquare, and $l=$ the length, $a=10944$, $b=14724$; then $x^2\times l=$ ſolidity, and, by the queſtion, $\overline{x+2}\times\overline{x+3}\times\overline{l+4}=lx\times\overline{x^2+5x+6}+4x^2+20x+24=lx^2+a$;
$\overline{x+3}\times\overline{x+4}\times\overline{l+2}=lx\times\overline{x^2+7x+12}+2x^2+14x+24=lx^2+b$.
By exterminating l out of theſe two equations, we get $l=\frac{b+2x^2-14x-24}{7x+12}=\frac{a-4x^2-20x-24}{5x+6}$; reduc'd is

$18x^3$

$18x^3+106x^2 \left.\begin{matrix} +204 \\ +\ 5b \\ -\ 7a \end{matrix}\right\} x = 12a-6b-114$. Therefore $18x^3+106x^2-2784x=42840$, whence $x=15$ inches; Th. $l=120$ inches.

Again, by Mr WILLIAM SUTTON.

LET $x=$ inches ſq. $y=$ length, $S=$ ſolidity; and for the three ſolid contents above S, put b, c, and d reſpectively.

Then is $\overline{x+2}\times\overline{x+3}\times\overline{y+4}=S+b$,

Alſo $\overline{x+3}\times\overline{x+4}\times\overline{y+2}=S+c$,

And $\overline{x+4}\times\overline{x+2}\times\overline{y+3}=S+d$;

which being duly involved, and the double of the latter equation taken from the ſum of the two former, gives $y=x+105$; which value of y ſubſtituted in the two former equations, and ſubtracting the firſt from the ſecond, gives this equation $210x=3150$. Th. $x=\frac{3150}{210}=15$. Theref. the piece of timber was 15 inches ſquare, and 10 foot long.

QUESTION LXXIV. *anſwered by Mr* JOHN COLERIDGE.

LET $x=$ ſide of the leſſer cube, when t is greater than unity, and $=$ ſide of the greater cube, when t is leſs than unity; Then *per Queſt.* $x^3 : b^3-x^3 :: 1 : t$; Th. $tx^3=b^3-x^3$, and $x=\frac{b}{\overline{t+1}^{\frac{1}{3}}}$.

Again putting x as before, when s is greater or leſs than unity, and *per Queſt.* $x : \sqrt[3]{b^3-x^3} :: 1 : s$; Th. $s^3x^3=b^3-x^3$, and $x=\frac{b}{\sqrt[3]{s^3+1}}$.

The ſame anſwered by Mr NICH. FARRER.

AS $1+t : b^3 :: 1 : \frac{b^3}{1+t}$, the ſolidity of one cube; and $1+t : b^3 :: t : \frac{tb^3}{1+t}$, the ſolidity of the complemental cube; hence their ſides are $\frac{b}{\sqrt[3]{1+t}}$ and $\frac{b\sqrt[3]{t}}{1+t}$. Now if A repreſent $\frac{1}{\sqrt[3]{1+t}}$, or $\frac{1}{\sqrt[3]{1+s}}$, or any other given ratio univerſally, and $B=\frac{\sqrt[3]{t}}{1+t}$, &c. then will bA and bB be the ſides of the two cubes whoſe ſum is the ſolidity of that given; hence the cube may be divided into innumerable pairs of cubes, according as the ratios are given.

QUESTION

Pl. 6. Fig. 119.

QUESTION LXXV. *answered by Mr* CHAPPLE.

LET the right angle ABC be biſected by the line BH, ſomewhere in which line will be λ the center of the inſcribed circle, and BD will be the neareſt diſtance given in the queſtion; let fall the perpendicular λE (= rad. of the inſcrib'd circle,) then will $\lambda E = BE$ (the $\angle \lambda BE$ being $= \angle B\lambda E = 45°$, and conſequently the $\Delta \lambda BE$ right-angled and iſoſcelar) which put $= r$, and $B\lambda = m$; then, *per* 47 *E.* 1. $m = \sqrt{2r^2}$. Th. $m - r = \sqrt{2r^2} - r = BD = 8$; and putting $8 = d$, and ſolving the equation, we have $r = d + \sqrt{2d^2} = 19.3137$ = radius of the inſcribed circle.

Put $AG = b$ = rad. of the circumſcribing ſemicircle, or half the hypotenuſe $= 50$. Then (*per Cor.* to *Prop.* 7. *p.* 123. *M. Curioſa*) $\sqrt{b^2 - 2br} = \lambda G$, and, *per* 47 *E.* 1. $\sqrt{b^2 - 2br - r^2} = GF = 13.98606$, which put $= e$. Then by the theorem in my ſolution of *Queſt.* 62. *Miſc. Cur.* $\sqrt{2rb + e^2} + r + e = AB$ the greater leg, and $\sqrt{2rb + e^2} + r^2 - e = BC$ the leſſer leg. But if the value of e^2 above found, *viz.* $b^2 - 2br - r^2$, be ſubſtituted inſtead of e^2 in thoſe values of AB and BC, it becomes $b + r + e = AB = 83.29976$, and $b + r - e = BC = 55.32764$. Hence the area of the triangle $= 2304.3898$, and we have this general theorem for finding the legs of *right-angled* triangles from the above Data, *viz.*

$$b + r \pm e = \left.\begin{matrix}\text{greater}\\ \text{leſſer}\end{matrix}\right\} \text{Leg.}$$

COROLLARY.

The ſum of the two legs of any right-angled triangle is = the ſum of the diameters of the inſcrib'd and circumſcribing circles. For $\sqrt{b + r + e} + \sqrt{b + r - e} = 2b + 2r$, and conſequently the ſum of the legs *minus* the hypotenuſe is = the diameter of the inſcrib'd circle; which may alſo be eaſily demonſtrated geometrically.

The ſame anſwered by Mr J. TURNER.

LET $x = \lambda E = BE = F\lambda$. As .7071 : x :: 1 : $x + 8 = B\lambda$; Th. $x = \frac{5.65685}{2929} = 19.3136 = c$. Let $AG = 50 = b$, $FG = \sqrt{z}$; then we ſhall have $AF = b + \sqrt{z}$, $FC = b - \sqrt{z}$, $AB = b + c + \sqrt{z} = d + \sqrt{z}$, $BC = d - \sqrt{z}$; and, by 47 *E.* 1. $2d^2 + 2z = 4b^2$; Th. $z = 2b^2 - d^2 = 196$ *ferè*. Conſequently, $AB = 83.3035$, $BC = 55.3237$, area $= 2304$.

QUESTION

QUESTION LXXVI. *anſwered by Mr* ROB. ROBINSON.

LET BC=m=14.4, and x=AB; then the area of the greateſt inſcrib'd parallelogram is $\frac{mx}{4}$, and that of the ſquare $=\frac{m^2x^2}{\overline{m+x}^2}$; Pl. 6. Fig. 120.

hence $\frac{mx}{4}-\frac{m^2x^2}{\overline{m+x}^2}=5.4=a$, which reduc'd is $x^3-2m-\frac{4a}{m}\times x^2+\overline{m^2-8a}\times x=4am$, in numbers $x^3-30.3x^2+164.16x-311.04=0$, or $x^2-6.3x+12.96\times x-24=0$; hence $x=24$=AB (the other two roots are imaginary, *viz.* $x=3.15\pm\sqrt{-3.0375}$;) hence the hyp. AC=27.98857.

The ſame anſwered by Mr THOMAS WORCESTER.

IN the right-angled $\triangle$ABC, there is given BC=14.4=b, and the difference between the greateſt inſcrib'd parallelogram and the greateſt ſquare abcB=5.4=c. For ab &c. put x, then AB=$\frac{bx}{b-x}$; but $\frac{BC}{2}\times\frac{AB}{2}$= greateſt parallelogram, Th. $\frac{bbx}{4b-4x}-xx$, that is, $bbx-4bx^2+4x^3=4bc-4cx$. Reduced $x=9$; Th. AB=24, and AC=28 nearly.

Again, by VASSAL MORPETHIENSIS.

LET abcB be the inſcrib'd ſquare, and defB the greateſt inſcrib'd parallelogram, in the triangle ABC; put BC=14.4=c, AB=x, Bb=ba, &c. =z; then $c-z$=Cb, and $c:x::c-z:z$; Here $z=\frac{cx}{c+x}$, th. $zz=\overline{\frac{cx}{c+x}}^2$. Let y=Ce, then $c-y$=Be, $c:x::y:\frac{xy}{c}$=de, and Be$\times$de$=\frac{cxy-xyy}{c}$ a *maximum*, which in fluxions, and reduced, gives $y=\frac{c}{2}$, and Be$\times$de$=\frac{1}{4}cx$; hence $\frac{1}{4}cx-\overline{\frac{cx}{c+x}}^2=5.4$, ſolved $x=24$; the hypotenuſe =27.98.

QUESTION LXXVII. *anſwered by Mr* ROB. ROBINSON.

LET A repreſent the place the ſhip ſailed from, I the iſland bound to, which ſince it bore *weſt* at A, muſt be in the right circle AC making right angles with the meridian of A; but ſince the ſhip conſtantly ſailed weſt, ſhe will deſcribe a portion of the Pl. 6. Fig. 121.

leſſer circle ASD, and will be at S, when ſhe has run 900 leagues. As rad. : co-ſine AI=45° :: co-ſine NA=30° : ſine nI=37° 45′ 40″, the lat. of the iſland; as rad. : co-tang. AI=45° :: ſine NA =30° : co-tan. ∠INA=63° 26′ 6″, the iſland's diff. of longitude from the meridian ſail'd from, weſterly. Again as ſine NA : rad. :: AS : QC= 1800 leagues =90°, the difference of longitude made good.

Pl. 6. Fig. 122. Then to find the courſe and diſtance on the rhumb leading between the iſland and ſhip at the beginning and end of ſailing,

Lat. departed from at A	=60° 00′ 00″ M.P.	=4527.4
Lat. iſland at I	=37 45 40	=2450.2
Diff. lat. =444.7778 leag.	=22 14 20	2077.2

Diff. longitude IB =63° 26′ 06″=1268.7 leagues
Diff. longitude made=90 00 00 =1800
Hence KI = 531.3

As AB=692.4 : rad. :: BI=1268.7 : tang. ∠A=61° 22′ 34″ =S.W.bW.¼W. 2° 19′ W, her courſe to the iſland at firſt ſetting out; and as co-ſine ∠A : Ab=444⅞ :: rad. : the diſtance on the above rhumb =928.442 leagues. Again, as SK=692.4 : rad. :: KI=531.3 : tang. ∠KSI=37° 30′=S.E.bS.¼E. 0° 56′ E. her courſe from where ſhe run too to the iſland; then as co-ſine ∠S : SK=444⅞ :: rad. : the diſtance on the laſt rhumb =560.63 leagues, and ſo far ſhe has to ſail to gain the iſland.

The ſame anſwered by Mr NICH. FARRER.

Pl. 6. Fig. 123. S Repreſents the ſhip in 60° of latitude, I the iſland, which muſt lie in a great circle at right-angles to the meridian paſſing thro' S; and as the ſhip ſails weſt, ſhe will deſcribe the parallel SA, and when ſhe has run 900 leagues will be at A, and conſequently muſt have miſs'd the iſland.

In the right-angled ſpherical triangle NSI, the ∠N will be found =63° 26′, the iſland's diff. of longitude from the meridian of S, and NI the complement of lat. =52° 14′, hence the iſland is in lat. 37° 46′. Now here is given the difference of latitude, and difference of longitude, to find by *Mercator*'s ſailing the courſe and diſtance from S to the iſland, which will be found SWbW¼W. 29′ ſoutherly 928.5 leagues; and the ſhip's difference of longitude, made by ſailing in the parallel SA=1800 leagues; hence the difference of longitude from A to the iſland is 531.3 leagues, and the difference lat. =444.7 leagues, and her courſe and diſtance from A to the iſland,

iſland, by *Mercator*'s ſailing, is SE ¼S 56′ eaſterly, 560.5 leagues, and the bearing of S from A is NE ¼E 1° 17′ eaſterly, and diſtance 828.7 leagues on the arc of a great circle.

QUESTION LXXVIII. *anſwered by Mr* EDW. HAUXLEY.

LET N repreſent the north pole, P the port ſail'd from, NPQ the meridian thereof, EQL an arc of the equinoctial, S the place one ſhip came to by ſailing 203 7/11 leagues on the arc of a great circle, which form'd an angle with the meridian, ſail'd from, of 11° 15′, and then arriv'd in the lat. of 20°N. Let NSL repreſent the meridian of the place S, and SL=20° the lat. thereof; Then is NS=70°, its co-lat. Hence in the oblique ſpherical triangle NPS, we have given PS=10° 11′ (=203 7/11 leagues) NS=70°, and ∠NPS =168° 45′ (the ſupplement of the angle of poſition QPS) from whence, by *ſpherical trig.* I find NP=60°, the co-lat. ſail'd from, and the ∠PNS=2° 07′=QL, the difference of longitude made eaſterly. Hence we have now given, in the right-angled ſpherical triangle PQE, PQ=30° the lat. ſail'd from, and the angle EPQ =22° 30′, the angle the arc of the great circle the weſtermoſt ſhip ſail'd along to E on the equator made with the meridian NP, to find the diſtance EP, and the arc of the equator EQ, the difference of longitude made; hence by right-angled *ſpherical trigonometry*, I find PE=32°, and EQ=11° 42′; then EQ+QL=13° 49′= EL. Laſtly, we have now given in the right-angled ſpherical triangle SLE, SL=20°, the lat. of the eaſtermoſt ſhip, and LE= 13° 49′, the diff. of long. made to gain the port E, to find SE the diſtance on the arc of a great circle, and the ∠ESL the angle of poſition at the place ſail'd from. Hence I find ES=24° 09′, the diſtance, and ∠ESL=35° 43′, the angle of poſition, which is S 35° 43′ weſterly. Pl. 6. Fig. 124.

The ſame anſwered by Mr ROB. ROBINSON.

I. In the triangle ABa.

AS rad. : s. AB=10° 10′ 45″ :: s. ∠BAa=11° 15′ : s. BA= 1° 58′ 33″. And rad. : t. AB :: co-s. ∠BAa : t. Aa=9° 59′ 15″, Pl. 6. Fig. 125.

II. In the triangle NBa.

As ſine NB=70° : rad. :: s. aB=1° 58′ 33″ : s. ∠aNB=2° 06′ 09′. And co-s. Ba : rad. :: co-s. NB : s. Da=20° 90′ 45″; to which add Aa, and the ſum is AD=30°, the lat. departed from.

III. In

III. In the triangle ADC,

Rad. : t. ∠DAC=22° 30′ ; : s. AD=30° : t. CD=11° 42′ 03″. Th. Cn=13° 48′ 12″ ; as rad. co-t. AD : : co-s. ∠DAC : co-t. CA=32° 00′ 08″=640.044 leagues, the weſtermoſt ſhip's diſtance run.

IV. In the triangle CBn,

As rad. : co-s. Bn=30° : : co-s. Cn=13° 48′ 12″ : co-s. CB= 24° 8′ 22″=482.789 leagues run by the eaſtermoſt ſhip upon her ſecond courſe, and her whole diſtance is 686.372 leagues.

QUESTION LXXIX. *anſwered by Mr* J. TODD.

Pl. 6. Fig. 126. LET AB repreſent the height of the ſteeple, AC and AD the two given ſhadows, the angles D and ACB the ſun's altitudes, their difference =41°. Let x and y= ſine and co-ſine of half the ſum, s and c= ſine and co-ſine of half the difference of the altitudes, a=287.9=DA, and b=56.3=CA ; then $\frac{cx+sy}{cy-sx}$=t. ACB, and $\frac{cx-sy}{cy+sx}$= t. ∠D; and by *trig.* 1 : a : : $\frac{cx-sy}{cy+sx}$: $\frac{acx-asy}{cy+sx}$=BA, and 1 : b : : $\frac{cx+sy}{cy-sx}$: $\frac{bcx+bsy}{cy-sx}$=AB alſo, Th. $\frac{acx-asy}{cy+sx}=\frac{bcx+bsy}{cy-sx}$. Th. $2xy=2cs\times\frac{a+b}{a-b}$=.97502= ſine of 77° 10′ ; hence ∠D=18° 05′, and AB=94 yards, the height of the ſteeple.

Mr *Robert Robinſon*, after an eaſy algebraical proceſs, finds the ſteeple's height to be 94.01, or 172.41 yards ; and then gives the following geometrical

CONSTRUCTION.

Make AC=56.3, CD=231.6 at A, and upon the middle of DC raiſe the perpendiculars AB and EF, draw DF, making an angle with DA=49°, the comp. of the angle DBC ; then upon F, with the radius FD, deſcribe an arc to cut AB in the points B, b; then will either AB or Ab repreſent the height of the ſteeple.

CALCULATION.

As s. ∠DFE=41° : DE=115.8 : : radius : DF=FB=176.5085; and rad. : DE : : t. ∠FDE=49° : EF=133.2126. Then having Fb=FB, and FG=172.1, find Gb=GB=39.2025 ; then FE+ Gb=

Gb=Ab=172.4151, and EF—GB=AB=94.0101 yards, either of which may be the height of the ſteeple.

Sodalis Mathematicus has given an elegant geometrical conſtruction.

QUESTION LXXX. *anſwered by* VASSAL MORPETHIENSIS.

LET c= ſine of 15°, the ſun's alt. at 6, s= ſine of his alt. when due eaſt =22° 58′, and x= ſine of the latitude; then, *per ſpherics*, $x : 1 :: c : \frac{c}{x}$= ſine of the ſun's declination; again, $1 : s :: x : sx$= ſine ſun's declination alſo. Th. $\frac{c}{x}=sx$; Th. $x=\sqrt{\frac{c}{s}}=$ 54° 31′, and the declination =18° 32′, anſwering to *May* 3, or *July* 19.

Again, by SODALIS MATHEMATICUS.

PUT s= ſine 15°, p= ſine 22° 58′, x the ſine of the latitude, radius =1. Then, by *ſpherics*, $x : s :: 1 : \frac{s}{x}$= ſine declination; and $1 : p :: x : px=\frac{s}{x}$. Th. $x=\sqrt{\frac{s}{p}}=54°\ 32'$, the latitude; and the obſervation was made on *May* 3, or *July* 19.

QUESTION LXXXI. *anſwered by Mr* ROB. ROBINSON.

THE propoſer made a miſtake with regard to the plane's declination, it ſhould have been 45° 54′ 17″, which being corrected, Mr *Robinſon*'s ſolution is as follows:

Here is given the ſtile's height, to the ſubſtile's diſtance from the meridian, as 42.1 : 63.15 :: 2 : 3; call the tangent of 45° 54′ 17″, the plane's declination, m; and put x and y, = ſine and co-ſine of half the ſtile's height; then we have, As tangent of the ſtile's height= $\frac{2xy}{1-2x^2} : 1 ::$ ſine of the ſubſtile's diſtance $=3x-4x^3 : m$. Th. $\frac{2my}{2y^2-1}=4y^2-1$; ſolved, $y=.939693$, the co-ſine of 20°; hence the ſtile's height =40°, ſubſtile's diſtance from the meridian =60, and latitude of the place =22° 31′ 16″ N.

The Propoſer's anſwer.

LET m= co-tangent of declination, x= ſine of half the ſtile's height; then the ſine of the ſtile's height $=2x\sqrt{1-x^2}$, the ſine

of the ſubſtile's diſtance from the meridian $=3x-4x^3$; and ſince rad. : co-tang. declination :: ſine ſubſtile's diſtance : tangent ſtile's height, we have $3m-4mx^2 \times \sqrt{1-4x^2+4x^4} = 2\sqrt{1-x^2}$; hence $x=.3420202$ = ſine of 20°; hence the ſtile's height =40°, diſtance of the ſub-ſtile from the meridian =60°, and the lat. =22° 31′N.

The ſame anſwered by TOM PROBLEM.

Pl. 6. Fig. 127. THE ratio of 42.1 and 63.15 is that of 2 to 3. In the right-angled ſpherical triangle ABC, let t= tang. A the complement of the declination, x= ſine of half the ſtile's height, $\frac{1}{4}$CB; then $2x\sqrt{1-xx}=s$, BC, and $\frac{2x\sqrt{1-xx}}{1-2xx}$= tang. BC; alſo $3x-4x^3=$ ſine of three times the arc that x is the ſine of, or $\frac{3}{4}$BC, that is, $3x-4x^3=s$, AC the ſubſtile's diſtance from the meridian; Th. rad.: s, AC :: tang. A, or cot. declination : tang. BC, that is, $1 : 3x-4x^3 :: t : \frac{2x\sqrt{1-xx}}{1-2xx}$; and $3t-4tx^2=\frac{2\sqrt{1-xx}}{1-2xx}$, which reduc'd gives x, and thence AC, CB; then s, A : s, BC :: rad. : s, AB, or co-s. lat.

Mr *Turner* has given a curious ſolution to this queſtion.

QUESTION LXXXII. *anſwered by Mr* W. KINGSTON.

PUT a=80 inches, n=267.3668 gallons, x= head diameter; then will $\sqrt{ax}$= bung diameter, and $\overline{2ax+x^2} \times \frac{a}{3} = n$; whence $x=\sqrt{\frac{3n}{a}+a^2}-a=20$, the head diameter, and 40= bung diameter.

Again, by VASSAL MORPETHIENSIS.

LET x= the head diameter; then $\sqrt{80x}$= the bung diameter, and $\frac{\overline{xx+160x} \times 80}{1077.1554}$= content in ale gallons =267.3668; hence x=20, and the bung diameter =40 inches.

QUESTION LXXXIII. *anſwered by* Vaſſal Morpethienſis.

Pl. 6. Fig. 128. LET $\overline{AO}^2+\overline{CO}^2=\frac{4}{9}=a$, TB=35=$p$, CB=25=$q$, s and c= ſine and co-ſine ∠EBD, and x=BD. Then *per conics*, $ax\overline{p+x} \times \overline{q-x}$ $(=a \times TD \times DC) = \overline{DE}^2$; again, by *Trigonometry*, $cc : xx :: ss : \frac{s^2x^2}{c^2} = \overline{DE}^2$ alſo; conſequently $a \times \overline{p+x} \times \overline{q-x} = \frac{s^2x^2}{c^2}$. Th. x=6.69, and $\overline{DE}^2$=336.6, OD=11.69, the diſtance of D from the

the focus $F=\sqrt{\overline{OC^2}-\overline{AO^2}}-OD=DF=10.66$, and $\sqrt{\overline{DF^2}+\overline{DE^2}}=FE=21.22$, the required distance.

The same answered by Mr CH. SMITH.

PUT $2t=TC=60$, $2c=2AO=40$, the conjugate, p and $q=$ fine and co-fine of the angle $EBD=70°$, $b=35=TB$, $g=25=BC$, $x=BD$; then $b+x=TD$, and $g-x=DC$. By *trig.* as $q:x::p:\frac{px}{q}=ED$; and by the property of the *ellipsis*, (*see* Steele's *Conic Sections*) as $\overline{ED^2}:CD\times DT::\overline{AO^2}:\overline{TO^2}$, *i. e.* $\frac{p^2x^2}{q^2}:\overline{b+x}\times\overline{g-x}::c^2:t^2$; Th. $\overline{bg-10x-x^2}\times c^2=\frac{p^2x^2}{q^2}$; reduc'd, $xx+.555323x=48.653207$; hence $x=BD=6.70304$ chains; whence $FE=21.27796$.

Queſt. LXXXIV. *anſwered by* Philoſluentimecanalgegeomaſtrolongo.

$s, D:EF::s, E:DF=\frac{s, E\times EF}{s, D}$; but the area of the triangle EDF $=EF\times DF\times s, F=\frac{\overline{EF^2}\times s, E\times s, F}{s, D}=$ *maximum*; Th. $\overline{EF^2}$ is a *maximum*, and $EF=$ *maximum*. Pl. 6. Fig. 129.

2. Draw DA; then $\angle ACF=CDA+CAD$, and $EBA=ADB+BAD$; whence the ſum $ACF+EBA=D+BAC$, and $EBA=D+BAC-ACF$.

3. Let the ſines of the angles, E, F, be s, t; the ſine and co-ſine of $D+BAC$ be p, q; s, $ACF=v$, $AC=a$, $AB=b$, $AF=x$, $AE=y$; then $x+y=$ *maximum*.

4. Then, by *trigonometry*, $a:s::x:v=\frac{sx}{a}$, and, by ſubtraction of ſines, $p\sqrt{1-vv}-qv=s, EBA$.

5. $t:b::p\sqrt{1-vv}-qv:y=\frac{pb\sqrt{1-vv}-bqv}{t}=\frac{pb}{t}\sqrt{1-\frac{s^2x^2}{a^2}}-\frac{bqsx}{at}$.

6. Whence $x+\frac{pb\sqrt{aa-ssxx}-bqsx}{at}=$ *maximum*, or $atx-bqsx+pb\sqrt{aa-ssxx}=$ *maximum*. In *Fluxions* $at\dot{x}-bqs\dot{x}-\frac{pbssx\dot{x}}{}$

$\frac{pbssx\dot{x}}{\sqrt{aa-ssxx}}=0$; and $ta-bqs=\frac{pbssx}{\sqrt{aa-ssxx}}$, $=n$, by ſubſtitution

7. $n\sqrt{aa-ssxx}=pbssx$; Th. $x=\frac{na}{s\sqrt{nn+ppbbss}}$.

The ſame anſwered by Mr W. JEPSON.

BEcauſe the triangle DEF is given in ſpecie, the ſides will always be in the ſame conſtant ratio, and therefore when any one ſide is a *maximum*, the triangle's area will be ſo too. From what is given in the queſtion, we get by *trig.* $\angle EBA+ACF=133°\ 19'$, s and $q=$ ſine and co-ſine of half the ſum, and x and $y=$ ſine and co-ſine of half the difference; then $sy+qx=$ ſine of the greater, and $sy-qx=$ ſine of the leſſer angle. Let $AC=a$, $AB=b$, ſine $\angle AFC=m$, of $AEB=n$, and we have $m:a::sy+qx:\frac{asy+aqx}{m}=FA$, and $n:b::sy-qx:\frac{bsy-bqx}{n}=EA$; Th. $\frac{asy+aqx}{m}+\frac{bsy-bqx}{n}=EF$ a *maximum*; ſubſtitute $c=\frac{as}{m}+\frac{bs}{n}$, $d=\frac{aq}{m}-\frac{bq}{n}$, $\sqrt{1-xx}=y$, and $c\sqrt{1-xx}+dx=$ *maximum*, in Fluxions $\frac{-cx\dot{x}}{\sqrt{1-xx}}+d\dot{x}=0$; Th. $d\sqrt{1-xx}-cx=0$, whence $x=\frac{d}{\sqrt{c^2+d^2}}=.021654$, the natural ſine of 1° 14′ 27″; and Th. ACF = 67° 53′ 57″, EBA = 65° 25′ 3″; hence EF = 100.978, ED = 93.094, DF = 82.348.

The ſame anſwered by Mr TURNER, *the Propoſer.*

Pl. 6. Fig. 130. SUPPOSE the triangle DEF that required, and that upon the ſides AB, BC, CA, there are ſegments of circles deſcribed, capable of containing the given angles E, D, F;

Let nat. ſine $\angle BAE=x$, its co-ſine $=\sqrt{1-x^2}$
$\angle AEB=b$ $=c$
$\angle AFC=d$ $=f$
$\angle BAC=m$ $=n$
} $AB=p$; $AC=q$;

Then $s,ABE=cx+b\sqrt{1-xx}$, ſine $CAF=nx+m\sqrt{1-xx}$, its co-ſine $=mx-n\sqrt{1-xx}$, $s,ACF=dmx-dn\sqrt{1-xx}+fnx+fm\sqrt{1-xx}$; as $b:p::cx+b\sqrt{1-xx}:AE$, and $d:q::dmx-dn\sqrt{1-xx}+fnx+fm\sqrt{1-xx}:AF$. But $\frac{pcx}{b}+p\sqrt{1-xx}+qmx-$

qn

$qn\sqrt{1-xx}+\frac{qfnx}{d}+\frac{qfm}{d}\sqrt{1-x^2}$ is to be a *maximum*; let $s=\frac{pc}{b}+$ $+qm+\frac{qfn}{d}$, and $t=p+\frac{qfm}{d}qn$, and we have $FE=sx+t\sqrt{1-xx}$, its Fluxion $s\dot{x}-\frac{txx}{\sqrt{1-xx}}=0$. Th. $x=\sqrt{\frac{ss}{ss+tt}}=.9032$, the fine of 64° 35′, conſequently AE=47.482.

QUESTION LXXXV. *anſwered by Mr* JOHN TURNER.

LET $2a=10$, the tranſverſe axis; $p=28.36334$, the periphery; $d=\frac{aa-xx}{aa}$; $x=$ ſemi-conjugate axis, and let $Q=7.854$, a quadrant of the circumſcribing circle; then, by *page* 174th of Mr *Emerſon's Fluxions*, $Q\times 1-\frac{d}{2.2}+\frac{3d}{4.4}A+\frac{3.5d}{6.6}B+\frac{5.7d}{8.8}C$, &c. is equal to the length of the elliptical quadrant; or for d putting its equal in the equation $Q\times.70312+\frac{11x^2}{32a^2}-\frac{3x^4}{64a^4}$, &c. $=\frac{p}{4}$, which by reverting the ſeries gives $x=3.999$, or when corrected $=4$ exactly. Hence the conjugate axis is $=8$ poles, and the *latus rectum* $=6.4$ poles. Now let $t=1.366$ poles the tree's height, $d=$ fine of the ſun's declination, $\sqrt{1-d^2}$ its co-fine, $c=$ coſine of the latitude. Then $\frac{2dt\sqrt{1-d^2}}{d^2-c^2}=$ $2a$, and $\frac{2t\sqrt{1-d^2}}{d}=6.4$. Th. $\frac{6.4}{2t}=\frac{\sqrt{1-d^2}}{d}$, the co-tangent of the declination $=2.3426$. So the declination was 23° 07′ N. anſwering to *May* 31, or *June* 21; and by the firſt equation $c=.2356$, the ſine of 13° 38; hence the latitude was 76° 22′ north.

The ſame anſwered by Mr ROBERT ROBINSON.

LET height of the tree $=22.539$ feet $=1.366$ poles $=H$, tranſverſe $=10$ poles $=b$, periphery $=28.36334=P$; then the conjugate is $\frac{b\sqrt{9P^2-35b^2}-3P}{35}=8.0531$ nearly $=A$; alſo put x and $y=$ ſine and co-ſine ſun's declination, and $v=$ co-ſine of the latitude; then $\frac{2Hxy}{x^2-v^2}=b$, and $\frac{2Hy}{\sqrt{x^2-v^2}}=A$; hence $x^2-v^2=\frac{4H^2y^2}{A^2}$

$=2Hxy$

$=\frac{2Hxy}{b}$; Th. $\frac{x}{y}=\frac{2Hb}{A^2}$ = tangent of the ſun's declination = .4212642, anſwering to 23° nearly; hence *May* 31, or *June* 21, are the days of obſervation. Again, putting $q=$ verſed ſine of double the declination, and $p=$ its co-ſine, we have $2v^2=q-\frac{2pH}{b}$ $=.1059109=$ verſed ſine of twice the co-latitude; Th. the latitude is 76° 41′ 45″ north.

The ſame again, by Mr W. JEPSON.

LET $a=$ half the tranſverſe, $p=$ *one fourth* of the periphery, x =half the conjugate, and $q=$ quadrantal arc of the circumſcribing circle; then, by 174th *page* of Mr *Emerſon's Doctrine of Fluxions*, we have this equation, $1-\frac{a^2-x^2}{4a^2}-\frac{3a^4-6a^2x^2+3x^4}{64a^4}-$ $\frac{45a^6-135a^4x^2+135a^2x^4-45x^6}{2304a^6}-$ &c. $=\frac{p}{q}$, which being reduced is $x^6-135x^4+12875x^2=175390.62$, &c. whence $x=4$ exactly. Now having the tranſverſe and conjugate diameters given, the diſtance of the foci is had $=6$, and therefore the diſtance of the tree from each end of the tranſverſe $=2$ and 8 reſpectively, whence the meridian co-altitudes of the ſun are had, from whence the latitude will be found as above.

QUESTION LXXXVI. *anſwered by* TOM PROBLEM.

LET $ax^2=y^3$, $c=3.1416$; then $x=\frac{y^{\frac{3}{2}}}{a^{\frac{1}{2}}}$, and $\dot{x}=\frac{3}{2}\times\frac{y^{\frac{1}{2}}\dot{y}}{a^{\frac{1}{2}}}$, and the Fluxion of the ſolid $=cyy\dot{x}=\frac{3c}{2}\times\frac{\overline{y^2}^{\frac{1}{2}}\dot{y}}{a^{\frac{1}{2}}}$, and the fluent $=\frac{3c}{7}\times$ $\frac{\overline{y^3}^{\frac{1}{2}}}{a^{\frac{1}{2}}}=\frac{3cy^2x}{7}$, the ſolidity required.

Again, the Fluxion of the curve $=\dot{y}\sqrt{1+\frac{9}{4a}y}$, and the Fluxion of the ſurface $=2cy\dot{y}\sqrt{1+\frac{9}{4a}y}$, and the fluent $\frac{-27a^2c+18acy}{10}\times$ $\overline{1+\frac{4}{9a}y}^{\frac{3}{2}}+\frac{27a^2c}{10}$, the ſurface of the paraboloid.

The

The ſame anſwered by Mr JAMES WAINE.

FROM the equation $ax^2=y^3$, its Fluxion is $\dot{y}\sqrt{1+\frac{9}{4a}y}$, and the correct fluent is $\frac{8a}{27}\times\overline{\frac{4a+9y}{4a}}\Big|^{\frac{3}{2}}-\frac{8a}{27}=z$, for the length of the curve, which muſt be obtained firſt; ſee *page* 170, of Mr *Emerſon*'s *Fluxions*. By exterminating y, we have $\frac{4a}{9}\times\overline{1+\frac{27z}{8a}}\Big|^{\frac{2}{3}}-\frac{4a}{9}=y$, which ſubſtituted for y in the expreſſion $cy\dot{z}$ makes $\frac{4ca\dot{z}}{9}\times\overline{1+\frac{27z}{8a}}\Big|^{\frac{2}{3}}-\frac{4ac\dot{z}}{9}$, for the fluxion of the ſurface, and its fluent is $=\frac{4.8caa}{5.9.9}\times\overline{1+\frac{27z}{8a}}\Big|^{\frac{5}{3}}-\frac{4acz}{9}-\frac{4.8caa}{5.9.9}$; and then for the ſolidity, write the value of $\dot{x}$ in $\frac{cyy\dot{x}}{2}$, and finding the fluent gives $\frac{3cxyy}{14}$ for the ſolidity, whence (making $c=3.1416$) the curve ſurface is $=581.9704$, and the ſolidity $=1551.04915$.

The ſame by Mr ROBERT ROBINSON.

LET $d=3.141592$, and ſince $ax^2=y^3$, Th. $x=\frac{y^{\frac{3}{2}}}{a^{\frac{1}{2}}}$, and $\dot{x}^2=\frac{9y\dot{y}^2}{4a}$; then will $2dy\sqrt{\dot{x}^2+\dot{y}^2}=2dy\dot{y}\times\overline{1+\frac{9}{4a}y}\Big|^{\frac{1}{2}}=\overline{\frac{9}{4a}+y^{-1}}\Big|^{\frac{1}{2}}\times 2dy^{\frac{1}{2}}\dot{y}$, whoſe fluent, by form 15th of Mr *Emerſon*'s *Fluxions*, is $\frac{16ad}{45}\times\overline{y-\frac{8a}{27}:\times\frac{9y}{4a}+1}\Big|^{\frac{5}{2}}$, which corrected, putting $A=\frac{128a^2d}{1215}$, is the curve ſurface $=\overline{\frac{16ady}{45}-A:\times\frac{9y}{4a}+1}\Big|^{\frac{5}{2}}:+A=582.61041$.

Again, ſince $\dot{x}=\frac{2}{3}a^{-\frac{1}{2}}y^{\frac{1}{2}}\dot{y}$, we have the fluent of $dy^2\dot{x}=\frac{3dy^3}{7}\sqrt{\frac{y}{a}}=\frac{3}{7}d\dot{x}y^2$; hence the ſolidity is $\frac{3}{7}$ of its circumſcribing cylinder $=1551.0528$.

QUESTION

QUESTION LXXXVII. *anſwered by Mr* SAM. DUNN.

Pl. 6. Fig. 131.

PUT AP=90=a, AB=40=c, VP=x; then will VA be $x-a$, whence by the nature of the curve $x-a : c^2 :: x : \frac{c^2x}{x-a}$, and $\sqrt{\frac{c^2x}{x-a}}$ = PM, which multiplied into $\frac{2}{3}x$ gives $\sqrt{\frac{4c^2x^3}{9x-9a}}$, the area, a *minimum*, in Fluxions $72c^2x^3\dot{x}-108ac^2x^2\dot{x}$ =0. Th. $x=\frac{3}{4}a$, that is, VA=$\frac{1}{4}$AP, hence PM=69.2820, whence A's part =16^A 3^R 20^P, and B's =22^A 0^R 15^P.

Mr *Dunn* has alſo given a very elegant and conciſe conſtruction to this Problem.

Mr ROBERT ROBINSON *gives the following*

CONSTRUCTION.

HAVING given AP=90, AB=40, and the area a *minimum*, biſect AP in C, and make AV=AC; draw CB, upon which raiſe the perpendicular BD; then will AD be the parameter, $\frac{1}{4}$ of which ſet from V to F and f; then with the extent Pf, and one foot in F, cut PM in M.

CALCULATION.

Since AV=$\frac{1}{4}$AP=45, th. VP=135, and 45 : $\sqrt{40\times40}$ :: 135 : 4800, whoſe ſquare root is PM=69.282032; hence the area of the triangle VBP=A's part =16^A 3^R 20^P, and B's part = 22^A 0^R 15^P.

DEMONSTRATION.

Let m=AB, n=AP, and y=AV; then $y^{\frac{1}{2}} : m :: \overline{y+n}^{\frac{1}{2}} : \text{PM} = m\sqrt{1+\frac{n}{y}}$. Th. area=$\frac{2}{3}m\sqrt{\frac{\overline{n+y}^3}{y}}$=a *minimum*, or $\overline{n+y}^3\times y^{-1}$, its Fluxion is $3y^{-1}\dot{y}-:y^{-2}\dot{y}\times\overline{n+y}=0$; Th. $y=\frac{1}{2}n$. The reſt of the conſtruction is evident. *Q. E. D.*

QUESTION LXXXVIII. *anſwered by Mr* NICH. FARRER.

LET m= diameter of the globe, q= its denſity, p= denſity of the medium, $n=15\frac{1}{172}$, v= the velocity projected with in *vacuo*, x= ſpace deſcribed from the beginning of motion, T= the time deſcribing x, and V= the velocity at the end of the time T; then when the globe aſcends, (by Sir *Iſaac Newton*'s *Principia*) T=

.0349066mq

$\frac{.0349066mq}{\sqrt{q-p}\times\sqrt{3mnp}}$ x degrees of that arch whoſe tangent is $\frac{v\sqrt{3p}}{q-p\times 16mn}$ $=7.23$ ſeconds, the time of aſcent. $x=\frac{9.21034mq}{3p}$ x log. $1+\frac{3pv^2}{q-p\times 16mn}=832.31$ feet, the height. Again, for the deſcent putting $N=$ log. $\frac{3px}{9.21034mq}$, we have $V=\sqrt{\frac{q-p\times 16mn\times N-1}{3NP}}$ $=205.5$ feet *per* ſecond, the velocity the deſcending ball ſtrikes the earth; hence T the time of deſcent $=\frac{2.30258mq}{\sqrt{3qnpy-3mnp^2}}$ x log. $\frac{\sqrt{q-p\times 16mn}+\sqrt{3pV^2}}{\sqrt{q-p\times 16mn}-\sqrt{3pV^2}}=7.651$ ſeconds.

The ſame anſwered by Mr ROBERT ROBINSON.

SINCE the perpendicular height in *vacuo* is 1000 feet, the velocity *per* ſecond is $2\sqrt{1000\times 15\frac{1}{1\frac{1}{2}}}=100\sqrt{6\frac{1}{3\frac{1}{6}}}=245.62845$ feet $=a$, diameter of the globe $=3$ inches $=0.25$ feet $=m$, its denſity $=7000=n$, the denſity of the medium $=1=d$, $15\frac{1}{1\frac{1}{2}}=r$; alſo let $x=$ the ſpace deſcribed in the time y, and $v=$ the velocity at the end of that time; likewiſe let $n-d=f$, $N=$ number anſwering log. $\frac{dx}{3.070114mn}$, $A=$ the degrees in the arc whoſe tangent is $\frac{a}{4}\sqrt{\frac{3d}{fmr}}$, and radius $=$ unity, $B=n\sqrt{\frac{m}{3fdr}}$, $D=4\sqrt{\frac{fmr}{3d}}$, $E=$ log. of $\frac{16fmr+3da^2}{16fmr}$; then, by Mr *Emerſon*'s *Fluxions*, $y=\frac{2mn\times.0174533}{\sqrt{3fmrd}}$ $A=7.232009$ ſeconds, the time of the aſcent.

$x=\frac{4mn\times 2.302585}{3d}E=832.310487$ feet, the height of the aſcent.

$v\ 4\sqrt{\frac{mrf\times N-1}{3Nd}}=205.499796$ feet *per* ſecond, the velocity, and the end of the fall.

$y=2.302585\,B\times$ log. $\frac{D+v}{D-v}=7.651572$ ſeconds, the time of deſcent.

Anſwered by the Propoſer.

THIS problem being exactly of the ſame nature as that propo in page 41. N° II. *Miſcellanea Curioſa Mathematica*, we h the ſolution of it page 43, as follows;

Let d= diameter of the globe, 3 inches $=\frac{1}{4}$ foot.

$p=1$.

$q=7000$.

$s=15\frac{1}{11}$.

$b=245.6$, the velocity generated in falling 1000 feet.

x= height deſcribed.

t= time of deſcribing x.

v= velocity at the end of the time t. Then

1. When the ball aſcends $t=\frac{2dq\times.01745}{\sqrt{q-p\times 3dsp}}\times$ degrees in the arc whoſe tangent is $\frac{b}{4}\sqrt{\frac{3p}{q-p.ds}}$, that is, $t=.217\times$ degrees in the arc, whoſe tangent is .655; therefore $t=.217\times 33.225=7.21$ ſeconds, the time of aſcent.

2. The whole height of the aſcent $x=\frac{2.30258\times 4dq}{3p}\times$ log. $\frac{\sqrt{q-p\times 16ds+3pbb}}{\sqrt{q-p\times 16ds}}=5372.7\times$ log. $1.4287=5372.7\times .15494=832.45$, the whole aſcent.

3. Put $G=q\sqrt{\frac{d}{q-p\times 3ps}}=6.2202$, $H=4\sqrt{\frac{q-p\times ds}{3p}}=375.18$; then $x=2.3025\times\frac{4dq}{3p}\times\log.\frac{HH}{HH-vv}$, or $\frac{3px}{2.30258\times 4dq}=\log.\frac{HH}{HH-vv}$, where x is known. Put $R=1.4287$, the number belonging to the logarithm of $\frac{3px}{2.30258\times 4dq}$; Th. $R=\frac{HH}{HH-vv}$, whence $v=H\sqrt{\frac{R-1}{R}}=205.5$, the velocity in falling to the earth again.

4. $t=2.3025G\times\log.\frac{H+v}{H-v}=14.322\times.53432=7.65$ ſeconds, the time of falling to the earth again.

5. The

5. The greateſt velocity the globe can acquire by an infinite deſcent, is H, or $\sqrt{\frac{q-p}{3p} \times 16ds} = 375.18$.

This problem was propoſed and ſolved by a conceited foreigner, who is perpetually boaſting of his own performances, and at the ſame time depreciating thoſe of others, eſpecially the *Engliſh* mathematicians (againſt whom he ſeems to have a particular ſpite,) not excepting Sir *Iſaac Newton* himſelf. The ſolution, on which he ſpends no leſs than 20 pages in quarto, is as follows.

The time of aſcent $= 5\frac{3}{4}$ ſeconds, the height $582\frac{3}{8}$ feet, the time of deſcent $6\frac{17}{25}$ ſeconds, and the greateſt velocity the globe can acquire is the ſame as that of a body falling freely thro' $583\frac{3}{8}$ feet; moſt of which are wide enough *from the truth.* H

New QUESTIONS *to be anſwered in our next.*

QUESTION LXXXIX. *By Mr* W. KINGSTON.

THERE is a number conſiſting of three places, whoſe digits are in a geometrical proportion; if this number be divided by the difference between the 3d and 2d digits, the quotient will be 62; and if to the number you add 594, the digits will be inverted. *Quere* the number?

QUESTION XC. *By Mr* W. CHAPPLE.

BETWEEN 8 and 9 o'clock, *Quere* the true time when the hour index and minute index are perpendicular to each other; with a general theorem for finding the time when the indices make any given angle in any given hour?

QUESTION XCI. *By* LOMBARD STREET.

THE diſcounting of a bill, which at a certain rate *per cent. per annum* came to 5*l.* 12*s.* would at 1*l. per cent.* more have coſt 6*l.* 6*s.* and at 1*l. per cent.* leſs, no more than 4*l.* 16*s.* The value of the bill, time when due, and rate of intereſt are required?

QUESTION XCII. *By Mr* JOHN HAMMOND.

IF a cubical foot of braſs, &c. were to be drawn into wire, of $\frac{1}{10}^{th}$ of an inch in diameter, is is required to determine the length of the ſaid wire, allowing no loſs in the metal?

QUESTION

QUESTION XCIII. *By Mr* W. HART.

SUPPOSE three ſpherical, perfectly elaſtic bodies, D, E, F; the weight of D=40 pounds, of F=50 pounds; 'tis required to find the velocity and weight of the body E, ſo that D, with 7 degrees of velocity, ſtriking E at reſt; and E, with the motion acquired by the ſtroke, ſtriking F at reſt, the motion that F acquires by the ſtroke, ſhall be =6 degrees of velocity?

QUESTION XCIV. *By Mr* ROBERT ROBINSON.

SUPPOSE a ball be projected from the earth's ſurface, admitting the medium non-reſiſting; and let

m=ſine, n=co-ſine, b=tangent of the angle of direction, rad.=r,
w=height of the range,
v=amplitude,
T=the time of projection,
d=the height of the perpendicular ſhot with the ſame *impetus*;

'Tis required, from theſe five quantities, to determine theorems for all poſſible *data*, and diſpoſe them into a table fit for uſe?

QUESTION XCV. *By Mr* JOHN TODD.

IN a right-angled triangle ABC, right-angled at B, there is given the right line AE, biſecting the angle A, and falling upon the oppoſite ſide BC=10, alſo the ſide of the inſcribed ſquare =4. Required the triangle's ſide and area?

QUESTION XCVI. *By Mr* NICH. FARRER.

FROM an eminence viewing an equilateral triangular field, in which ſtood a large tree, one of my companions told me, he once meaſured the diſtance from thence to the corners, and found them 30, 40 and 50 chains. *Quere* the content in acres?

QUESTION XCVII. *By Mr* THOMAS COWPER.

AT a certain place the ſun's altitude at 5 and 6 o'clock in the afternoon was found 24° 14′, and 14° 31′ reſpectively. Required the latitude of the place, and ſun's declination, by a ſimple equation?

QUESTION XCVIII. *By Mr* SAMUEL DUNN.

IF a cone's ſlant height be 40, what is its ſolidity when a *Maximum*?

QUESTION

QUESTION CXIX. *By Mr* W. BROWN.

IF a circle's diameter be 24, what is the length of the chord and verſed ſine of ſuch a ſegment, whoſe area ſhall be juſt one third of the circle's area?

QUESTION C. *By Mr* SAMUEL DUNN.

IF a cylinder's whole ſurface be 100, what is its ſolidity when a *maximum*?

QUESTION CI. *By Mr* EDW. HAUXLEY.

WHAT is the value of x, when $\sqrt[x^{\frac{1}{3}}]{x^{\frac{6}{13}}}$ is a *minimum*?

QUESTION CII. *By Mr* JAMES WAINE.

ADMIT there be a punch-bowl in form of a parabolic conoid, that when it has ſuch a quantity of liquor in it whoſe ſurface riſeth juſt one half of the bowl's altitude, hath ſo much of the internal ſuperficies immerſed therein, as is in proportion to the whole ſuperficial concavity as 1 is to 2.4261136; and if the focus of the generating parabola be 2 inches diſtant from the vertex; I would know the dimenſions of the bowl?

QUESTION CIII. *By Mr* WILLIAM SUTTON.

A Gentleman having a trilinear piece of land, BVE, to divide into two equal parts, whoſe content is known to be 9.9795 acres, two ſides whereof are curves, the baſe BE a right line, and the nature of each curve is expreſs'd by the two following equations, *viz.* BV by $ax^3-y^2=0$, and VE by $ax^2-y^3=0$, when $a=1$, the perpendicular VD was the abſciſſa to both curves, and being deſirous of having it exactly divided by a line VM drawn ſomewhere upon the baſe BE, has made his application to the contributors of the *Miſcellanea Curioſa Mathematica*, hoping in the next number to know the length of the lines BM and VM, and the method of finding the ſame? Pl. 6. Fig. 132.

According to the advices from our correſpondents, we obſerve how well the method for finding Fluents is received; the following is a continuation, from page 148. *and each ſucceeding number, if room per-*

 mit,

mit, will contain ſome further examples, till we have gone thro' Mr Emerſon's *excellent table.*

EXAMPLE VII.

To find the *Fluent* of $\frac{c\dot{x}\sqrt{x}}{ab^3+cx^3}$.

Here, by *Form* 5, $z=x$, $n=3$, $\alpha=ab^3$, $\beta=c$, and φ, or *Fluent* of $\frac{x^{\frac{1}{2}}\dot{x}}{ab^3+cx^3}=\frac{2N}{3\sqrt{ab^3c}}\times$ degrees whoſe tangent is $\sqrt{\frac{cx^3}{ab^3}}$, and *Fl.* $\frac{c\dot{x}\sqrt{x}}{ab^3+cx^3}=\frac{2N}{3b\sqrt{abc}}\times$ degrees in the arc whoſe tangent is $\frac{x}{b}\sqrt{\frac{cx}{ab}}$.

EXAMPLE VIII.

To find the *Fluent* of $\frac{11y^{-2}\dot{y}}{1+aay^{-2}}$.

Here, by *Form* 5, $z=y$, $n=-2$, $\alpha=1$, $\beta=aa$, and *Fl.* $\frac{y^{-2}\dot{y}}{1+aay^{-2}}=\frac{2N}{-2\sqrt{aa}}\times$ degrees whoſe tangent is $\sqrt{\frac{aay^{-2}}{1}}$, or $\frac{a}{y}$, and *Fl.* $\frac{11y^{-2}\dot{y}}{1+aay^{-2}}=\frac{11N}{a}\times$ degrees in the arc whoſe tangent is $\frac{a}{y}$.

EXAMPLE IX.

Let the *Fluxion* $\frac{6\dot{x}}{aa-xx}$ be propoſed.

Comparing this with *Form* 6. you'll have $z=x$, $n=2$, $\alpha=aa$, $\beta=1$, and *Fluent* of $\frac{\dot{x}}{aa-xx}=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{\sqrt{\alpha}+\sqrt{\beta}z^n}{\sqrt{\alpha}-\sqrt{\beta}z^n}=\right)\frac{L}{2\sqrt{aa}}\times\log.\frac{\sqrt{aa}+\sqrt{xx}}{\sqrt{aa}-\sqrt{xx}}=\frac{L}{2a}\times\log.\frac{a+x}{a-x}$, and *Fluent* $\frac{6\dot{x}}{aa-xx}=\frac{6L}{2a}\times\log.\frac{a+x}{a-x}$.

Or *Fl.* $\frac{\dot{x}}{aa-xx}=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{z+\beta z^n+2\sqrt{\alpha\beta z^n}}{z-\beta z^n}=\right)\frac{L}{2a}\times\log.\frac{aa+xx+2\sqrt{a^2x^2}}{aa-xx}$; Therefore *Fluent* $\frac{6\dot{x}}{aa-xx}=\frac{3L}{a}\times\log.\frac{aa+xx-2ax}{aa-xx}$.

Or *Fl.* $\frac{\dot{x}}{aa-xx}=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}}=\right)\frac{L}{2a}\times\log.\frac{x+a}{x-a}$; and *Fluent* $\frac{6\dot{x}}{aa-xx}=\frac{3L}{a}\times\log.\frac{x+a}{x-a}$.

Or,

Or, lastly, *Fl.* $\frac{\dot{x}}{aa-xx} = \left(\frac{L}{n\sqrt{a\beta}} \times \log. \frac{a-\beta x^n}{a+\beta x^n-2\sqrt{a\beta x^n}}\right) \frac{L}{2a}$ $\times \log. \frac{aa-xx}{aa+xx-2\sqrt{a^2x^2}}$, and *Fl.* $\frac{6\dot{x}}{aa-xx} = \frac{3L}{a} \times \log. \frac{aa-xx}{aa+xx-2ax}$.

So that the *Fluent* may be expreſſed any of theſe ſeveral ways. Now, ſuppoſe you want to know the value of the *Fluent* in numbers, Let $a=20$, $x=43$; then as x is greater than a, we will take the *Fluent* $= \frac{3L}{a} \times \log. \frac{x+a}{x-a}$, and $\frac{x+a}{x-a} = \frac{63}{23} = 2.73913$; ſeek for this number in the table of logarithms, and you will find its log. $=$ 0.4376127. Therefore the *Fluent* of $\frac{6\dot{x}}{aa-xx}$, or, which is the ſame thing, of $\frac{-6\dot{x}}{xx-aa}$, is $\frac{3L}{20} \times .4376127$.

Or if you ſuppoſe $a=20$, and $x=$ infinity, then $\frac{x+a}{x-a}=1$, whoſe logarithm is 0, and then the *Fluent* of $\frac{-6\dot{x}}{xx-aa} = \frac{3L}{20} \times 0 = 0$.

But if you will ſuppoſe $a=20$, $x=12$, then $\frac{a+x}{a-x} = \frac{32}{8} = 4$, whoſe logarithm is 0.60206, and then the *Fl.* $\frac{6\dot{x}}{aa-xx}$ is $= \frac{6L}{2a} \times 0.60206$ $= 2.30258 \times \frac{3 \times 0.60206}{20}$.

And if you ſuppoſe $a=20$, $x=20$, then $\frac{x+a}{x-a} = \frac{40}{0} =$ infinity, whoſe logarithm is infinite; therefore in this caſe *Fl.* $\frac{6\dot{x}}{aa-xx} = \frac{6L}{2a}$ $\times$ infinity $=$ infinity.

EXAMPLE X.

To find the *Fluent* of $\frac{ax^{-\frac{1}{2}}\dot{x}}{a-6x}$.

If this be compared with *Form* 6. we have $z=x$, $n=1$, $a=a$, $\beta=6$, and *Fluent* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta x^n}}{\sqrt{a}-\sqrt{\beta x^n}}\right) \frac{L}{\sqrt{6a}} \times \log.$ $\frac{\sqrt{a}+\sqrt{6x}}{\sqrt{a}-\sqrt{6x}}$ for the *Fluent*, when a is greater than $6x$.

Or

Or φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{\sqrt{6x}+\sqrt{a}}{\sqrt{6x}-\sqrt{a}}$, for the *Fluent*, when $6x$ is greater than a.

Or φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\alpha+\beta z^n+2\sqrt{\alpha\beta z^n}}{\alpha-\beta z^n} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{a+6x+2\sqrt{6ax}}{a-6x}$, the *Fluent* when a is greater than $6x$.

Or, laſtly, φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\alpha-\beta z^n}{\alpha+\beta z^n-2\sqrt{\alpha\beta z^n}} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{a-6x}{a+6x-2\sqrt{6ax}}$, being the *Fluent* if a is greater than $6x$; then multiply any of theſe by a, you have the *Fluent* required.

EXAMPLE XI.

Let the *Fluent* of $\frac{7bx\dot{x}}{d^4-x^4}$ be required.

By *Form* 6. $z=x$, $n=4$, $\alpha=d^4$, $\beta=1$, and φ, or *Fluent* of $\frac{x\dot{x}}{d^4-x^4}$ $= \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\alpha}+\sqrt{\beta z^n}}{\sqrt{\alpha}-\sqrt{\beta z^n}} =\right) \frac{L}{4dd} \times \log. \frac{dd+xx}{dd-xx}$, the *Fluent* when dd is greater than $\dot{x}x$.

Or *Fl.* $\frac{x\dot{x}}{d^4-x^4}$, or *Fl.* $\frac{-x\dot{x}}{x^4-d^4} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}} =\right) \frac{L}{4dd} \times \log. \frac{xx+dd}{xx-dd}$, being the *Fluent* when xx is greater than dd.

Therefore the *Fluent* of $\frac{7bx\dot{x}}{d^4-x^4}$ is $= \frac{7bL}{4dd} \times \log. \frac{dd+xx}{dd-xx}$, or $= \frac{7bL}{4dd} \times \log. \frac{xx+dd}{xx-dd}$

EXAMPLE XII.

To find the *Fluent* of $\frac{c\dot{x}\sqrt{x}}{ab^3-cx^3}$.

Comparing this with *Form* 6. there is $z=x$, $n=3$, $\alpha=ab^3$, $\beta=c$, and φ, or *Fl.* $\frac{x^{\frac{1}{2}}\dot{x}}{ab^3-cx^3} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\alpha}+\sqrt{\beta z^n}}{\sqrt{\alpha}-\sqrt{\beta z^n}} =\right) \frac{L}{3\sqrt{ab^3c}} \times \log. \frac{\sqrt{ab^3}+\sqrt{cx^3}}{\sqrt{ab^3}-\sqrt{cx^3}}$, when ab^3 is greater than cx^3.

Or

Or φ, or *Fl.* $\frac{x^{\frac{1}{2}}\dot{x}}{ab^3-cx^3}=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}}=\right)\frac{L}{3b\sqrt{abc}}$ $\times\log.\frac{x\sqrt{cx}+b\sqrt{ab}}{x\sqrt{cx}-b\sqrt{ab}}$, when cx^3 is greater than ab^3.

Therefore *Fl.* $\frac{cx\sqrt{x}}{ab^3-cx^3}=\frac{cL}{3b\sqrt{abc}}\times\log.\frac{b\sqrt{ab}+x\sqrt{cx}}{b\sqrt{ab}-x\sqrt{cx}}$, or $=$ $\frac{cL}{3b\sqrt{abc}}\times\log.\frac{x\sqrt{cx}+b\sqrt{ab}}{x\sqrt{cx}-b\sqrt{ab}}$.

EXAMPLE XIII.

To find the *Fluent* of $\frac{11y^{-2}\dot{y}}{1-aay^{-2}}$.

Comparing this with *Form* 6th, $z=y$, $n=-2$, $\alpha=1$, $\beta=aa$, and φ, or *Fl.* of $\frac{y^{-2}\dot{y}}{1-aay^{-2}}$, $=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{\sqrt{\alpha}+\sqrt{\alpha\beta z^n}}{\sqrt{\alpha}-\sqrt{\alpha\beta z^n}}=\right)\frac{L}{-2a}$ $\times\log.\frac{1+\frac{a}{y}}{1-\frac{a}{y}}=\frac{L}{-2a}\times\log.\frac{y+a}{y-a}$, when y is greater than a.

Or *Fl.* $\frac{y^{-2}\dot{y}}{1-aay^{-2}}=\left(\frac{L}{n\sqrt{\alpha\beta}}\times\log.\frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}}=\right)\frac{L}{-2a}\times\log.$ $\frac{\frac{a}{y}+1}{\frac{a}{y}-1}=\frac{L}{-2a}\times\log.\frac{a+y}{a-y}$, when a is greater than y.

Th. *Fl.* $\frac{11y^{-2}\dot{y}}{1-aay^{-2}}=\frac{11L}{-2a}\times\log.\frac{y+a}{y-a}$, or $=\frac{11L}{-2a}\times\log.\frac{a+y}{a-y}$.

EXAMPLE XIV.

To find the *Fluent* of $\frac{\dot{z}}{a^3+z^3}$.

Compare this with *Form* the 7th, and we have $z=z$, $n=3$, $\alpha=a^3$, $\beta=1$, $\frac{\delta}{\lambda}=\frac{1}{1}$; $K=\frac{180^\circ}{3}=60^\circ$, $x=\sqrt[3]{\frac{z^3}{a^3}}=\frac{z}{a}$; $a=$ s. 60° $\sqrt{\frac{3}{4}}$, $b=0$, &c. $s=$ cos. $60^\circ=\frac{1}{2}$, $t=$ cos. $180=-1$, $u=0$, &c. $P=$ degrees in the arc whose fine is $\frac{\sqrt{\frac{3}{4}}}{\sqrt{1-\frac{z}{a}+\frac{zz}{aa}}}$, which is easi-

Or φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{a}}{\sqrt{\beta z^n}-\sqrt{a}} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{\sqrt{6x}+\sqrt{a}}{\sqrt{6x}-\sqrt{a}}$, for the *Fluent*, when $6x$ is greater than a.

Or φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{a+\beta z^n+2\sqrt{a\beta z^n}}{a-\beta z^n} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{a+6x+2\sqrt{6ax}}{a-6x}$, the *Fluent* when a is greater than $6x$.

Or, laſtly, φ, or *Fl.* $\frac{x^{-\frac{1}{2}}\dot{x}}{a-6x} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{a-\beta z^n}{a+\beta z^n-2\sqrt{a\beta z^n}} =\right) \frac{L}{\sqrt{6a}} \times \log. \frac{a-6x}{a+6x-2\sqrt{6ax}}$, being the *Fluent* if a is greater than $6x$; then multiply any of theſe by a, you have the *Fluent* required.

EXAMPLE XI.

Let the *Fluent* of $\frac{7bx\dot{x}}{d^4-x^4}$ be required.

By *Form* 6. $z=x$, $n=4$, $\alpha=d^4$, $\beta=1$, and φ, or *Fluent* of $\frac{x\dot{x}}{d^4-x^4} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta z^n}}{\sqrt{a}-\sqrt{\beta z^n}} =\right) \frac{L}{4dd} \times \log. \frac{dd+xx}{dd-xx}$, the *Fluent* when dd is greater than xx.

Or *Fl.* $\frac{x\dot{x}}{d^4-x^4}$, or *Fl.* $\frac{-x\dot{x}}{x^4-d^4} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{a}}{\sqrt{\beta z^n}-\sqrt{a}} =\right) \frac{L}{4dd} \times \log. \frac{xx+dd}{xx-dd}$, being the *Fluent* when xx is greater than dd.

Therefore the *Fluent* of $\frac{7bx\dot{x}}{d^4-x^4}$ is $= \frac{7bL}{4dd} \times \log. \frac{dd+xx}{dd-xx}$, or $= \frac{7bL}{4dd} \times \log. \frac{xx+dd}{xx-dd}$

EXAMPLE XII.

To find the *Fluent* of $\frac{c\dot{x}\sqrt{x}}{ab^3-cx^3}$.

Comparing this with *Form* 6. there is $z=x$, $n=3$, $\alpha=ab^3$, $\beta=c$, and φ, or *Fl.* $\frac{x^{\frac{1}{2}}\dot{x}}{ab^3-cx^3} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta z^n}}{\sqrt{a}-\sqrt{\beta z^n}} =\right) \frac{L}{3\sqrt{ab^3c}} \times \log. \frac{\sqrt{ab^3}+\sqrt{cx^3}}{\sqrt{ab^3}-\sqrt{cx^3}}$, when ab^3 is greater than cx^3.

Or

Or φ, or *Fl.* $\frac{x^{\frac{1}{2}}\dot{x}}{ab^3-cx^3} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}}\right) \frac{L}{3b\sqrt{abc}}$ $\times \log. \frac{x\sqrt{cx}+b\sqrt{ab}}{x\sqrt{cx}-b\sqrt{ab}}$, when cx^3 is greater than ab^3.

Therefore *Fl.* $\frac{c\dot{x}\sqrt{x}}{ab^3-cx^3} = \frac{cL}{3b\sqrt{abc}} \times \log. \frac{b\sqrt{ab}+x\sqrt{cx}}{b\sqrt{ab}-x\sqrt{cx}}$, or $=$ $\frac{cL}{3b\sqrt{abc}} \times \log. \frac{x\sqrt{cx}+b\sqrt{ab}}{x\sqrt{cx}-b\sqrt{ab}}$.

EXAMPLE XIII.

To find the *Fluent* of $\frac{11y^{-2}\dot{y}}{1-aay^{-2}}$.

Comparing this with *Form* 6th, $z=y$, $n=-2$, $\alpha=1$, $\beta=aa$, and φ, or *Fl.* of $\frac{y^{-2}\dot{y}}{1-aay^{-2}} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\alpha}+\sqrt{\alpha\beta z^n}}{\sqrt{\alpha}-\sqrt{\alpha\beta z^n}}\right) \frac{L}{-2a}$ $\times \log. \frac{1+\frac{a}{y}}{1-\frac{a}{y}} = \frac{L}{-2a} \times \log. \frac{y+a}{y-a}$, when y is greater than a.

Or *Fl.* $\frac{y^{-2}\dot{y}}{1-aay^{-2}} = \left(\frac{L}{n\sqrt{\alpha\beta}} \times \log. \frac{\sqrt{\beta z^n}+\sqrt{\alpha}}{\sqrt{\beta z^n}-\sqrt{\alpha}}\right) \frac{L}{-2a} \times \log.$ $\frac{\frac{a}{y}+1}{\frac{a}{y}-1} = \frac{L}{-2a} \times \log. \frac{a+y}{a-y}$, when a is greater than y.

Th. *Fl.* $\frac{11y^{-2}\dot{y}}{1-aay^{-2}} = \frac{11L}{-2a} \times \log. \frac{y+a}{y-a}$, or $= \frac{11L}{-2a} \times \log. \frac{a+y}{a-y}$.

EXAMPLE XIV.

To find the *Fluent* of $\frac{\dot{z}}{a^3+z^3}$.

Compare this with *Form* the 7th, and we have $z=z$, $n=3$, $\alpha=a^3$, $\beta=1$, $\frac{\delta}{\lambda}=\frac{1}{1}$; $K=\frac{180^\circ}{3}=60^\circ$, $x=\sqrt[3]{\frac{z^3}{a^3}}=\frac{z}{a}$; $q=$ s. 60° $\sqrt{\frac{3}{4}}$, $b=0$, &c. $s=$ cos. 60° $=\frac{1}{2}$, $t=$ cos. $180=-1$, $u=0$, &c. $P=$ degrees in the arc whose fine is $\frac{\sqrt{\frac{3}{4}}}{\sqrt{1-\frac{z}{a}+\frac{zz}{aa}}}$, which is easi-

ly found from the table of fines, in the ſame manner as the degrees were found from the table of tangents in *Example* 4. alſo Q=0, &c. and, by *Caſe* 1. A=−1, B=$\sqrt{3}$, C=1; whence the *Fluent* is $\frac{1}{3aa}$ × into −2.30258 log. $\sqrt{1-\frac{z}{a}+\frac{zz}{aa}}$ +.01745$\sqrt{3}$ × P+ 2.30258 × log. $1+\frac{z}{a}$.

EXAMPLE XV.

To find the *Fluent* of $\frac{dx^5\dot{x}}{a^8-x^8}$?

Compare this with *Form* 8. $z=x$, $y=8$, $\alpha=a^8$, $\beta=-1$, $\frac{\delta}{\lambda}=\frac{1}{4}$, K=45, $x=\frac{xx}{aa}$; alſo a= s. 90=1, b= s. 180=0, c=0, &c. And s= cos. 20=0, t= cos. 180=−1, u=0, &c. P= degrees in the arc whoſe ſine is $\frac{xx}{aa\sqrt{1+\frac{x^4}{a^4}}}$, or $\frac{xx}{\sqrt{a^4+x^4}}$, Q=0, &c, Alſo by *Caſe* 4. A=−1, B=0, C=−2, D=1. All theſe being found, the *Fluent* is $\frac{d}{8aa}$ × into $-2.30258\log.\sqrt{1-\frac{xx}{aa}}-2\times.017453$ P+2.30258 log. $\sqrt{1+\frac{xx}{aa}}$.

Now ſuppoſe $a=5$, $x=4$; then $\frac{xx}{aa}=.64$, and log. $\sqrt{1-\frac{xx}{aa}}$ = log. .36=−1.556302=−.443698 and log. $\sqrt{1+\frac{xx}{aa}}$ = logarithm 1.64=0.214844; alſo $\frac{xx}{\sqrt{a^4+x^4}}=.539053$=s. P. Th. P=32.6192 degrees, whence the *Fluent* is =2.302585×.658542−.017453× 65.2384×$\frac{d}{200}$=d×.013275; and this being wrought out at length, will ſerve as an example how to proceed in other ſuch caſes.

EXAMPLE XVI.

Let $\frac{2ax\dot{x}}{\sqrt{a^4+x^4}}$ be propoſed.

By *Form* 9. $z=x$, $n=4$, $\alpha=a^4$, $\beta=1$; whence *Fluent* = 2×2.30258

$\frac{2\times 2.30258}{2}\times$ log. $\sqrt{x^4}+\sqrt{a^4+x^4}$; that is, the φ, or *Fl.* of $\frac{x\dot{x}}{\sqrt{a^4+x^4}}$ $=2.302585$ log. $xx+\sqrt{a^4+x^4}$; Th. $2.30258\times 2a$ log. $xx+\sqrt{a^4+x^4}$ is the *Fluent* required.

EXAMPLE XVII.

To find the *Fluent* of $\frac{-4a\dot{x}}{\sqrt{2ax+xx}}$, or of $\frac{-4ax^{-\frac{1}{2}}\dot{x}}{\sqrt{2a+x}}$.

By *Form* 9. $z=x$, $n=1$, $\alpha=2a$, $\beta=1$; whence *Fluent* $=2.30258\times 2$ log. $\sqrt{x}+\sqrt{2a+x}$; or rather, by the 2d way, *Fluent* $=2.30258$ log. $2a+2x+2\sqrt{2ax+xx}$; or, by the 3d way, *Fluent* $=-2.30258$ log. $2a+2x-2\sqrt{2ax+xx}$:

Multiply by $-4a$, and we ſhall have the *Fluent* of $\frac{-4a\dot{x}}{\sqrt{2ax+xx}}$ $=-2.3025\times 8a$ log. $\sqrt{x}+\sqrt{2a+x}=-2.30258\times 4a$ log. $2a+2x+2\sqrt{2ax+xx}=2.30258\times 4a$ log. $2a+2x-2\sqrt{2ax}=\dot{x}x$.

EXAMPLE XVIII.

To find the *Fluent* of $\frac{r\dot{y}}{y\sqrt{rr-yy}}$:

This quantity $\frac{r\dot{y}}{y\sqrt{rr-yy}}$, as it ſtands, not agreeing with any of the *Forms*, therefore divide both numerator and denominator by yy; then $\frac{r\dot{y}}{y\sqrt{rr-yy}}=\frac{ry^{-2}\dot{y}}{\sqrt{-1+rry^{-2}}}$, which belongs to *Form* 9. where $z=y$, $n=-2$, $\alpha=-1$, $\beta=rr$; whence *Fluent* $=\frac{-2\times 2.302585}{2\sqrt{rr}}$ x log. $rry^{-2}+\sqrt{-1+rry^{-2}}=-\frac{2.3025}{r}\times$ log. $\frac{rr}{yy}+\sqrt{\frac{rr-yy}{yy}}$, and *Fluent* $\frac{r\dot{y}}{y\sqrt{rr-yy}}=-2.30258$ log. $\frac{rr}{yy}+\sqrt{\frac{rr-yy}{yy}}$.

H.

A LETTER *from a gentleman near* Cambridge, *concerning the right aſcenſions and declinations of the late* COMET.

SIR,

I HAVE herewith ſent the right aſcenſions and declinations of the late comet, as calculated from the longitudes and latitudes which were publiſhed in your laſt *Miſcellanea Curioſa Mathematica*: If you think proper to publiſh theſe, it will render the other table more compleat.

I am, yours, &c. S. B.

P. S. The times are according to vulgar computation.

Equal Time. *Dec.* 23. 10′ 30″ P.M.			*Dec.* 27. 6ʰ 20′			*Dec.* 28. 6ʰ 0′			*Dec.* 31 6ʰ 30′			*Jan.* 6. 6ʰ 15′			*Jan.* 10. 6ʰ 45′		
R. A.	5°	42	20	3	39	26	3	10	02	1	45	40	359	05	23	357	21	56
Decl. N.	21	44	17	21	07	29	20	59	04	20	32	50	19	55	00	19	28	19
Jan. 12. 6ʰ 0′			*Jan.* 13. 6ʰ 0′			*Jan.* 14. 8ʰ 45′			*Jan.* 16. 6ʰ 15′			*Jan.* 23. 6ʰ 0′			*Jan.* 29. 6ʰ 0′		
R. A.	356	39	00	356	17	44	355	53	20	355	05	44	352	10	54	349	15	27
Decl. N.	19	19	23	19	14	21	19	08	33	18	56	33	18	05	25	17	07	35
Feb. 9. 6ʰ 32′ 30″			*Feb.* 12. 6ʰ 30′			*Feb.* 13. 6ʰ 0′ A. M.			*Feb.* 13. 6ʰ 0′ P. M.			*Feb.* 18. 6ʰ 22′ A. M.					
R. A.	341	14	00	338	01	59	337	33	16	336	58	48	332	42	26			
Decl. N.	12	04	00	8	59	30	8	06	29	7	19	06	S. 1	55	50			

The End of Number VI.

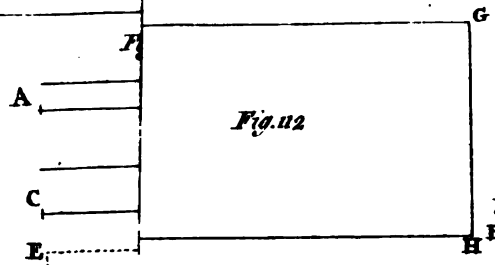


Fig. 112

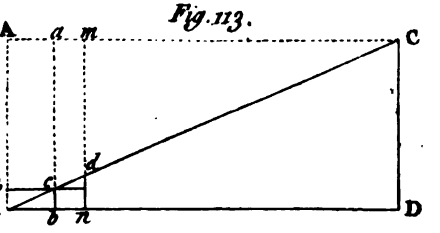
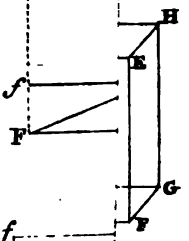


Fig. 113.



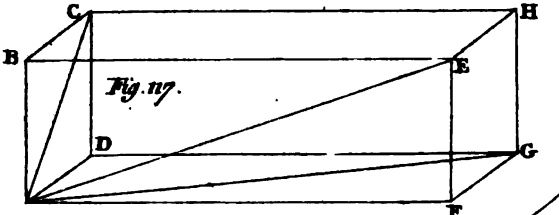


Fig. 117.

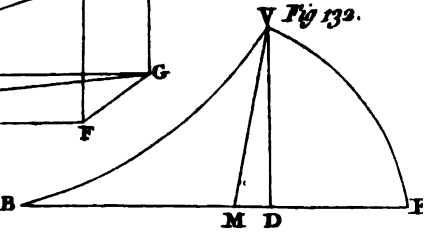
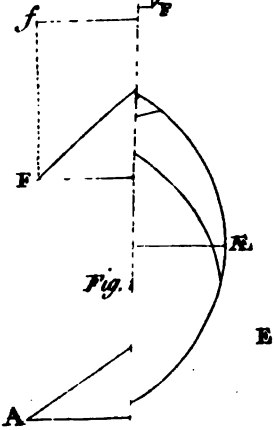


Fig 132.



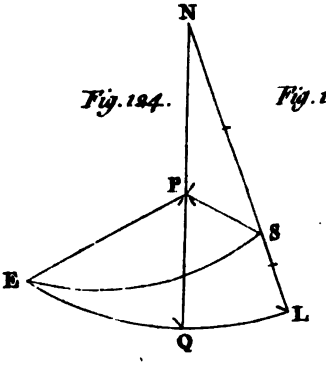


Fig. 124.

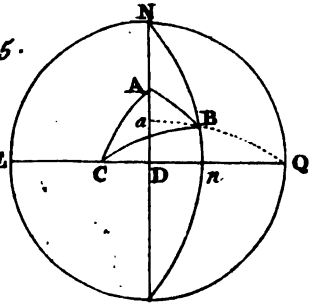
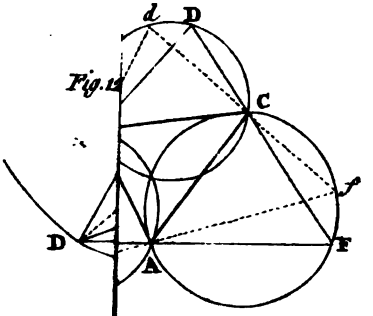


Fig. 125.



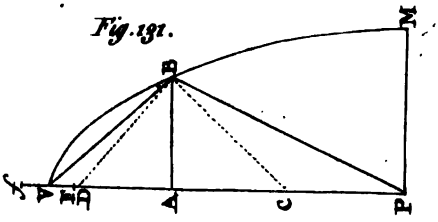


Fig. 131.

Let $\left\{\begin{matrix}S\\Z\\X\end{matrix}\right\}$ be the $\left\{\begin{matrix}\text{Linear sides}\\\text{Superficies}\\\text{Solidity}\end{matrix}\right\}$ of a regular solid. and $\left\{\begin{matrix}R\\r\end{matrix}\right\}$ th[e] $a=\frac{6-2\sqrt{5}}{3}$; $n=\sqrt{\frac{1+2\sqrt{,2}}{2+2\sqrt{,2}}}$; $m=\frac{1}{\sqrt{2+2\sqrt{,2}}}$; $cc=4m^2-a$.

Given.	Requir'd.	Tetraedron. Solution.	Hexaedron. Solution.	Icosaedron. Solution.
If S=1	R=	$\frac{1}{4}\sqrt{\frac{3}{2}}$=0,6123724	$\frac{1}{2}\sqrt{3}$=0,8660254	$2m\sqrt{\frac{1}{3a}}$=0,9510565
	r=	$\frac{1}{6}\sqrt{\frac{1}{2}}$=0,2041241	$\frac{1}{2}$ =0,5	cn =0,7557613
	Z=	$\sqrt{3}$=1,7320508	6 =6,	$5\sqrt{3}$=8,660254
	X=	$\frac{1}{6}\sqrt{\frac{1}{2}}$=0,1178511	1 =1,	$\frac{5c}{3\sqrt{a}}$=2,1816951
If R=1	S=	$2\sqrt{\frac{2}{3}}$=1,6329932	$2\sqrt{\frac{1}{3}}$=1,1547006	$\frac{\sqrt{3a}}{2m}$=1,0514622
	r=	$\frac{1}{3}$ =0,3333333	$\sqrt{\frac{1}{3}}$=0,5773503	$\frac{c}{2m}$=0,7946545
	Z=	$8\sqrt{\frac{1}{3}}$=4,6188023	8 =8,	$\frac{15a\sqrt{3}}{4mm}$=9,5745413
	X=	$\frac{8}{9}\sqrt{\frac{1}{3}}$=0,5132002	$\frac{8}{3}\sqrt{\frac{1}{3}}$=1,5396006	$\frac{5ca\sqrt{3}}{8m^3}$=2,5361507
If	S=	$6\sqrt{\frac{2}{3}}$=4,8989795	2 =2,	$\frac{1}{cn}$=1,3231691
	R=	3 =3,	$\sqrt{3}$=1,7320508	$\frac{2m}{c}$=1,2584086

is $=\frac{\quad}{73320}$ days $=29$ [illegible]

4. *Note*, That the quantity of a synodical month is not the same at all times; for in the summer solstice, when the sun seems to move slowest, the synodical month appeareth less, being about 29d 06h 42'. But in the winter solstice, when the sun's motion seems fastest, the

Miſcellanea Curioſa, &c.

NUMBER VII.

An Essay *on the Calendar, with regard to the correction of the* Julian *and* Gregorian *accounts in the celebration of* Easter; *being deſigned for an introduction to Chronology.*

Of the lunar cycle, prime or golden number.

1. THE lunar cycle is a ſyſtem of 19 *Julian* years both *ſolar* and *lunae-ſolar*, of which latter ſort 12 have 12 months a piece, and 7 have 13 months a piece; that is to ſay, the moon is ſuppoſed to make ($12\times12+7\times13=$) 235 lunations in 19 *Julian* years. Theſe 19 years being elapſed, the mean new moons were ſuppoſed to return upon the ſame inſtant.

2. But this ſuppoſition is not juſt, 235 lunations being leſs than 19 *Julian* years: For from the council of *Nice*, which was held in the year of our Lord 325, to the year 1573, it was found that there was an error of 4 days, very near; and conſequently every 312 *Julian* years, from the year 1573, will cauſe a farther anticipation of one day.

$$\text{For } 1573-325=1248 \overset{\text{Jul. years}}{:} 4 \overset{\text{days}}{::} 312 \overset{\text{Jul. years}}{:} 1 \text{ day.}$$

Wherefore every 19 *Julian* years will cauſe an anticipation of $\frac{19}{312}$ days $=1^h\ 27'\ 41''\ 32\frac{4}{13}'''$ very near.

3. Hence the number of days, &c. in one mean ſynodical month may be found thus, *viz.* ſuppoſing the ſaid number $=x$, then $235x$ (or 235 lunations) $+\frac{19}{312}$ days $=\overset{\text{days}}{365\frac{1}{4}}\times19=\overset{\text{days}}{6939\frac{3}{312}}$; therefore x is $=\frac{2165181}{73320}$ days $=29^d\ 12^h\ 44'\ 03''\ 08\frac{33}{611}'''$.

4. *Note*, That the quantity of a ſynodical month is not the ſame at all times; for in the ſummer ſolſtice, when the ſun ſeems to move ſloweſt, the ſynodical month appeareth leſs, being about $29^d\ 06^h\ 42'$. But in the winter ſolſtice, when the ſun's motion ſeems faſteſt, the

moon does not fetch up the ſun ſo ſoon; for which reaſon the ſynodical month then ſeems greater, *viz.* about $29^d\ 19^h\ 37'$, as ſome aſtronomers have obſerved: So that what is above ſaid is to be underſtood of one mean ſynodical month $=29^d\ 12^h\ 44'\ 03''\ 09'''$, according to the ſaid aſtronomers obſervations and calculations, and is very near the ſame with the above found value of x.

5. Hence a ſeries of the beſt primes may be found thus:

By what is above ſaid one mean ſynodical month is $=\frac{2165183}{73320}$ Days, wherefore 2165183 Days : 1 Lunation :: $365\frac{1}{4}\times 73320$ Days : $12\frac{797934}{2165183}$ lunations, conſequently the neareſt ratios, in the leaſt integers, are *Jul.* year Lunations *Jul.* year Lunations 1 12, and 1 13; which, when truly compleated, are $1..12+\frac{797934}{2165183}$, and $1..13-\frac{1367249}{2165183}$. Now, omitting to write the common denominator of the fractions, but inſerting their numerators in ſmaller figures, to diſtinguiſh them from the integers, the operation will ſtand thus:

	1	1	$13-1367249$
and	2	1	$12+0797934$
$1+2$	3	2	$25-0569315$
$2+3$	4	3	$37+0228619$
	5	0228 &c.)0569 &c.(2+	
$4\times\bar{2}$	6	6	$74+0457238$
$3+6$	7	8	$99-0112077$
	8	0112 &c.)0228 &c.(2+	
$7\times\bar{2}$	9	16	$198-0224154$
$4+9$	10	19	$235+0004465$
	11	0004465)0112077(25+	
$10\times\overline{25}$	12	475	$5875+0111625$
$7+12$	13	483	$5974-0000452$
	14	0000452)0004465(9+	
$13\times\bar{9}$	15	4347	$53766-0004068$
$10+15$	16	4366	$54001+0000397$
$13+16$	17	4849	$59975-0000055$
		&c.	

Of all the foregoing ratios the integer parts of that in the 10th ſtep are the moſt convenient for practice, the next following diviſion, in the 11th ſtep, giving $25+$ for the quotient, which is more than any other quotient in the whole operation.

Now

Now theſe integer parts are 19..235; ſo that 19 *Julian* years (which is ſaid to be firſt invented by *Meton* an *Athenian*) may be taken for a good prime; and then the number of the lunations will be 235. And though theſe are not as near the truth as the integer parts of the above ſubſequent ratios, *viz.* 483..5974, 4366..54001, 4849..59975, yet the former are preferable, becauſe they are leſs, and the numbers commonly uſed, and that the error they produce may be nearly aton'd for by means of the aforementioned anticipation.

§ I. *To find the lunar cycle, which is alſo called the prime or golden number, for any propos'd year of the chriſtian æra.*

Note, The number of years allow'd for the revolution of the lunar cycle is 19, of which 1 year was paſt when CHRIST was born: Therefore

Rule. Divide the propos'd year of Chriſt by 19, and to the remainder add 1, and the ſum is the lunar cycle ſought.

Example. I demand the lunar cycle for the year of Chriſt 1748?

19)1748(92
0 remainder
1
Sum 1 is the lunar cycle required.

Of the EPACTS.

1. Under the name of epacts, we uſually underſtand the difference between a lunar and the common *Julian* year.

2. The civil epacts are days intercepted between the common *Julian* year of 365 days, and the lunar of 354 days, in the neareſt integers. For it being inconvenient to account for any thing but whole days in the uſual year, the parts of days belonging to the lunar and *Julian* years, though really unequal, are however, in common uſe, ſuppoſed equal till the laſt year of the lunar cycle, at which time, the leap of the moon, as it is called, is ſuppoſed to make all even.

3. The leap of the moon happens the 19th year of every cycle, by reaſon that, in the term of 19 years, the exceſs of the common *Julian* years above the lunar is computed at (19×11=) 209 days, which being divided by 30 (which is the number of days in an *emboliſmal* month, and the greateſt number of the common epacts) gives 6 *emboliſmal* months and 29 days; whence it follows that, inſtead

ſtead of 11 for the epact, we make uſe of 12 in the 19th year.

4. Hence the annual epacts conſiſt of (365—354=) 11 days, excepting every 19th year, in which there are 12 days.

5. The uſe of the epacts is to ſhew by their places the moon's age, and chiefly the full moon before *Eaſter*; which probably is the uſe for which the firſt authors intended them.

§ II. *To find the* Engliſh *epact for any propoſed year of the chriſtian aera.*

Rule. The lunar cycle of the propoſed year being firſt found, by § I. multiply it by 11, the product divide by 30, and the remainder is the *Engliſh* epact for that year.

Example. I demand the *Engliſh* epact for the year of Chriſt 1748? The lunar cycle of the year 1748 is 1, by § I.

1
11
30)11(0
remainder 11 the epact required.

§ III. *To find the* Engliſh *Dominical* (*or* Sunday) *letter or letters.*

Rule. Add to the propoſed year, one fourth part thereof (omitting the fraction, if there be any) and 4 together, and divide the ſum by 7, then ſubtract what remains from 7, and this remainder ſhews the thing required, counting A for 1, B for 2, C for 3, D for 4, E for 5, F for 6, and G for 7.

But in dividing the propoſed year by 4 (in order to find the fourth thereof) 0 remains; then that year is a leap year, and the letter found by this rule is the dominical letter for all the months of that year, *January*, and *February* to the 24th day only excepted; for which the dominical letter is the next following to that found in the foregoing manner, counting A to be the next letter following G.

Example. I demand the *Engliſh* dominical letter, or letters, for the year 1748.

1748
437; ¼ exactly, the leap year
4
7)2189(312
5

For the year 1748 the remainder is 5, and 7—5=2, or B is the dominical letter for all the months in the year 1748, excepting

January

January and *February* to St. *Mathias*'s day, for which the next following letter, *viz.* C, is the dominical letter that is to say, the *Engliſh* dominical letters for all that year are C B.

§ IV. *To find the moon's age, old ſtyle.*

Rule. The *Engliſh* epact of the propoſed year being firſt found by § II. add to it the propoſed day of the month, and the number of the propoſed month (counting from the year of *Chriſt* 1573, to the year (1573+312=) 1885, for *March* 1. for *April* 2, for *May* 3, for *June* 4, for *July* 5, for *Auguſt* 6, for *September* 8, for *October* 8, for *November* 10, for *December* 10, for *January* 0, and for *February* 2; and from the year 1885 to the year 2197 adding 1 more to each of the ſaid months: and thus proceeding by the continual addition of 1 day to each month (for every 312 years more) the ſum if it exceeds not 30, is ſuppoſed to be the number of days of the moon's age on the propoſed day, month and year; but when the ſaid ſum exceeds 30, divide it by 30, and the remainder is that which is required, nearly.

Example. The 9th day of *March*, old ſtyle, in the year of *Chriſt* 1748, I deſire to know the moon's age?

The *Engliſh* epact for the year 1748 is 11, then

Epact	11
The propoſed day of the month	9
The number of the month	1
	21

Anſwer. The moon is 21 days old for the year 1748.

§ V. *To find the moon's ſouthing, or the time of her coming to the ſouth.*

Rule. The moon's age on the day, month and year propoſed being firſt found by § IV, multiply it by 4, then divide the product by 5, and the quotient is the number of hours required.

Example. The 9th day of *March*, old ſtyle, in the year 1748, I demand the moon's ſouthing?

By § IV. the moon is 21 days old.

Then $21\times4=84$, and $\frac{84}{5}=16\frac{4}{5}$ hours $=16^h\ 48'$ for 1748.

§ VI. *To find the time of full ſea, or high water, at any time and place propoſed.*

Rule. The moon's ſouthing being found by § V, add to it the

number of the point of the compaſs, belonging to the place propoſed, found in the tide table of the ſeaman's calendar, and the ſum is the number of hours in which it will be high water in that place, the day, month and year propoſed.

Example. The 9th day of *March*, old ſtyle, in the year 1748, I demand the time of high water in the paſſage of *Cork*?

Note, *Cork* harbour ſets E. N. E. and W. S. W. which is $4\frac{1}{2}$ points $=4\frac{1}{2}$ hours.

But it is always ſooner high water at the mouth of any harbour, than farther in; and therefore 5 points, or 5 hours, may be allowed for the paſſage in *Cork* harbour.

The moon's ſouthing is $16^h\ 48'$, by § V.
Point of the compaſs $\underline{\quad 5 \quad}$
Sum $21\ \ 48$ anſwer.

That is to ſay, it is 48 minutes paſt 9 of the clock high water at the time and place above propoſed.

§ VII. *To find the day of the* Engliſh *Eaſter-limit from the firſt day of* March *inclusive.*

Rule. When for the propoſed year the *Engliſh* epact (which is never 27) is $\left.\begin{matrix}\text{leſs}\\\text{greater}\end{matrix}\right\}$ 27, ſubtract it from $\left.\begin{matrix}47\\77\end{matrix}\right\}$, and the remainder is the *Engliſh Eaſter* limit, for that year; and the next following *Sunday* is the *Engliſh Eaſter-day*.

Example. For the year 1748, I demand the *Engliſh Eaſter limit*?

By § II. the *Engliſh* epact for the year 1748 is 11; wherefore $47-11=36$ is the *Engliſh Eaſter* limit required.

§ VIII. *To find the* Engliſh Eaſter-day.

Rule. The *Engliſh* dominical letter, and *Eaſter* limit of the propoſed year, being found by § III, and VII. take the number of the ſaid letter, more by 4, from the ſaid limit, and the reſidue from the neareſt greater ſum of ſevens; and this laſt remainder, added to the ſaid limit, the ſum, or its exceſs above 31, is accordingly in *March* or *April* the *Engliſh Eaſter-day* that year, according to the old ſtyle.

Example. I deſire to know on what month and day the *Engliſh Eaſter-day* happens in the year 1748?

The

The *Engliſh* dominical letter for the year 1748 is 2=B or BA by §3.

add 4

Sum 6

The *Engliſh Eaſter* limit for the ſaid year is 36 by § VII.

Reſidue 30

To which the neareſt greater ſum of ſevens is 35

Remainder 5

The *Engliſh Eaſter* limit 36

Sum 41, which is greater than 31; therefore

The number of days in *March* is 31

Exceſs 10

That is to ſay, the *Engliſh Eaſter-day*, in the year 1748, falls on the 10th of *April*, old ſtyle.

§ IX. *To find what day of* March *the firſt* Sunday *of the month falls on, in any year propoſed, in purſuance of the old ſtyle.*

Rule. To the propoſed year add the one fourth part thereof, omitting the fraction when there is one, the ſum divide by 7, what remains ſubſtract from 7, and this laſt remainder ſhews the day required, counting for 1, 2, 3, 4, 5, 6 and 7, the 1ſt, 2d, 3d, 4th, 5th, 6th and 7th days of *March* reſpectively.

Example. In the year of *Chriſt* 1748, I demand on what day of *March* the firſt *Sunday* of that month happens.

The propoſed year 1748

its 4th 437

7)2185(1 remains.

312

And 7−1=6; Th. the 6th day of *March*, old ſtyle, is the firſt *Sunday* of that month, in the year 1748.

Of the Gregorian *or* New Style.

1. THE *Gregorian*, or new, ſtyle was deduced from the *Julian*, or old, ſtyle, and founded upon this inſtitution, *viz.* that after the 1600th *Julian* year of *Chriſt*, of every four years terminating four centuries, the three firſt ſhould be common, and the fourth *Biſſextile*, according to the *Gregorian*; whereas all four are leap years in the *Julian* form. Moreover that there ſhould be 10 days difference

difference between the *Julian* and *Gregorian* accounts in the ſaid 1600th year, and but the ſame difference every year after to the 1700th year excluſive; conſequently there were and will be 11, 12, 13, 14, 15, 16, 17, 18, 19, &c. difference between both accounts in the *Julian* years 1700, 1800, 1900, 2000, 2100, 2200, 2300, 2400, 2500, 2600, 2700, &c. reſpectively, due regard being thus had to every fourth century.

Hence 400 *Julian* years are, by 3 days, more than 400 *Gregorian* years; and conſequently each *Gregorian* year conſiſts of 365^d 5^h $49'$ $12''$. For 400 *Julian* years : 400 *Jul. years* − 3 *days* = 146097 *days* :: 1 *Jul. year* : 365^d 5^h $49'$ $12''$; but this is more than the true tropical year.

2. What gave riſe to the *Gregorian* ſtyle is, that in the time of the council of *Nice*, which was held in the year of *Chriſt* 325, the vernal equinox then fell on the 20th day of *March*: And pope *Gregory* the 13th found, that in the year of *Chriſt* 1583, the vernal equinox fell on the 10th day of *March*.

Hence 1583 − 325 = 1258 *Julian* years.

Then 1258 *Jul. years* : 20 − 10 = 10 *days* :: 1 *Jul. year* : 00^h $11'$ $26''\frac{506}{619}$;

And 365^d 06^h $00'$ $00''$ = one *Julian* year.

00 11 $26\frac{506}{619}$ = difference

365 05 48 $33\frac{113}{619}$ = one tropical year, according to this computation.

3. From *parag.* 1 and 2, the vernal equinoxes in the *Julian* years of *Chriſt* 1700, 1800, 1900, 2000, 2100, 2200, &c. are ſuppoſed to fall on the 20 − 11 = 9th; 20 − 12 = 8th; 20 − 13 = 7th; 20 − 14 = 6th; 20 − 15 = 5th, &c. days of *March* reſpectively; rather than on the days, which in ſome years may be found by ſtrict obſervations and calculations to differ by near about 1 day from thoſe.

4. In the above computation, in *parag.* 2. it is ſuppoſed that the vernal equinoxes which fell, in the years 325, and 1583, on the 20th and 10th days of *March* reſpectively, happen'd nearly at the ſame time of the ſaid days. But this ſuppoſition was ſo uncertain to the mathematicians of thoſe years, that ſome of them aſſerted the vernal equinox in the year 325, to have fallen on the 21 of *March*; in purſuance of which aſſertion and ſuppoſition that the vernal equinox in the year 1583 fell on the 10th day of *March*, at the ſame time of day with the former, there would have been a further deduction of $\frac{1}{1258}$ days $= 01'$ $08''\frac{428}{619}$; and thus the tropical year would conſiſt but of 365^d 05^h $47'$ $24''\frac{324}{619}$, which is certainly leſs than the truth.

5. But

5. But to determine the value of the tropical year to a greater degree of certainty and exactneſs than any of the precedent ones, I ſhall here have recourſe to ſome modern obſervations, and calculations: In order to which, obſerve that the word *minute* is here applicable to two different things, *viz.* 1ſt a minute is a part of the circumference of a circle, and 2ndly, a minute is part of a *Julian* year, or of time. Thus, 1ſt, one revolution, or the whole circumference of a circle being (1×12=) 12 *ſigns*, or (12×30=) 360 degrees, is =(360×60 or) 21600 minutes; and

2ndly, One *Julian* year being $365\frac{1}{4}$ days, or ($365\frac{1}{4}\times 24=$) 8766 hours, is =(8766×60=) 525960 minutes.

Therefore 525960=21600 is —24.35, and conſequently 24.35 multiplied into one minute of the circumference of a circle, produces one minute of a *Julian* year.

6. Thus much being premiſed, I am now to obſerve, from the great *Newton*'s theory of the moon, that the mean motion of the ſun from the vernal equinox, at noon, on the laſt day of *December*, old ſtyle, in the years

	ſigns	degrees	min.	ſec.
1680 was	9	20	34	46,
and 1700 was	9	20	44	50.

Hence in (1700—1680=) 20 *Julian* years, the ſun made more than 20 revolutions, by ($9^{ſ}\ 20^{d}\ 43'\ 50''$ *minus* $9^{ſ}\ 20^{d}\ 34'\ 46''=$) $0^{d}\ 09'\ 04''=544''$; conſequently in one *Julian* year, he (the ſun) made more than one revolution, by ($\frac{544''}{20}=$) 27.2″.

And 27.2″ reduc'd to time, by multiplying it by the above 24.35, produces ($27.2''\times 24.35=$) $662.32''=11'\ 02''\frac{8}{25}$. And this taken from the *Julian* year of $365^{d}\ 06^{h}$, leaves $365^{d}\ 5^{h}\ 48'\ 57''\frac{17}{25}$ for the tropical year.

7. I will here add (though it be foreign to my preſent purpoſe) that by the beſt obſervations, the *receſſion* of the equinoctial points motion *in antecedentia*, is found to be at the rate of 50 ſeconds in one *Julian* year; wherefore 50″ reduc'd to time, by multiplying them by the above 24.35, produces 1217.5″=20′ 17″½, which added to the tropical year of $365^{d}\ 5^{h}\ 48'\ 57''\frac{17}{25}$, give $365^{d}\ 6^{h}\ 9'\ 15''\frac{9}{50}$ for the ſydereal year.

8. But as the number of days, &c. in either the tropical or ſydereal year, cannot be exactly determined from what obſervations have been hitherto made, or probably that ever will, or can be made, ſo Sir *Iſaac Newton* counted the tropical year to be but $365^{d}\ 05^{h}\ 48'\ 57''$, and the ſydereal year to be but $365^{d}\ 06^{h}\ 09'\ 14''$.

9. Now, if (instead of subtracting 3 days out of every 400 *Julian* years, as in the *Gregorian* reformation) 3 days be subtracted out of every 391 *Julian* years (or one day out of each $130\frac{1}{3}$ *Julian* years) the remainder will give $365^d\ 5^h\ 48'\ 57''\frac{33}{391}$ for the tropical year sought: For

$$\overset{\text{Jul. year}}{391} : \overset{\text{Jul. year}}{391-3} = \overset{\text{days}}{142809\frac{3}{4}} :: \overset{\text{days}}{1} : \overset{\text{Jul. year}}{365^d\ 05^h\ 48'\ 57''\frac{33}{391}},$$

which being less than what I deduced from Sir *Isaac Newton*'s method, and greater than as he designed it, may perhaps be more exact than either of them; but it is certainly nearer the truth than the *Gregorian* year, though their difference is so small as to amount but to one day in $5792\frac{16}{17}$ *Julian* years. For

$$365^d\ 05^h\ 49'\ 12'' - 365^d\ 05^h\ 48'\ 57''\tfrac{33}{391} = 0'\ 14''\tfrac{358}{391} : \overset{\text{Jul. year}}{1} :: \overset{\text{day}}{1} : 5792\tfrac{16}{17}$$ *Julian* years.

10. The *Gregorian Easter-day*, in pursuance to the institution of the council of *Nice*, is the *first* Sunday *after the first full moon, which happens next after the vernal equinox:* and, by what hath been said in *parag.* 3, the sun is suppos'd to be on the vernal equinox every $20-11=9^{th}$ day of *March* during this century, *viz.* from and after the year 1699 to the year 1800 exclusive.

§ X. *To find the* Gregorian Easter-day.

Rule. The day of the year proposed that the sun is on the vernal equinox being found, in pursuance of the precedent *parag.* 3. then by the help of § IV. find the day in which the moon is supposed to be full, or 15 days old, next after the day of the said equinox; and then, by the help of § IX. find the *Sunday* next after the day of that full moon, and this *Sunday*, old style, shall be the *Gregorian Easter-day* in that year.

This *rule* will be farther explained by the two following cases.

Case 1. In any one year of the christian *aera*, the moon cannot be full, or 15 days old, next after the day of the sun's being on the vernal equinox, sooner than the next following day after that vernal equinox; consequently, in this case, her age, on the day of the sun's vernal equinox, cannot exceed $(15-1=)$ 14 days.

Case 2. But if on the day that the sun is on the vernal equinox, the moon's age be 15 days or more, then that full moon is not that which happens next after the day of the vernal equinox, but the next following full moon.

In *case* 1. The sun, during this present century, being on the vernal

nal

nal equinox every (20=11) 9th day of *March*, if the moon's age on any ſuch 9th day of *March* be leſs than 15 days, ſuppoſe her age to be $=x$, and then the firſt full moon after will happen on the $9+\overline{15-x}=24-x$ day of *March*.

In *Caſe* 2d. But if the moon's age, on any ſuch 9th day of *March*, be equal to, or greater than 15 days, ſuppoſe her age to be $=y$; then that moon will be 30 days old, or ſpent on the $9+\overline{30-y}=30-y$ days; and then the next following full moon will happen on the $\overline{39-y}+15=54-y$ day.

Now, the value of $24-x$, or $54-y$ (according as it happens) being thus found by *Caſe* 1ſt, or 2d, then, by § IX. find the day of *March* on which the firſt *Sunday* of that month happens, and then, putting $z=$ the leaſt integer number making the number of this laſt mentioned day of *March* $+7z$ (not equal, but) greater than the ſaid $24-x$ or $54-y$ day, the reſult, or its exceſs above 31 (which is the number of days in the month *March*) will accordingly give, in *March* or *April*, old ſtyle, the *Gregorian Eaſter-day*; to which 11 being added will give the ſame *Eaſter-day*, according to the new ſtyle, in the year propoſed, during the preſent century.

EXAMPLES.

1. In the year of *Chriſt* 1748, I demand the day and month on which the *Gregorian Eaſter-day* will happen?

In the year 1748 the ſun will be on the vernal equinox *March* 9th, on which day the moon's age, by § IV. will be 21 days, which is greater than 15; and therefore, by *Caſe* the 2d, equal to y, conſequently ſhe will be full, or 15 days old, the $54-21$ $(=54-y)$ $=2^{d}$ day of *April*.

Now, by § IX. The firſt *Sunday* in *March*, in the year 1748, will fall on the 6th day of that month; wherefore putting $z=$ leaſt integer number that will make $6+7z$ greater than 15, we ſhall have $z=2$, and conſequently $6+7z=20$, *viz.* the 20th day of *March*, old ſtyle, or the $20+11=31^{ſt}$ day of *March*, new ſtyle, will be the *Gregorian Eaſter-day* in the year 1748.

2. In the year of *Chriſt* 1728, I demand the day and month on which the *Gregorian Eaſter-day* will happen?

In the year 1728 the ſun was on the vernal equinox *March* 9th, on which day the moon's age, by § IV. will be 9 days, which is leſs than 15, and therefore, by *Caſe* 1. $=x$; conſequently ſhe will be full, or 15 days old, the $24-9$ $(=24-x)$ $=15^{th}$ day of *March*.

Now, by § IX. the firſt *Sunday* in *March*, in the year 1728, fell

on

on the 3d day of that month; wherefore putting z = leaſt integer number that will make $3+7z$ greater than 15, we ſhall have $z=2$; and conſequently $3+7z=17$, *viz.* the 17th day of *March*, old ſtyle, or the $17+11=28$ day of *March*, new ſtyle, will be the *Gregorian Eaſter-day*.

Note. The moon's age being corrected by the addition of one day to the month of *March*, old ſtyle, for every 312 *Julian* years from the year of *Chriſt* 1573, as by *Rule* to § IV. the eaſieſt correction of the *Julian* calendar, for finding the feſtival of *Eaſter*, is by ſubtracting 3 days out of every 400 *Julian* years, as by § X. and the next foregoing examples. The more exact correction would be by ſubtracting 3 days out of every 391 *Julian* years, as in *Parag.* 9. preceding § X.

But perhaps the beſt correction, if well methodized, would be by ſubtracting 1 day out of every 130 (or 3 days out of every 390) *Julian* years, the *tropical* year, in purſuance hereof, conſiſting of $365\ 05^h\ 48'\ 55''\frac{5}{11}$, which differs from that in the ſaid *Parag.* 9. but by *one* day in 50830 *Julian* years.

March 5, $174\frac{6}{7}$. H.

Solutions to Queſtions in Number VI.

QUESTION LXXXIX. *anſwered by Mr* Ch. Smith.

LET x = the firſt digit, r = common ratio; then $x\times\overline{100+10r+r^2}$ is the number, and $\frac{x\times\overline{100+10r+r^2}}{x\times r^2-r}=62$; Th. $61r^2-72r=100$, whence $r=2$; alſo $x\times\overline{100+10r+r^2}+594=x\times\overline{100r^2+10r+1}$; Th. $99x\times\overline{r^2-1}=594$, and $x=\frac{594}{99\times\overline{r^2-1}}=2$, whence $x\times\overline{100+10r+r^2}=248$.

The ſame anſwered by Mr John Todd.

LET x, xy, and xy^2 be the 3 digits required; then, by the queſtion, $\frac{100x+10xy+xy^2}{xy^2-xy}=62$, and $100x+10xy+xy^2+594=100xy^2+10xy+x$. From the firſt of theſe we get $y=2$, which ſubſtituted in the ſecond we get $x=2$ alſo, whence the required number is 248.

QUESTION

QUESTION XC. *anſwered by Mr* R. ROBINSON.

THE ratio of the motion of the two hands is as 1 : 12$=r$. Let $a=$ hour at the beginning, $y=$ time paſt the given hour, and $m=$ the diſtance of the two hands at that inſtant; then will $ry=$ the hours moved over by the minute hand on the dial-plate, and $a+y=$ hours from 12 by the hour hand; hence $ry-a-y=m$, or $a+y-ry=m$; Th. $y=\frac{a\pm m}{r-1}$, and the time from 12 will be expreſs'd by this general theorem,

$$\frac{ar\pm m}{r-1}.$$

Where the negative ſign takes place when the minute hand is behind, and the affirmative when it is before the hour hand.

Example. Where $m=\frac{90^\circ}{30}=3$ (ſince 30° is 1 hour on the dial-plate) and $a=8$; hence the theorem gives $8\frac{5}{11}$ hours, the time required, when the two hands are perpendicular to each other.

The ſame anſwered by the Propoſer, Mr CHAPPLE.

IN order to give a general ſolution to ſuch queſtions as theſe, it will be moſt convenient, firſt to find the time when the two indices *ſtand together* in a given hour. Thus,

Put $d=$ the diſtance (taken in minute diviſions of the dial) of the two indices at the laſt *compleat hour*, reckoning always from the minute index to the right-hand; and this being the arc to be gained by the minute index on the hour index before it can be in conjunction therewith, and becauſe 55 diviſions in 60, or 11 in 12, are gain'd by the former on the latter; ſay, If 11 diviſions be gain'd in 12 minutes, in how long time will d diviſions be gain'd? which gives $\frac{12d}{11}$, and is a general expreſſion for the time elaps'd ſince the laſt compleat hour, when the indices were in conjunction.

Now to find the time when the indices in any given hour make any given angle, it will be proper that this angle be previouſly reduc'd from degrees to minute-diviſions as above, by dividing by 6; and let the arc ſo reduc'd be call'd a. Now becauſe the minute index may be either approaching towards the hour index, when it makes the given angle therewith, or elſe having overtaken it may be receding from it, the ſame angle may be made by the indices twice

in every hour, and conſequently this problem admits of two anſwers. In the former caſe the arc a is the ſpace which the minute index muſt gain on the hour index before they are in conjunction, the time required for which, *viz.* $\frac{12a}{11}$ (found by the above method) muſt be ſubtracted from the time of the conjunction; but in the latter caſe, where a is the ſpace already gain'd by the minute hand ſince the conjunction, $\frac{12a}{11}$ muſt be added to the time of conjunction. Hence we have this

General Theorem $\frac{12d \pm 12a}{11}$ = the minutes elaps'd ſince the laſt compleat hour; where $12a$ is affirmative when the minute hand is paſt the conjunction, and negative when it is ſhort of it.

To apply this to the preſent queſtion, where the angle made by the indices is 90 degrees; here $a=15$, $d=40$, and from the above theorem we have the time either $27\frac{3}{11}$ paſt eight, or elſe 60′ paſt 8, that is, preciſely 9 o'clock; ſo that here the indices can be perpendicular but once within the given hour, *viz.* at $27\frac{3}{11}$ minutes paſt eight. Q. E. I.

N. B. If the time of the oppoſition of the indices be required, a is $=30$, *i. e.* 180°.

QUESTION XCI. *anſwered by Mr* ROBERT ROBINSON.

LET a, m, and $n=5.6$; 6.3; and 4.8 pounds, the given diſcounts reſpectively; $x=$ time in years, $v=$ rate *per cent. per annum*, and $y=$ the value of the bill; then its preſent value is $y-a$, and $1 : v :: x : vx$, the intereſt *per cent.* for x time.

$100 : vx :: y-a : \frac{vx \times \overline{y-a}}{100}$ = the diſcount; hence we have

$$vx \times \overline{y-a} = 100a$$
$$x \times \overline{v+1} \times \overline{y-m} = 100m.$$
$$x \times \overline{v-1} \times \overline{y-n} = 100n.$$

The ſecond and third of theſe equations give $x \times \overline{v+1} = \frac{100m}{y-m}$, and $x \times \overline{v-1} = \frac{100n}{y-n}$, whoſe ſum $2ux = \frac{100m}{y-m} + \frac{100n}{y-n}$ $(= \frac{200a}{y-a}$ by the

the first equation;) hence $\frac{m}{y-m}+\frac{n}{y-n}=\frac{2a}{y-a}$; Th. $y=\frac{an+am-2mn}{2a-m-n}$ $=16.8$, $x=\frac{100y\times m-a}{y-m\times y-a}=10$, and $v=\frac{100a}{x\times y-a}=5$. Theref. the value of the bill is 16 *l.* 16 *s.* rate is 5 *l. per cent. per annum*, and the time 10 years.

The same answered by Mr SAMUEL DUNN.

PUT 5 *l.* 12 *s.* $=a$, 6 *l.* 6 *s* $=b$, 4 *l.* 16 *s.* $=c$, $x=$ rate *per cent.* $y=$ time, $z=$ value of the bill at the end of y years.

Then $\overset{\text{Year}}{1} : x :: y : xy$.

And $100+xy : xy :: z : \frac{xyz}{100+xy}=a$.

Again $\overset{\text{Year}}{1} : x+1 :: y : xy+y$;

And $100+xy+y : xy+y :: z : \frac{xyz+yz}{100+xy+y}=b$.

Again $\overset{\text{Year}}{1} : x-1 :: y : xy-y$;

And $100+xy-y : xy-y :: z : \frac{xyz-yz}{100+xy-y}=c$.

Hence $\begin{cases} xyz=100c+cxy-cy+yz \\ xyz=100b+bxy+by-yz \\ xyz=100a+axy; \end{cases}$

From which three equations, x y and z may be found answering to the same as above.

QUESTION XCII. *answered by* BRETOSITERAB.

IF 1728 be divided by .0019635, the area of the base gives 880061.115 inches = length of the wire.

The same answered by Mr SAMUEL DUNN.

$.7854\times\overline{\frac{1}{20}}^{2})1728(=\overset{\text{Feet}}{73338}\ \overset{\text{Inches}}{115.115}$, the length required.

The same answered by Mr WILLIAM DENT.

LET $a=$ length of the wire; then $\frac{1}{20}=.05$ the diameter of the wire, and $.0019635=$ area. Th. $ax.0019635=1728$, and $\frac{1728}{.0019635}=x=880061.115$, the length of the wire.

QUESTION

QUESTION XCIII. *anſwered by Mr* WILLIAM JEPSON.

FOR the quantities of matter, or weights of the bodies, put a, x, b; and for their reſpective velocities c, y, d. Then, from what has been demonſtrated, about the motions of elaſtic bodies, we have $x+a : 2a :: c : \frac{2ac}{x+a} = y$, $x+b : 2x :: \frac{2ac}{x+a} : \frac{4acx}{x+a \times x+b} = d$. Therefore $x^2 + \overline{a+b} \times x - \frac{4acx}{d} = -ab$, and by ſubſtitution $x^2 - 2nx = -ab$; Th. $x = n \pm \sqrt{n^2 - ab} = \begin{cases} 66.666, \text{ \&c.} \\ 30 \end{cases}$ the weights, whence $y = \begin{cases} 5.25 \\ 8. \end{cases}$ their reſpective velocities. Conſequently the body E may weigh 30 lb, and have 8 degrees of velocity; or it may weigh $66\frac{2}{3}$ lb, and have $5\frac{1}{4}$ degrees of velocity.

The ſame anſwered by Mr WILLIAM DENT.

LET the three bodies be $d=40$, $f=50$, and e. Then, by the nature of elaſtic Bodies, it is, $d+e : 2d :: 7 : \frac{14d}{d+e}$ = the velocity that e approacheth f; and $f+e : 2e :: \frac{14d}{d+e} : \frac{28de}{df+fe+de+ee}$. But $\frac{28de}{df+fe+de+ee} = 6$ by the queſtion; Theref. the equation ſolv'd, e will be found $= 66\frac{2}{3}$, its weight, and its velocity $= 5\frac{1}{4}$ degrees.

QUESTION XCIV. *anſwered by the Propoſer, Mr* ROBINSON.

SOME problems in gunnery, wherein are demonſtrated the ſeveral theorems, in anſwer to *queſtion* 94.

The THEORY. PROPOSITION I.

TO determine the paths of projectiles near the earth's ſurface, ſuppoſing the medium non-reſiſting, and the direction horizontal.

SOLUTION.

Pl. 7. Fig. 133. LET the body be projected from A in the direction AC, with a velocity that will carry it 9 feet *per* ſecond, and $p = 16.092$, the feet deſcended by a heavy body in that time; alſo put x = AD, y = DE. Then

Then $q : 1 :: y : \frac{y}{q}$ = time of deſcribing Ac, or AE; and $1 : p ::$ $\frac{yy}{qq}$: the diſtance deſcended by the force of gravity in that time = Ec $= \frac{py^2}{q^2} = x$; Th. $\frac{q^2}{p} x = y^2$. Hence the curve will be the common parabola, whoſe vertex is A, axis AD, and parameter $\frac{q^2}{p}$.

PROPOSITION II.

TO determine the paths of projectiles near the earth's ſurface, ſuppos'd to move in a non-reſiſting medium, *and the direction obliquely elevated to the horizon.*

Suppoſe the body projected from A, in the direction AC, with a velocity that will carry it 9 feet *per* ſecond, $p = 16.092$ as above, m and n = ſine and coſine of the $\angle$CAD, radius $= r$; Alſo let AD $= v$, AD = DE $= y$, and BD $= x$. Then Pl. 7. Fig. 134.

$$n : v :: r : \frac{rv}{n} = \text{Ac}; \text{ And } n : v :: m : \frac{mv}{n} = \text{dc}.$$

Again $q : 1 :: \frac{rv}{n} : \frac{rv}{nq}$ ſeconds = the time of deſcribing Ac, or Ab; $1 : p :: \frac{r^2v^2}{n^2q^2} : \frac{pr^2v^2}{n^2q^2}$ feet = cb, the diſtance deſcended by a heavy body in that time. Th. $\text{db} = \frac{mv}{n} - \frac{pr^2v^2}{n^2q^2}$, which is a *maximum* when d coincides with D; Th. its fluxion $\frac{mv}{n} - \frac{2pr^2v\dot{v}}{n^2q^2} = 0$. Th. $v = \frac{mnq^2}{2pr^2} = \text{AD} = y$; and $\text{BD} = x = \frac{m^2q^2}{4pr^2}$, Th. $m^2 = \frac{4pr^2x}{q^2} = \frac{4p^2r^4y^2}{n^2q^4}$, and $\frac{n^2q^2}{pr^2} x = y^2$; hence the curve deſcribed is a *parabola*, whoſe vertex is B, axis BD, and parameter $\frac{n^2q^2}{pr^2}$.

COROLLARY I.

WHEN the direction is horizontal, or $\angle A = 0$, then the coſine is = to the *radius*, that is, $n = r$; hence the equation is $\frac{q^2}{p} x = y^2$; the ſame as was inveſtigated in *Propoſition* I.

COROLLARY II.

WHEN the direction is perpendicular, or $\angle A=90°$, the ſine $m=r$ the radius; in which caſe the perpendicular height $=\frac{q^2}{4p}$, and the time of flight $=\frac{q}{p}$ ſeconds.

COROLLARY III.

DRAW BG parallel to AE, and AH to DC; and through the points A, F, draw the ſemi-circle AFH; then will AH expreſs the perpendicular height to which the given *impetus* will throw the projectile, which is thus proved.

Pl. 7. Fig. 135. Since by the nature of the curve BD=BC, th. $AF=FC=\frac{mq^2}{4pr}$; then by *Trigonometry*

$$m : \frac{mq^2}{4pr} :: r : \frac{q^2}{4p}=AH, \text{ per Corollary 2. } \mathcal{Q}.E.D.$$

COROLLARY IV.

Fig. 135. SINCE BD=BC, th. FB=FG, then will GF be equal to one fourth part of the *amplitude* AE.

Hence it is evident that any cannon, &c. elevated to an angle of 45°, will throw a ball the fartheſt poſſible; the amplitude will then be equal to twice the height of the perpendicular ſhot, and the height of the projection DB will be equal to one half of the ſaid perpendicular ſhot.

LEMMA.

Fig. 136. IF a line be drawn parallel to the axis of any parabola, and a tangent drawn to the point of contact, putting m and n =ſine and coſine $\angle G$, $x=AB$, $y=BC$, parameter $=a$; Then I ſay $ax=y^2+\frac{an}{m}y$. For put $v=EF$, $z=EA$, then $2mv=nz$, and $av=z^2$; Th. $v=\frac{nz}{2m}=\frac{z^2}{a}$, and $z=\frac{an}{2m}$.

Again, $av+ax=z^2+2zy+y^2$, Th. $ax=y^2+2zy$, hence $ax=y^2+\frac{an}{m}y$. $\mathcal{Q}.E.D.$

PROPOSITION III.

TO determine the paths of projectiles near the earth's ſurface, ſuppos'd to move in a non-reſiſting medium, and the elevation obliquely inclined to the horizon.

SOLU-

SOLUTION.

LET the body be projected from A, in the direction AC, with a velocity that will carry it 9 feet *per* ſecond, $p=16.092$ feet, the diſtance deſcended by a heavy body in the firſt ſecond, m and $n=$ ſine and coſine $\angle BAC$, radius $=r$, $x=AD$, $y=DE$; then $m:y::r:\frac{ry}{m}=AC$; and $m:y::n:\frac{ny}{m}=AB$; and $q:1::\frac{ry}{m}:\frac{ry}{qm}$ ſeconds, the time of deſcribing AC or AE; again $1:p::\frac{r^2y^2}{q^2m^2}=EC$ the diſtance deſcended by a heavy body in that time; Th. $x=\frac{ny}{m}+\frac{pr^2y^2}{q^2m^2}$, and $\frac{q^2m^2}{pr^2}x=\frac{q^2mn}{pr^2}y+y^2$; hence the curve deſcribed is a parabola, whoſe parameter is $\frac{q^2m^2}{pr^2}$, by the *Lemma*. Pl. 7. Fig. 139.

COROLLARY I.

HENCE it is prov'd that projectiles deſcribe parabolas in non-reſiſting mediums, and that in all oblique and horizontal directions.

COROLLARY II.

WHEN the $\angle A=90°$, then $m=r$, and the equation is $\frac{q^2}{p}x=y^2$, where A becomes the vertex, as by *Propoſition* I.

To what I have already premiſed, I hope it won't be amiſs if I add a few problems in gunnery.

PROBLEM I.

ADMIT a gun be fired directly upwards, and let there be given by obſervation the time of its flight $=a$ ſeconds; I demand to what height it will aſcend, and the velocity wherewith it was projected.

SOLUTION.

BY *Cor.* 2. to *Prop.* 2. we have $q^2=4pd$, and $q=pa$; hence the height of the perpendicular ſhot $d=\frac{1}{4}pa^2$ } feet.
and the velocity *per* ſecond $q=ap$ }

PROBLEM II.

SUPPOSE a ball be ſhot directly upwards, with a velocity that will carry it 9 feet *per* ſecond; I demand the time of the flight, and how high it will aſcend.

SOLU-

SOLUTION.

LET $p=16.092$ feet, $a=$ time of the flight in seconds, $d=$ height; then by *Cor.* 2. *Prop.* 2. the time of flight $a=\frac{q}{p}$, and the perpendicular height $d=\frac{q^2}{4p}=\frac{1}{4}qx$.

PROBLEM III.

LET a ball be thrown directly upwards the height of d feet; I demand the time of flight, with the velocity wherewith it was projected.

SOLUTION.

SINCE, as above, $a=\frac{q}{p}$, $d=\frac{q^2}{4p}$, we have the time in motion $a=2\sqrt{\frac{d}{p}}$, and the velocity *per* second $q=\sqrt{2dp}$ feet.

PROBLEM IV.

HAVING given the height of the perpendicular shot, to find the amplitude, height of the range, and time in motion, at any given elevation?

SOLUTION.

LET $m=$ sine, $n=$ cosine, $b=$ tangent of the $\angle$ of direction, radius $=r$, $w=$ height of the range, $v=$ amplitude, $T=$ time of flight in seconds, $d=$ height of the perpendicular shot with the same *impetus*.

Pl. 7. Fig. 137.

Then $r : d :: m : \frac{md}{r} =$ AF.

And $r : \frac{md}{r} :: m : w$.

And $r : \frac{md}{r} :: 4n : v$. Hence we have

Theorem 1. $w=\frac{m^2d}{r^2}$.

Theorem 2. $v=\frac{4mnd}{r^2}$; and by *Prop.* 2. we have

Theorem 3. $T=\frac{m\sqrt{d}}{2.006r}$ seconds.

PROBLEM V.

GIVEN the elevation and amplitude, to find the height of the range, perpendicular shot, and the time of flight. SOLU-

SOLUTION.

As $r : \frac{v}{4} :: b : w = \frac{bv}{4r} = AG.$

And $\frac{bv}{4r} : \frac{v}{4} :: \frac{v}{4} : \frac{rv}{4b} = GH.$ Hence we have

Pl. 7. Fig. 137.

Theorem 4. $w = \frac{bv}{4r}$ feet = height of the range.

Theorem 5. $d = \frac{1}{4}v \times \sqrt{\frac{r}{b} + \frac{b}{r}}$ = height of the perp. ſhot.

Theorem 6. $T = \frac{\sqrt{bvr^{-1}}}{4.012}$ ſeconds, by *theorem* 3.

PROBLEM VI.

GIVEN the height of the perpendicular ſhot, and the amplitude, to find the time, elevation and height of the range.

SOLUTION.

$IG = Ig = \frac{1}{4}\sqrt{4d^2 - v^2}$; Th. $AG = \frac{d}{2} - \frac{1}{4}\sqrt{4d^2 - v^2}$, $Ag = \frac{d}{2} + \frac{1}{4}\sqrt{4d^2 - v^2}$:

And $\frac{1}{4}v : r :: \frac{d}{2} - \frac{1}{4}\sqrt{4d^2 - v^2}$: tangent $\angle AFG = \frac{2dr - r\sqrt{4d^2 - v^2}}{v}$

and :: Ag : tangent $\angle Afg = \frac{2dr + r\sqrt{4d^2 - v^2}}{v}$. Hence

Theorem 7. $w = \frac{d}{2} \pm \frac{1}{4}\sqrt{4d^2 - v^2}$.

Theorem 8. $b = \frac{2dr \pm r\sqrt{4d^2 - v^2}}{v}$.

Theorem 9. $T = \frac{\sqrt{2d \pm \sqrt{4d^2 - v^2}}}{4.012}$ ſeconds.

PROBLEM VII.

GIVEN the height of the range, and height of the perpendicular ſhot, to find the time, direction, and amplitude.

SOLUTION.

$GF = \sqrt{w \times d - w}$, and $\sqrt{wd - ww} : r :: w : b$. Hence Fig. 137.

Theorem 10. $v=4\sqrt{dw-w^2}$.

Theorem 11. $b=r\sqrt{\frac{w}{d-w}}$

Theorem 12. $T=\frac{\sqrt{w}}{2.006}$ ſeconds.

PROBLEM VIII.

IT is required to determine the angle of direction, height of the range, and amplitude, by having the time of deſcription and height of the perpendicular ſhot.

SOLUTION.

By *Theorem* 12, $T=\frac{\sqrt{w}}{2.006}$; Th. $AG=w=4.023T^2$; hence $GF=\sqrt{d-4.023T^2\times 4.023T^2}$, the amplitude $AE=v=8.024T\sqrt{d-4.023T^2}$, and then will the tangent of the angle of direction $b=\frac{2.0054rT}{\sqrt{d-4.023T^2}}$. Hence

Theorem 13. $w=4.023T^2$.

Theorem 14. $v=8.024T\sqrt{d-4.023T^2}$.

Theorem 15. $b=\frac{2.0054rT}{\sqrt{d-4.023T^2}}$.

PROBLEM IX.

HAVING given the amplitude and height of the range, I demand the time, and angle of direction, with the height of the perpendicular ſhot.

SOLUTION.

As $\frac{v}{4}:r::w:b=\frac{4rw}{v}$, and $w:AF=\sqrt{w^2+\frac{v^2}{16}}::AF:AH=d=w+\frac{v^2}{16w}$; whence $T=\frac{\sqrt{w}}{2.006}$ ſeconds, by *Theo.* 12. Hence

Theorem 16. $b=\frac{4rw}{v}$.

Theorem 17. $d=w+\frac{v^2}{16w}$.

PROBLEM X.

GIVEN the angle of elevation, and height of the range; I demand

mand the amplitude, and time of flight, with the height of the perpendicular ſhot.

SOLUTION.

As $b : w :: r : \frac{rw}{b} = GF$; Th. amp. $AE = v = \frac{4rw}{b}$, and $\frac{r^2w}{b^2} = GH$; Th. $AH = d = w + \frac{r^2w}{b^2}$; the time of projection is found by *Theorem* 12, *viz.* $T = \frac{\sqrt{w}}{2.006}$ ſeconds. Hence

Theorem 18. $v = \frac{4rw}{b}$.

Theorem 19. $d = w + \frac{r^2w}{b^2}$.

PROBLEM XI.

GIVEN the amplitude, and time of flight; I demand the height of the range, angle of elevation, and the height of the perpendicular ſhot.

SOLUTION.

BY *Theorem* 13. the height of the range $AG = w = 4.023T^2$; then $\frac{v}{4} : r :: 4.023T^2 :$ tangent $\angle$ direction $b = \frac{16.092rT^2}{v}$;

Again $4.023T^2 : \frac{v}{4} :: \frac{v}{4} : GH = \frac{v^2}{64.368T^2}$. Hence

Theorem 20. $b = \frac{16.092rT^2}{v}$.

Theorem 21. $d = 4.023T^2 + \frac{\frac{1}{16}v^2}{4.023T^2}$.

PROBLEM XII.

GIVEN the angle of direction, and time of flight, to find the reſt.

SOLUTION.

BY *Theorem* 13. we have *Theorem* 22. for the height of the range $w = 4.023T^2$, and $b : 4.023T^2 :: r : GF = \frac{4.023T^2}{b}$; Th. $AE = v = \frac{16.092rT^2}{b}$, and the height of the perpendicular ſhot $AH = 4.023T^2 \times \sqrt{\frac{r^2}{b^2} + 1}$. Hence

Theorem 23. $v = \frac{16.092rT^2}{b}$.

Theorem

Theorem 24. $d=4.023T^2\times\sqrt{\frac{r^2}{b^2}+1}$.

Since m, n, and $b=$ fine, cofine, and tangent $\angle$ of direction, radius $=r$, $w=$ height of the range in feet $v=$ amplitude in feet, $T=$ time of projection in seconds, and d the height of the perpendicular fhot with the fame velocity, we have theſe equations:

$$\left.\begin{array}{l} r^2v=4mnd \\ m^2d=r^2w \\ 4.023r^2T^2=dm^2 \\ nb=mr \end{array}\right\}$$

Let $A=$ the angle of direction, then from the foregoing operations are deduced the following *Theorems*:

	Given	THEOREMS.
1	A, d	$w=m^2dr^{-2}$
2		$v=4mnd^{r-2}$
3		$T=\frac{m\sqrt{d}}{2.006r}$.
4	A, v	$w=\frac{bv}{4r}$.
5		$d=\frac{1}{4}v\times\sqrt{\frac{r}{b}+\frac{b}{r}}$.
6		$T=\frac{\sqrt{bv}r^{-1}}{4.012}$.
7	d, v	$w=\frac{d}{2}\pm\frac{1}{4}\sqrt{4d^2-v^2}$.
8		$b=\frac{2dr\pm r\sqrt{4d^2-v^2}}{v}$.
9		$T=\frac{\sqrt{2d\pm\sqrt{4d^2-v^2}}}{4.012}$. 'Tis impoſſible to hit the mark, when v is greater than $2d$.
10	w, d	$v=4\sqrt{dw-w^2}$.
11		$b=r\sqrt{\frac{w}{d-w}}$.
12		$T=\frac{\sqrt{w}}{2.006}$.
13	T, d	$w=4.023T^2$.
14		$v=8.024T\sqrt{d-4.023T^2}$
15		$b=\frac{2.0054rT}{\sqrt{d-4.023T^2}}$.
16	v, w	$b=4rwv^{-1}$.
17		$d=w+\frac{v^2}{16w}$.
		T= as per *Theorem* 12.
18	A, w	$v=4rwb^{-1}$.
19		$d=w+r^2wb^{-2}$.
		T= as per *Theorem* 12.
		$w=$ as per *Theorem* 13.
20	T, v	$b=16.092rT^2v^{-1}$.
21		$d=4.023T^2+\frac{\frac{1}{16}v^2}{4.023T^2}$.
22	A, T	$w=4.023T^2$.
23		$v=16.092rT^2b^{-1}$.
24		$d=4.023T^2\times\frac{r^2}{b^2}+1$.

ROBERT ROBINSON.

We hope it won't be thought diſagreeable to our readers, if we adjoin a table of gunnery, to Mr *Robinſon*'s theory, though ſome thing foreign to our preſent purpoſe, wherein are ſhewn the names of the ſeveral pieces of ordnance, length of the cannons, their weight in pounds avoirdupois, their bore, bullets diameter and weight, the ladles length and breadth, together with the quantity to charge them.

A TABLE of GUNNERY.

The names of the ſeveral pieces of ordnance.	Gun's length.	Gun's weight	Gun's bore.	Bullet's diame.	Bullet's weight	Ladle's length	Ladle's bread.	Powder's weight.	Shot point blank
	feet inc.	Pounds	Inches	Inches	lb. oz.	Inches	Inches	lb. oz.	Paces
A Baſe	4 6	200	1 1/2	1 1/2	0 5	4	2	0 8	
A Rabinet	5 6	300	1 1/2 1/4	1 1/2 1/4	0 8	4 1/2	2 1/2	0 12	70
A Fauconet	6	400	2 1/4	2 1/4	1 5	7 1/2	4	1 4	90
A Faucon	7	750	2 1/2	2 1/2	2 8	8 1/2	4 1/2	2 4	200
Minion ord.	7	800	3	2 7/8	3 4	8 1/2	5	2 8	120
Minion large	8	1000	3 1/4	3	3 12	9	5	3 4	125
Saker large	8	1400	3 1/2 1/4	3 1/4	4 12	9 1/2	6 1/2	3 6	150
Saker ord.	9	1500	3 1/2	3 1/4	6 0	10 1/2	6 1/2	4 0	160
Saker old	10	1800	4	3 1/4	7 5	11	7 1/2	5 0	163
Demi cul. leaſt	10	2000	4 1/4	4	9 0	12	8	6 4	174
Demi cul. ord.	11	2700	4 1/2 1/4	4 1/4	10 11	12 1/2	8	7 4	175
Demi cul. old	11	3000	4 1/2	4 1/4	12 11	13 1/2	8 1/2	8 8	178
Culverin leaſt	11	4000	5	4 1/4	15 0	14 1/4	9	10 0	180
Culverin ord.	12	4500	5 1/4	5	17 5	16	9 1/2	11 6	
18 pounder	12	4600	5 1/2 1/4	5 1/4	18 0	16	9 3/4	11 7	
Culver. large	12	4800	5 3/4	5 1/2	20 0	16	10	11 8	
24 Pounders	12	5000	5 3/4 1/4	5 1/2	24 0	16 1/2	10 1/4	12 4	
Demi can. leaſt	11	5400	6	6	30 0	20	11 1/2	14 0	136
Demi can. ord.	12	5600	6 1/4	6 1/4	32 0	22	12	17 8	162
Demi can. large	12	6000	6 1/2 1/4	6 1/2	36 0	22 3/4	12	18 0	180
Cannon royal	12	8000	8	7 1/2	48 0	24	14 1/2	32 8	185

The ſeven laſt are commonly called *battering pieces*; tho a 24 pounder, as I was informed by the late honourable the lord *George Graham*, is the beſt in common uſe for land ſervice. Thoſe that would ſee more of the *arms* and *machines* uſed in war, ſince the invention of *gun-powder*, may conſult M. *Le Blond*'s elements of war, printed for *E. Cave* at St *John*'s *Gate*, being eſteemed the beſt work of its kind for the inſtruction of young gentlemen in the army, yet extant.

QUESTION XCV. *anſwered by Mr* W. JEPSON.

Pl. 7. Fg. 140. LET AE=10=a, BD=4=c, ſine ∠BEA=x, and ſine ∠BAC=$2x\sqrt{1-xx}$, its coſine =$2x^2-1$; then $1 : a :: x : ax$=AB, and $2x\sqrt{1-xx} : c :: 2x^2-1 : \frac{2cx^2-c}{2x\sqrt{1-xx}}$=AD; Th. $ax-\frac{2cx^2-c}{2x\sqrt{1-x^2}}=c$, or $\overline{2ax^2-2cx}\times\sqrt{1-xx}=2cx^2-c$, which, being involved and brought into numbers, is $x^6-.8x^5-.68x^4+.8x^3-.32x^2=-.04$; whence, by converging ſeries, $x=.951056$, &c. Th. AB=9.51056; BC=6.90624, and the triangle's area=32.8336.

The ſame anſwered by VASSAL MORPETHIENSIS.

LET AE=10=c, DF=DB=4=2a, DA=$\frac{a}{x}-ax$; then AF =$\frac{a}{x}+ax$, AB=$a\times\frac{1-x^2+2x}{x}$, $\overline{BE}^2=c^2-a^2\times\frac{\overline{1-x^2+2x}^2}{x^2}$; and $\frac{4a^2}{x^2}=\overline{DA+AF}^2 : 4a^2\ (:: \overline{AD}^2 : \overline{DG}^2) :: a^2\times\frac{\overline{1-x^2+2x}^2}{x^2}$ $: c^2-a^2\times\frac{\overline{1-x^2+2x}^2}{x^2}$. This analogy, properly order'd, and in numbers, gives $x^6-4x^5+3x^4-22x^2+4x+1=0$; and, by converging ſeries, $x=.3246$. Th. AB=9.512, BC=6.92, AC=11.782, and the area =32.91.

QUESTION XCVI. *anſwered by Mr* R. ROBINSON.

Pl. 7. Fig. 141. LET AT=a=30, CT=n=40, BT=m=50, v=TE, and $2y$=AB=BC=CA the ſide of the field; then CF=$y\sqrt{3}$=ry, and $4y : m+a :: m-a : \frac{m^2-a^2}{4y}=\frac{s}{y}$=EF, Th. AE=$y-\frac{s}{y}$, and by 47 *E.* 1. $v^2=a^2-\overline{y-\frac{s}{y}}^2$, and $n^2=\frac{s^2}{y^2}+\overline{ry-v}^2$; in which ſubſtitute the value of v, and we have $r\sqrt{a^2y^2-y^4+2sy^2-s^2}=y^2+\frac{a^2+2s-n^2}{2}=y^2+q$, and ſquaring both ſides $y^4+\frac{3q-3a^2-6s}{4}y^2+\frac{3s^2+q^2}{4}=0$; wherefore $2y=67.6643$ chains the ſide of the field,

and

and its area $y^2\sqrt{3}=1982.5308$ square chains = Ac. 198 R. 1 P. 0.49.

The same answered by Mr SAMUEL SMITH.

LET BT=50=d, AT=40=b, CT=30=c, let m and n= sine and cosine of 60°, and for the sine and cosine ∠BAT put x and y, and let AB=BC=CA=z. Then as $1 : b :: x : bx$= TE; also $1 : b :: y : by$=AE; but $z-by$=BE, Th. $zz-3byz+b^2y^2+b^2x^2=dd$; hence $y=\frac{zz-900}{80z}$. And, by the same way of reasoning, we have $\frac{zz+700}{80z}=ny+mx$, whence $x=\frac{zz+2300}{160z}$; but $xx+yy=1$, Th. by reduction we have $z^4+1930000=5000z^2$; Th. $z=67.664$, and the content = 198A. 1R. 0.49P.

QUESTION XCVII. *answered by Mr* CH. SMITH.

LET a and e represent the sine and cosine of the latitude, u and y those of the declination, c the cosine of the hour from noon at 5, n the sine of the altitude at 5, and m that of the altitude at 6 o'clock. Then $au=n-cey$, and $au=m$, by *Spherics*; Th. $n-cey=m$, and $ey=\frac{n-m}{c}$; hence $ey-au=\frac{n-cm-m}{c}=.3667273$ = cosine of 68° 29′ 10″, the sum, and $ey+au=\frac{n+cm-m}{c}=$ .8680505 = cosine of 29° 46′, the difference, of latitude and declination. Th. the latitude is 49° 7′ 35″, and the declination 19° 21′ 35″.

The same answered by the Proposer Mr T. COWPER.

LET a= sine of the sun's altitude at 5= 24° 14′, b= sine of the hour from 6=15°, c= sine of the sun's altitude at 6, = 14° 31′, and let x and y express the sine and cosine of half the sum of the co-lat. and co-declination, and v and w the sine and cosine of half the difference of those arcs. Then is $wy+vx$= sine of the latitude, and $wx-vy$= cosine thereof; and $wy-vx$= sine, and $wx+vy$= cosine of the sun's declination. Now, by a common theorem, $w^2y^2-v^2x^2+bw^2x^2-by^2v^2=a$; and, by *Trigonometry*, as $1 : wy+vx :: wy-vx : w^2y^2-v^2x^2=c$. The difference of these is $bw^2y^2-bv^2y^2=a-c$; and putting $1-w^2$ for v^2, and $1-y^2$ for x^2, and the reverse, we have $w^2-y^2=\frac{a-c}{b}$, and $x^2-v^2=\frac{a-c}{b}$; and

substi-

ſubſtituting the ſame again in the firſt equation above, and writing values of x^2 and y^2 in the laſt equations, I find $w^2=\frac{a+b-c+bc}{2b}$, and $v^2=\frac{b+c-a-bc}{2b}$; Th. $w^2-v^2=\frac{a-c+bc}{b}=.8680467$, the ſine of 60° 13′ 56″, the ſun's meridian altitude. By a like proceſs, I find $x^2-y^2=\frac{a-c-bc}{b}=.3667235$, the ſine of 21° 30′ 49″, the ſun's depreſſion at midnight; the complement of half the ſum of which is 49° 7′ 37″, the latitude of the place, and half the difference thereof 19° 21′ 34″, the ſun's declination.

THEOREM.

If the difference between the ſines of the ſun's altitude at the reſpective times be divided by the ſine of the hour from fix, and the quotient be augmented and diminiſhed with the ſine of the ſun's altitude at fix, the ſum will be the ſine of the ſun's meridian altitude, and the difference that of his ſeptentrional depreſſion.

The ſame anſwered by Philoſluentimecanalgegeomaſtrolongo.

Pl. 7. Fig. 142.

LET x = ſine ½ ſum of lat. and declination.
y = ſine of half their difference.
$s=s$, AB the altitude at 6.
$n=s$, altitude at 5 = cof. Z ⊙.
c = cof. hour ∠ at 5 = cof. ZP⊙.

Then $x\sqrt{1-yy}\pm y\sqrt{1-xx}=\begin{cases}s, \text{ lat. ACB.}\\ s, \text{ declin. AC.}\end{cases}$

And $\sqrt{1-xx}\sqrt{1-yy}\mp xy=\text{cof.}\begin{cases}\text{lat.}\\ \text{decli.}\end{cases}$

In the right-angled triangle ACB, $1:x\sqrt{1-yy}-y\sqrt{1-xx}::x\sqrt{1-yy}+y\sqrt{1-xx}:s$; whence $xx\times\overline{1-yy}-yy\times\overline{1-xx}=s=x^2-y^2$. In the triangle ⊙PZ, $c\times\overline{1-xx}\times\overline{1-yy}-xxyy+xx\times\overline{1-yy}-yy\times\overline{1-xx}=n$, by *Anderſon*'s theorem, that is, $c\times\overline{1-xx-yy}+xx-yy=n$, or $\overline{c\times 1-xx-yy}+s=n$, and $1-xx-yy=\frac{n-s}{c}$, whence $1-2xx=\frac{n-s}{c}-s=$ cof. ſum of the lat. and declination, and $1-2yy=\frac{n-s}{c}+s=$ cof. diff. lat. and declination, whence the latitude and declination are known.

QUES-

QUESTION XCVIII. *answered by Mr* WM BROWN.

PUT $b=40$, the slant height of the cone, and $x=$ semidiameter of the cone's base; theref. its area is $3.1416xx$, its altitude, by 47 *E.* 1. $\sqrt{bb-xx}$, and the cone's solidity $=1.0472xx\sqrt{bb-xx}$, which, by the *question*, must be a *maximum*, this in fluxions is $2.0944x\dot{x}\times\overline{bb-xx}-1.0472x^3\dot{x}=0$; Th. $2\times\overline{bb-xx}=0$, and consequently $x=b\sqrt{\frac{2}{3}}=32.65984$. Hence the diameter of the cone's base is 65.31968; its altitude $=23.09443$, and its solidity $=25796.75$.

The same answered by R. R. H.

PUT $b=$ slant height, $x=$ radius of the base, $c=.7854$; then $\sqrt{bb-xx}=$ perpendicular height, and $\frac{4cx^2}{3}\sqrt{bb-xx}=$ the solidity; therefore its Fluxion $4b^2x^3\dot{x}-6x^5\dot{x}=0$. Hence $x=b\sqrt{\frac{2}{3}}$, and the solidity is $\frac{8cb^3}{9\sqrt{3}}=25796.75$.

The same again by Mr W. JEPSON.

LET $40=a$, $3.1416=c$, radius of the base $=x$; then the altitude $=\sqrt{a^2-x^2}$, area of the base $=cx^2$, and Th. $\frac{cx^2}{3}\sqrt{a^2-x^2}$, or $a^2x^4-x^6=$ *maximum*, which in Fluxions is $4a^2x^3\dot{x}-6x^5\dot{x}=0$; Th. $x=\frac{2a}{\sqrt{6}}=32.66$ &c. and the altitude $=\frac{a}{\sqrt{3}}=23.0946$. Hence the solidity $=25797.2916$; and the slant side, radius of its base, and altitude, are as $\sqrt{3}$, $\sqrt{2}$, and 1, universally.

QUESTION XCIX. *answered by Mr* JOHN WILLIAMS.

LET $d=$ diameter of the circle $=24$, $a=\frac{1}{3}$ of its area $=150.7968$, and $v=$ the versed sine required. Then (by Sir *Isaac Newton*'s theorem to find a segment of a circle) $\frac{4v}{15}\sqrt{dv+4\sqrt{dv-\frac{1}{4}v^2}}=a$; Th. $3600dv^3+14400v^2\sqrt{dv-\frac{1}{4}v^2}=a^2$. Put $m=3600$, and $n=14400$; then $2a^2mdv^6-\frac{1}{4}n^2v^6+n^2dv^5=a^4$, Th. $v=8.82025$. Hence the required chord $=23.14208$.

The ſame anſwered by Mr W. JEPSON.

Pl. 7. Fig. 143. LET radius = 1, and area of the portion $EbBH = \frac{3.14156}{12} = .2618 = a$, $EA = BH = x =$ area $EABH$, $AB = y = 1 - \sqrt{1 - xx}$, area $EAb = z$; then $\dot{z} = y\dot{x} = \dot{x} - \dot{x}\sqrt{1-xx} = \dot{x} - \dot{x} + \frac{x^2\dot{x}}{2} + \frac{x^4\dot{x}}{8} + \frac{x^6\dot{x}}{16}$ &c. and $z = \frac{x^3}{6} + \frac{x^5}{40} + \frac{x^7}{112}$ &c. $= x - a$; Theref. $x - \frac{x^3}{6} - \frac{x^5}{40} - \frac{x^7}{112}$ &c. $= a$; and, by reverſion, $x = a + \frac{a^3}{6} + \frac{13a^5}{120} + \frac{493a^7}{5040}$, &c. $= .2649322$. Hence the required chord $= 2bF = 23.1424$, and verſed ſine $FG = 8.8208$.

The ſame anſwered by Mr JOHN HAMMOND.

PUT D = 24, the diameter, the verſed ſine $FG = v$, and $c = 452.3904$, the area of the circle. Then $D : v :: 1 : \frac{v}{D}$, and $1 : 1000$ (= rad.) $:: \frac{v}{D} : \frac{1000v}{D} = m$, the verſed ſine of ſuch a ſegment when the diameter is = 1. Again, by 12 *E.* 2. ſay $1^2 : m :: D^2 : D^2 \times m = 1000Dv = \frac{c}{3}$ by the queſtion; hence $v = \frac{c}{3000D} = .0062832$ ($= \frac{1}{3}c$) when the diameter is 1, and 0.0062832×1000 (the rad. of *Sbirtcliff*'s Tables of Segments) $= 6.2832$, which divided by $D = .2618 =$ the ſegment (when the diameter = 1) the verſed ſine anſwering thereto in the Tables is 367. Now $1000 : 24$ ($= GC$ or EH) $:: 367 : 8.808$, the verſed ſine required. Laſtly, put $x =$ ſemichord Fb, or FB; then by *E.* 6. 13. ſay $D - v : x :: x : v$; hence $x = \sqrt{Dv - vv} = 482$ ſuch parts as the diameter is 1000; Th. $24 : 1000 :: 482 : 11.568 = bF$; Hence the whole chord is 23.136.

QUESTION C. *anſwered by* R. R. H.

PUT $b = 100$, $c = 3.1416$, $x =$ length, $y =$ baſe's diameter; then the area of the baſe $= \frac{cy^2}{4}$, circumference $= cy$, and the whole ſurface $= cxy + \frac{cy^2}{2} = b$; whence $x = \frac{2b - cy^2}{2cy}$, and the ſolidity $= \frac{cy^2x}{4} = \frac{2by - cy^3}{8}$, a *maximum*, in Fluxions $2b\dot{y} - 3cy^2\dot{y} = 0$. Th. $y = \sqrt{\frac{2b}{3c}}$, and the ſolidity $= \frac{b}{6}\sqrt{\frac{2b}{3c}} = 76.77$.

The

The ſame anſwered by VASSAL MORPETHIENSIS.

LET $c=100$, $s=.7854$, $x=$ diameter; then $\frac{c-2sx^2}{4sx}=$alt. and $\frac{cx-2sx^3}{4}=$ ſolidity, a *maximum*, in Fluxions $c\dot{x}-6sx^2\dot{x}=0$.

Th. $x=\sqrt{\frac{c}{6s}}=4.606$, and ſolidity$=\sqrt{\frac{c^3}{216s}}=76.776$.

QUESTION CI. *anſwered by* TOM PROBLEM.

THE Propoſer means no more than to find $\overline{x^{\frac{1}{3}}}\,|^{x^{\frac{30}{91}}}$ a *minimum*, and therefore, by Mr *Hauxley*'s rule, *p.* 194, the hyp. log. of x is $\frac{-1}{\frac{30}{91}}=3\frac{1}{30}$, and th. $x=.048153$; but it will be a quite different thing to ſuppoſe $\overline{x^{\frac{1}{3}}}\,|^{x^{\frac{6}{13}}}$ to be the index of $x^{\frac{1}{3}}$.

The ſame anſwered by Mr ROB. ROBINSON.

IF $\overline{x^{\frac{1}{3}}}\,|^{\overline{x^{\frac{5}{7}}}\,|^{x^{\frac{6}{13}}}}$ be taken $\overline{x^{\frac{1}{3}}}\,|^{\overline{x^{\frac{5}{7}}}\,|^{x^{\frac{6}{13}}}}=\overline{x^{\frac{1}{3}}}\,|^{x^{\frac{30}{91}}}=$ (univerſally) $\overline{x^n}\,|^{x^n}=\overline{x}\,|^{ax^n}$ a *minimum*, or $\overline{x}\,|^{x^n}$, its hyp. log. is $\dot{X}x^n+nX\dot{x}x^{n-1}=0$. But ſince X is the hyp. log. of x, we have $\dot{X}=\frac{\dot{x}}{x}$; hence the equation becomes $1+nX=0$, and $X=\frac{-1}{n}$ (which is Mr *Hauxley*'s Theorem; ſee *p.* 193.) $=-3.0$, and this multiplied by .424294 gives the common negative log. $=-1.317385$, whoſe correſponding binomial log. is $\bar{8}.682642$, anſwering to $0.04815=x$.

QUESTION CI. *anſwered by* Philoſluentimecanalgegeomaſtrolongo.

LET AB$=x$, BG$=y$, AC$=2x$, $b=2.42611$, $c=3.1416\times 2$; $a=8$, $ax=yy$; then the ſurface FAG$=\frac{c}{12a}\times\overline{aa+4yy}\,|^{\frac{3}{2}}-\frac{caa}{12}$ Pl. 7. Fig. 144.

$=\frac{c}{12a}\times\overline{aa+4ax}\,|^{\frac{3}{2}}-\frac{caa}{12}$, and the ſurface DAE$=\frac{c}{12a}\times\overline{aa+8ax}\,|^{\frac{3}{2}}-\frac{caa}{12}$; then $\frac{c}{12a}\times\overline{aa+4ax}\,|^{\frac{3}{2}}-\frac{caa}{12}:\frac{c}{12a}\times\overline{aa+8ax}\,|^{\frac{3}{2}}-\frac{caa}{12}::1:b$, and

b, and $b \times \overline{aa+8ax}^{\frac{1}{2}} - b^2 b = \overline{aa+8ax}^{\frac{1}{2}} - a^2$, and $b \times \overline{aa+4ax}^{\frac{1}{2}} - \overline{aa+8ax}^{\frac{1}{2}} = b-1 \times a^3 = da^3$, by the reduction of which x is had, and consequently y.

The same answered by Mr ROBERT ROBINSON.

LET parameter $=a=8$, $n=3.1416$, and $x=CA$; then $DC=\sqrt{ax}$, and the fluxion of the surface is $nx\sqrt{a^2+4ax}$, whose Fluent is $\frac{n}{6a}\sqrt{a^2+4ax}^3 - \frac{na^2}{6}=$ surface DAE; and since AB=BC, the surface FGA is $\frac{n}{6a}\sqrt{a^2+2ax}^3 - \frac{na^2}{6}$; as 1 : 2.4261136 $=r$, then is $\frac{rn}{6a}\sqrt{a^2+2ax}^3 - \frac{rna^2}{6} = \frac{n}{6a}\sqrt{a^2+4ax}^3 - \frac{na^2}{6}$, Th. $r\sqrt{a+2x}^3 - ra^{\frac{3}{2}} = \sqrt{a+4x}^3 - a^{\frac{3}{2}}$. Let $v^2=a+2x$, and the equation becomes $\overline{2v^2-a}^{\frac{3}{2}} = rv^3 + a^{\frac{3}{2}} - ra^{\frac{3}{2}} = rv^3 - d$; Th. $v^6 - \frac{12a}{8-r^2}v^4 + \frac{2rd}{8-r^2}v^3 + \frac{6a^2}{8-r^2}v^2 = \frac{d^2+a^3}{8-r^2}$. Hence the altitude $=8$, and diameter at top $=16$.

QUESTION CIII. *answered by* TOM PROBLEM.

Pl. 6. Fig. 132.

THE fluxion of the area BVD$=a^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}$, of EVD$=a^{\frac{1}{2}}x^{\frac{3}{2}}\dot{x}$, and the area BVD$=\frac{2a^{\frac{1}{2}}x^{\frac{5}{2}}}{5}$, EVD$=\frac{3a^{\frac{1}{2}}x^{\frac{5}{2}}}{5}$ whence $\frac{2a^{\frac{1}{2}}x^{\frac{5}{2}}+3a^{\frac{1}{2}}x^{\frac{5}{2}}}{5}$ $=9.9795$, th. x or VD is known, and consequently y, or BD and DE. Let DM$=v$, then $\frac{2a^{\frac{1}{2}}x^{\frac{5}{2}}}{5} - \frac{1}{2}xv = \frac{3a^{\frac{1}{2}}x^{\frac{5}{2}}}{5} + \frac{1}{2}xv$, or $xv = \frac{2xy-3xY}{5}$, whence $v=\frac{2BD-3DE}{5}=DM$.

The same answered by Mr ROBERT ROBINSON.

LET area BVE$=A=9.9795$ sq. chains; and since $ax^3=y^2$, Th. $y\dot{x}=a^{\frac{1}{2}}x^{\frac{3}{2}}\dot{x}$, whose Fluent is $\frac{2a^{\frac{1}{2}}x^{\frac{5}{2}}}{5}=$ area VBD. Again, since $ax^3=y^2$. Th. $y\dot{x}=a^{\frac{1}{2}}x^{\frac{3}{2}}\dot{x}$, and its Fluent is $\frac{3a^{\frac{1}{2}}x^{\frac{5}{2}}}{5}=$ area VED, then

is

Plate VII

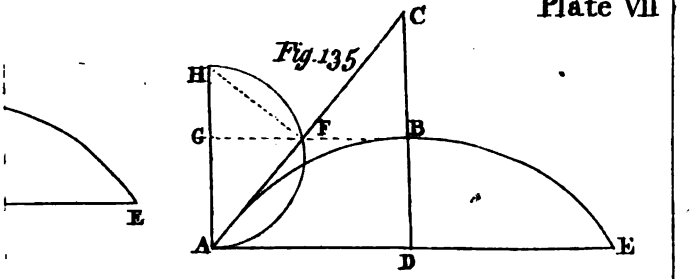
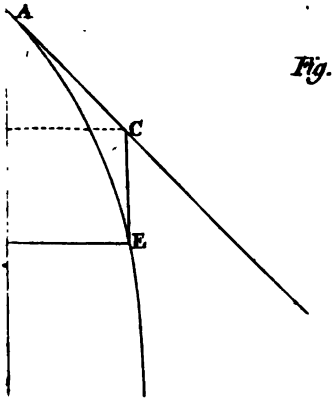


Fig.135
C
H
G
F
B
A
D
E
E


A
C
E

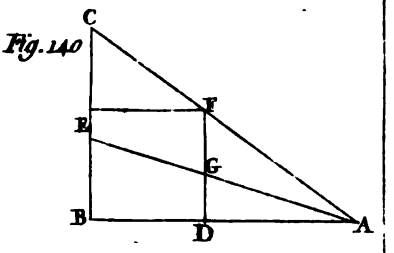


Fig.140
C
F
E
G
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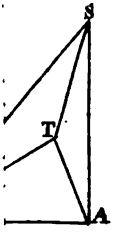
D
A


S
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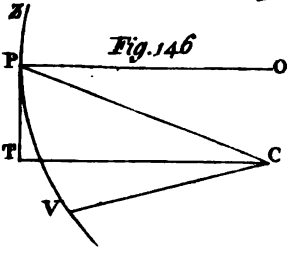

Z
Fig.146
P
O
T
C
V

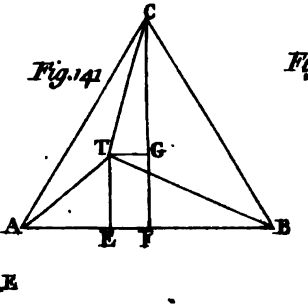


Fig.141
C
T
G
A
E
F
B
E

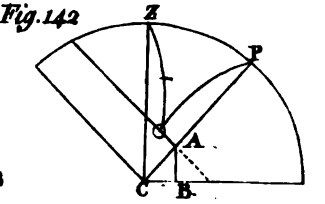


Fig.142
Z
P
G
A
C
B

is $\frac{2a^{\frac{1}{2}}x^{\frac{1}{2}}+3a^{\frac{1}{2}}x^{\frac{1}{2}}}{5}=A$. Let $v^{\frac{1}{2}}=x$, and the equation is $2a^{\frac{1}{2}}v^{\frac{1}{2}}+3a^{\frac{1}{2}}v^{\frac{1}{2}}=5A$; and when $a=1$, it is $2v^{\frac{1}{2}}+3v^{\frac{1}{2}}=5A$; Th. $v=5.8329192$, whence $x=8.299715=VD$, and the area $BDV=79.3812$, and that of $VDE=20.4137$; the difference of these areas, divided by VD, gives $MD=7.10477$. Th. $BD=23.91083$, $BM=16.80606$, and $MV=10.92487$ chains.

New QUESTIONS *to be answered in our next.*

QUESTION CIV. *By Monsieur* LE FETE.

HOW shall I find a canon for discovering the discount of money? And also a Theorem for discovering the present worth of money due at the end of any days?

QUESTION CV. *By Mr* GEORGE REED.

REquired the least number, which being divided by all the nine digits, shall leave no remainder?

QUESTION CVI. *By Mr* JOHN HAMMOND.

ON the sea shore, I observed from a ship the flash of a gun, at the same instant I set my cane a swinging, whose length from the string hole was $33\frac{1}{4}$ inches, and it made 22 vibrations, before I heard the report; it is proposed to find the distance of the ship?

N. B. *For the motion of sound, and the length of a royal pendulum, see* p. 71.

QUESTION CVII. *By Mr* WILLIAM CHAPPLE.

MEasuring a piece of ground that lies off at some distance from other grounds already measured and mapp'd, and being willing to connect this with the other at its proper bearing and distance; I took the angle (ASB) which my station S made with two known points (A and B) in the grounds already measured, and found it 25 degrees; also I found the angle (TSB) which my station made with a tower T (in the said grounds) and the said known point B 22° 30′. Now the distance AB was 47 chains, TA=40, and TB=67.5 (as appear'd by my former map.) It is required to find, from these data, the distances SA, ST, and SB, by a geometrical construction. Pl. 7. Fig. 145.

QUESTION CVIII. *By Mr* J. WAINE.

HAVING a desire to get the latitude of a certain place, I made use of the following method to obtain it, *viz.* by erecting

three ſtaves each 10 feet high, perpendicular to the horizon, two of which were placed *North* and *South* of each other, and the third 43,83146 feet diſtant from either of them; and at a time that the ſun had 10 degrees of ſouth declination, made my obſervations, and in the firſt place found that the ſhadow of the ſouthermoſt ſtaff fell juſt upon the bottom of that ſtanding north thereof; and afterwards, the ſame day, obſerved that the ſhadow of the third ſtaff fell perpendicular to the firſt ſhadow, and terminated therein; whence I preſumed I might find the latitude, but meeting with greater difficulties than I expected, deſire the aſſiſtance of ſome ingenious gentlemen, that may have leiſure to aſſiſt a friend?

QUESTION CIX. *By Mr* SAMUEL DUNN.

HOW many degrees is that angle, whoſe tangent is double its arc?

QUESTION CX. *By Mr* JOHN BULMAN.

ADMIT a fruſtum of a cone to ſtand on its leſſer baſe, whoſe diameter is 45 inches, the greater diameter is a geometrical mean betwixt the leſſer and the perpendicular height, and the content is =904.14376875 ale gallons; required the other dimenſions, and alſo to what part of this terraqueous globe muſt the ſaid tun be carried, ſo that when plac'd in the former poſition, it ſhall hold 5 gallons more?

QUESTION CXI. *By Mr* SAMUEL SMITH.

HOW many pints of ale may be drawn from a cylinder, whoſe diameter and diagonal (when made of the leaſt wood poſſible) are equal to the following expreſſions, $x^1 = z =$ diameter, $\overline{z^x \times x}^{\frac{1}{x}} =$ diagonal, and $x^x = y$?

QUESTION CXII. *By Mr* PAUL SHARP.

IF the difference of the tranſverſe and conjugate diameters of an ellipſis be =4, and the area of the greateſt inſcribed parallelogram =48; required the ſaid diameters, and area of the ellipſis?

QUESTION CXIII. *By Mr* JOHN TODD.

ADMIT a ſhip ſail from a port in a certain north latitude, due eaſt 400 miles, to another port in the ſame latitude, from which, ſometime after, ſhe ſets ſail again to come the neareſt way ſhe poſſibly can back again to the former port, and finds her diſtance

run

ſun = 3 8p miles. Required the latitude, and difference of longitude, of the ſaid ports?

QUESTION CXII. *By* GAMSTON RETFORD.

IT is propoſed to determine the quantity of degrees, whereby the moon is retarded by the action of the ſun, whilſt the earth deſcribes about the ſun the leaſt given arc poſſible?

QUESTION CXIV.

THE Fluent of $\dot{x}x^{n-1}$ (when $x=1$) is $=\frac{1}{n+1}+\frac{1}{n+1.n+2}+\frac{1.2}{n+1.n+2.n+3}+\frac{1.2.3}{n+1.n+2.n+3.n+4}+\frac{1.2.3.4}{n+1.n+2.n+3.n+4.n+5}$ &c. $=\frac{1}{n}$. Quere the inveſtigation?

QUESTION CXV.

2. THE Fluent of $\frac{n\dot{x}x^{\frac{m-n-2r}{r}}}{r}$ (when $x=1$) is $=\frac{n}{m}+\frac{n.n+r}{m.m+r}+\frac{n.n+r.n+2r}{m.m+r.m+2r}+\frac{n.n+r.n+2r.n+3r}{m.m+r.m+2r.m+3r}$, &c. $=\frac{n.}{m-n-r}$. Quere the inveſtigation?

QUESTION CXVI.

3. THE Fluent of $\frac{m-r.a\dot{x}x^{\frac{m-n-2r}{r}}}{r}$ (when $x=1$) is $=a+\frac{an}{m}+\frac{a.n.n+r}{m.m+r}+\frac{a.n.n+r.n+2r}{m.m+r.m+2r}+\frac{a.n.n+r.n+2r.n+3r}{m.m+r.m+2r.m+3r}=\frac{a.m-r}{m-n-r}$. Quere the inveſtigation of theſe three problems, they being of great uſe in the ſummation of ſeries? *G—t.*

QUESTION CXVII.

SUPPOSE the centripetal force tending to a given point C, in any place P, be as CP^n the power of the diſtance CP reciprocally, and the reſiſtance of the medium be as the denſity of the medium, and any power of the velocity conjunctly; it is propoſed to find the denſity of the medium in each place, which ſhall make the body move in any given curve VPZ, alſo the velocity of the body, and the reſiſtance of the medium in each place? F. HOLLIDAY. Pl. 7. Fig. 146.

The Method for finding Fluents according to Mr Emerson's *Table, continued from page* 219.

EXAMPLE XIX.

To find the *Fluent* of $\frac{2axx}{\sqrt{a^4-x^4}}$?

By reaſon of the negative quantity $-x^4$, compare this with *Form* 10th, then $z=x$, $n=4$, $\alpha=a^4$, $\beta=1$, and the *Fluent* of $\frac{xx}{\sqrt{a^4-x^4}}=\frac{2N}{4}\times$ degrees is the arc, whoſe ſine is $\sqrt{\frac{x^4}{a^4}}$ or $\frac{xx}{aa}$; and the *Fluent* of $\frac{2axx}{\sqrt{a^4-x^4}}$ is $=.017453ax$ degrees in the arc whoſe ſine is $\frac{xx}{aa}$, and radius 1.

EXAMPLE XX.

To find the *Fluent* of $\frac{-4ax}{\sqrt{2ax-xx}}$, or of $\frac{-4ax^{-\frac{1}{2}}\dot{x}}{\sqrt{2a-x}}$?

By *Form* 10. $z=x$, $n=1$, $\alpha=2a$, $\beta=1$; whence the *Fluent* of $\frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{2a-x}}=\frac{N}{1}\times$ degree in the arc whoſe ſine is $\frac{2}{2a}\sqrt{2a-x\times x}$; and the *Fluent* of $\frac{-4dx^{-\frac{1}{2}}\dot{x}}{\sqrt{2a-x}}$, or of $\frac{-4ax}{\sqrt{2ax-xx}}$, is $=-4ax.017453\times$ degrees in the arc whoſe ſine is $\frac{1}{a}\sqrt{2dx-xx}$, and radius 1.

H;

The End of Number VII.

Miſcellanea Curioſa, &c.

NUMBER VIII.

A Letter from the Reverend Mr Bolton, *A. M. ſhewing an eaſy Conſtruction and Calculation of a Problem for determining the Places of a Comet, when it firſt appears, and when it is going away.*

PROBLEM.

FOUR Lines being given in poſition, *viz.* FC, GA, HB, IC, 'tis required to find a fifth (FI) which ſhall cut be in the Points G and H in any given Ratio. Pl. 8. Fig. 147.

GEOMETRICAL CONSTRUCTION.

ON FC (produced if neceſſary) make GH : HI :: AB : BK. Then produce GA and HB indefinitely, and they will interſect in D. Make FG : GI :: BD : DL; join the points K, L, by the line KL, which produc'd will cut CI given in I, which will be one extremity of the line required.

On IL produc'd make HI : GH :: IL : LM; draw MG parallel to LB, and that will cut AG in G, which is another point of the line required. Through G draw IF, and that ſhall be the line required.

TRIGONOMETRICAL CALCULATION.

FROM the ſides BK, BL, and angle included, find KL and BKL, which, when K falls within C, is equal to IKC; but when K falls without C, then IKC is the complement of BKL. In the triangle IKC, from all the angles, and ſide KC (which is the difference between BK and BC) find IK, which added to KL gives IL. But (as *per* conſtruction) HI : GH :: IL : LM; and KL : LM :: BK : BC, by ſimilar triangles; and, laſtly, GH : FG :: BQ : QF;

QF; to which add AQ, and you will have AF; then draw IF, and that ſhall be the line required; whoſe length may be found from IK, KF, and angle included.

Yours, &c. STEPH. BOLTON.

Solutions to the Queſtions in Number VII.

QUESTION CIV. *anſwered by* R. R. H.

LET t = time in years, r = rate *per cent. per annum*, x = value of the principal p, at the end of any given time, c = 100, a = 365, n = any number of days to come; then rt = intereſt of 100 *l.* for t years, and $c+rt : rt :: x : \frac{rtx}{c+rt}$ = diſcount of x for t years. Again $1 : \frac{r}{a} :: n : \frac{rn}{a}$ = intereſt of 100 *l.* for n days; and $c+\frac{rn}{a} : x :: \frac{rn}{a} : \frac{rnx}{ca+rn}$ = diſcount for n days. Again $c : p :: \frac{rn}{a} : \frac{rnp}{ac}$ = intereſt of the ſum p for n days.

The ſame anſwered by Mr R. ROBINSON.

LET r = rate *per cent. per annum*, t = times in days, m = pounds due at t time, n = 365 days, and x = the diſcount; then $m-x$ = preſent worth, and $n : r :: t : \frac{rt}{n}$, = intereſt *per cent.* for t days; and $100 : \frac{rt}{n} :: m-x : x$; hence we have theſe *Theorems*.

I. $x = \frac{rmt}{100n+rt}$ = the diſcount.

II. $m-x = \frac{100mn}{100n+rt}$ = the preſent worth.

EXAMPLE.

WHAT is the diſcount and preſent worth of 151 pounds due at the end of 250 days, at 5 *l. per cent. per annum*?

Here $r=5$, $m=151$, $t=250$; hence the diſcount $x=5l$; preſent worth $m-x=146l$.

QUESTION CV. *anſwered by Mr* ROB. ROBINSON.

LET a, b, c, d, e, f, g and h be the quotients reſpectively, produced by dividing x by 2, 3, 4, 5, 6, 7, 8 and 9; then will $x =$

$2a =$

$2a=3b=4c=5d=6e=7f=8g=9h$. Th. $a=b+\frac{b}{2}$, hence $\frac{b}{2}=$ m a whole number, then is $b=2m$, and $c=m+\frac{m}{2}$, hence $\frac{m}{2}=n$ a whole number, then $m=2n$, $c=3n$; and proceeding in this manner, I find $b=280$ A, where A is any whole positive number; hence this *Theorem*,

$$x=2520\text{A}.$$

And when A = 1, then $x=2520$, the least number required.

The same answered by Mr R. GIBBONS.

IT is evident that $1\times2\times3\times4\times5\times6\times7\times8\times9=362880$, will leave nothing remaining when divided by the respective divisors; and $362880-1=362879$, will leave one less than the divisor remaining. But 4 is the square of 2, 6 is a multiple of 2 and 3, also 8 is the cube of 2, and 9 the square of 3. Consequently $2\times3\times2\times5\times7\times2\times3=2520$, the required number. But if it had been required to have left the remainders 8, 7, 6, &c. to unity, then $2520-1=2519$ would have been the required number. Hence this general rule: *Multiply all the prime numbers and lowest roots of their powers progressively together (rejecting all the composed numbers) to the height of the greatest divisor, the last product divided by the several divisors will leave nothing remaining; but being made less by unity, will always leave one less than the divisor remaining.*

The Rule exemplified by the 61st *Question*, Gentleman's Diary.

"Required the least number, which being divided by 20, 19, 18, "&c. to unity, shall leave 19, 18, 17, &c. respectively to unity re- "maining."

Here $2\times3\times2\times5\times7\times2\times3\times11\times13\times2\times17\times19-1=232792559$, the number required. Hence we prove Mr *Rob. Robinson*'s answer true, tho' not so general as by this rule.

Note, That s, v, x and z are the lowest roots, or primes of 4, 8, 9 and 16.

QUESTION CVI. *answered by* N. FARRER.

BY the nature of pendulums, as 33.5 : 39.13 :: 3600 : 4205.01492537, whose square root = 64.846087 = the cane's vibrations in one minute; hence the time of 22 vibrations = .33926488

.33926488 min. then $\frac{1106 \times 60 \times .33926488}{5280} = 4.2639427$ miles, the ſhip's diſtance.

QUESTION CVII. *anſwered by Mr* N. FARRER.

Pl. 8. Fig. 148. PRoject the triangle ABT, by the *data*; draw a ſegment of a circle on AB to contain an angle of 25°, and another on BT to contain an angle of 22° 30′, and they will interſect at the ſtation S. Draw SB, ST and SA, which, being meaſured from the ſame ſcale the triangle ABT is projected, will give SA=85.8, ST=46, SB=107.5.

The ſame anſwered by Mr ROBERT ROBINSON.

CONSTRUCTION.

SINCE AB=47, BT=67.5, and AT=40, project the triangle ABT, make the angle ABD=2° 30′, and ∠BAD=22° 30′, drawing the lines BD, AD till they interſect at D; then thro' the three points B, D, A, draw a circle, and thro' D and T draw the right line DTS; then will S be the place of ſtation.

CALCULATION.

IN the triangle ABD, having AB=47, and the angles, find BD=42.5588. In the triangle ABT, having the three ſides, find the ∠ABT=35° 30′ 18″. Th. ∠TBD=38° 00′ 18″. In the triangle TBD are known BT, BD and ∠TBD; find ∠TDB=∠BAS=104° 20′ 07″. In the triangle ABS, given AB and the angles, find BS=107.7485, and AS=86.0166. In the triangle BTS, there are known BT and the angles, find TS=46.1268 chains.

Again by Mr WILLIAM JEPSON.

BY trigonometry, ∠ATB=43° 2′, and 43° 2′−25°=18° 2′= ſum ∠∠'s SAT+SBT; let s and q= ſine and coſine of half the ſum, and x and y= ſine and coſine of half the difference; then $sy+qx$= ſine of the greater, and $sy-qx$ = ſine of the leſſer angle. Put BT=a, AT=b, ſine of 22° 30′=m, of 2° 30=n; then $m : a :: sy+qx : \frac{asy+aqx}{m}$= ST. $n : b :: sy-qx : \frac{bsy-bqx}{n} = \frac{asy+aqx}{m}$, which reduc'd, gives

$y=$

$y=\frac{mbqx+naqx}{mbs-nas}=\sqrt{1-xx}$, and by ſubſtitution $cx=d\sqrt{1-xx}$, whence $x=\frac{d}{\sqrt{c^2+d^2}}$; and now the diſtances are eaſily found.

QUESTION CVIII. *anſwered by Mr* JOHN TODD.

PUT c=EN=ES=43.83146, a=10, s and q = to the ſine and coſine of the ſun's declination, v and z= ſine and coſine of his meridian altitude, and x and y = ſine and coſine ∠WEN; then cy=EW, cx=NW, and $2cx$=NS, $qz-sv$= ſine latitude; now $v:a::z:\frac{az}{z}=2cx$, and $qz-sv:s::1:\frac{s}{qz-sv}$ = ſine altitude when due eaſt, and $cy:1::a:\frac{a}{cy}$=tangent altitude when due eaſt, hence the ſine $=\frac{a}{\sqrt{c^2y^2+a^2}}=\frac{s}{qz-sv}$; but from the equation $\frac{az}{v}=2cx$, we get $z=\frac{2cx}{\sqrt{a^2+4c^2x^2}}$, and $v=\frac{a}{\sqrt{a^2+4c^2x^2}}$, whence $\frac{a}{\sqrt{a^2+c^2y^2}}=\frac{s\sqrt{a^2+4c^2x^2}}{2cqx-as}$, and by reduction $x^4-1-\frac{3a^2}{4c^2}+\frac{a^2q^2}{c^2s^2}\times x^2-\frac{a^3q}{c^3s}x=\frac{a^2}{4c^2}$. In numbers, $x^4+.586657x^2-.067348x=.0126272$, whence by converging ſeries $x=.205066$=S∠WEN=11° 50′; hence the latitude =49° 55′. Pl. 8. Fig. 149.

QUESTION CIX. *anſwered by Mr* EDWARD HAUXLEY.

LET x= length of the required arc, radius =1; then will $x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7$ &c. = its tangent, which, by the queſtion, muſt be $=2x$; when reduced we have $x^6+2.4706x^4+6.17x^2$, &c. =18.53. By reverſion of ſeries, x=1.1668 the arc, and therefore $2x$=2.3336= tangent of 66° 48′, the degrees of that arc.

The ſame anſwered by Mr R. ROBINSON.

LET rad. =1, tangent $=v$; then the Fluxion of the arc is $\frac{\dot{v}}{1+v^2}$, whoſe Fluent, by the 5th form of Mr *Emerſon*'s tables, is $.0174532925x$, where x= degrees in the arc required; then, by the queſtion, $v-.0349065850 4x=0$, which is eaſily ſolved by a

table of natural tangents, thus:

Assume - - - x $=66.7°$, and the equation gives $-.0062946$

66.8 - - - - - - - - - - - - - $+.001415$

Then $\frac{66.7\times.00145+66.8\times.0062946}{.00145+.0062946}=66.781$ - - - - - - - - - - $-.0000568$

66.782 - - - - - - - - - - $+.0000206$

Hence $x=66.78173=66°\ 46'\ 54''$, the angle required.

QUESTION CX. *answered by Mr* N. FARRER.

Pl. 8. Fig. 150. GIVEN AB$=45=a$, let $x=$CD, then $a : x :: x : \frac{xx}{a} =$ EF, and the content in ale gallons $=xx+ax+aa\times\frac{.7854x^2}{846a}=$ 904.141376875; hence $x=67.5=$CD, for which put $2m$, FE$=$ 101.25. Let $z=$ radius of that sphere whose segment upon CD will contain 1410 inches $=5$ gallons, then the height of that segment $=z-\sqrt{z^2-m^2}$, which call n, and its solidity $=\overline{3m^2+n^2}\times$ $5236n=1410$ inches; hence $n=.7879$ inches, the height of the segment, and $z=723.241075$, from which subtracting the height of the tun and segment, leaves 621.203175 inches $=51.85$ feet, the distance the tun's bottom must be placed from the earth's center.

The same answered by Mr EDWARD HAUXLEY.

LET 45 inches the lesser diameter $=b$, $c=.7854$, $x=$ the greater diameter; and, by the question, $b : x :: x : \frac{xx}{b}=$the depth; then will $\frac{bcx+cx^2+b^2c}{3}\times\frac{x^2}{b}=254968.54278$, the given content in inches; and being further reduced, we have $x^4+45x^3+2025x^2=$ 43825761.74, &c. whence x, or the greater diameter, is $=67.472$ inches, and $\frac{x^2}{b}$, or the depth, $=101.16$. Now the earth's diameter being $=504725760$ inches, and the top diameter of the tub $=$ 67.472, which must be look'd upon to be a chord of the earth, and the liquor above it part of the earth's convex surface, therefore by putting $x=$ versed sine of the liquor sought, then $504725760-x=$ the other versed sine of the earth; hence $504725760x-x^2=$ $33.736\times33.736=1138,117696$; whence $x=.00000001$. Therefore the content of the liquor above the tub on the earth's surface, on account of its being small, it not amounting to $\frac{1}{100}$th part of a cubic

cubic inch, may be safely neglected in the remaining part of the question. Now to find the diameter of that globe the segment whereof shall contain 1410 solid inches, having the diameter of the segments of the base 67.472 inches = top diameter of the tub; Let x = versed sine of such segment, b=33.736 = half the diameter; then will $\overline{3b^2+x^2}\times 5236x = 1410$, whence x=.78858 inches, the versed sine of such segment; then, by 35 *E.* 3. $\frac{33.736\times 33.736}{.78858}$ =1443.25 inches, the other versed sine, and $\frac{1443.25+0.78858}{2}$ =722.019 inches, the radius of the required globe, or the distance the top edges of the tub are to be removed from the earth's center, or 25236215 7.9 from its surface; where it will evidently hold 5 gallons more than upon the earth's surface.

QUESTION CXI. *answered by Mr* W. JEPSON.

BEcause the whole surface is a *minimum*, the diameter and length are equal (see *Emerson*'s Fluxions p. 125.) Therefore (by 47*E.* 1.) the diagonal $=\overline{2z^2}^{\frac{1}{2}} = \overline{z^2\times x}^{\frac{1}{2}}$ by the Question; consequently x=2, y=4, z=16, and the content =91.262, &c. ale pints, supposing the dimensions given in inches.

The same answered by the Proposer, Mr SAMUEL SMITH.

ASsume the content =1, or any other number at pleasure; let d=3.1416, then will $\frac{4}{dzz}$= height. As $1 : d :: z : dz$= the circumference, then will $\frac{4}{z}$= curve superficies, and $\frac{dzz}{4}$= area of each end, whence $\frac{4}{z}+\frac{dzz}{2}$= *minimum*; in Fluxions, $3dz^2\dot{z}\times 2z - 2\dot{z}\times dz^3+8=0$. Th. $z=\sqrt{\frac{4}{d}}$=1.08385, and the height will be equal to the same. Therefore of all cylinders, when made of the least wood possible, the diameter and height are equal; hence we have $x^2y\times 2=z^2\times x$, or $\overline{x}^{2xx}\times 2=\overline{x}^{3}\times x$, whence $\overline{x}^{2}\times 2=\overline{x}^{2}\times x$. Th. x=2, y=4, and the diameter=16. Wherefore 91.2624 =number of pints.

QUE-

QUESTION CXII. *answered by Mr* SAMUEL DUNN.

PUT t= transverse diameter of any ellipsis, c= its conjugate, x = semilength of the greatest inscribed parallelogram; then, by the nature of the Ellipsis, $t^2 : c^2 :: t^2-x^2 : \frac{t^2c^2-c^2x^2}{t^2}$, and $\sqrt{\frac{t^2c^2-c^2x^2}{t^2}}$= its semibreadth, which multiplied into x gives $\sqrt{\frac{t^2c^2x^2-c^2x^4}{t^2}}=\frac{1}{4}$ of its area, a *maximum*. In Fluxions, $2t^2c^2x\dot{x}-4t^2c^2x^3\dot{x}=0$, Th. $x=t\sqrt{\frac{1}{2}}$, and $2x=2t\sqrt{\frac{1}{2}}$, the parallelogram's length; after the same manner is $2c\sqrt{\frac{1}{2}}$ the breadth. To apply this to the present case, put $\sqrt{\frac{1}{2}}=.70716=n$, z= half sum of the transverse and conjugate diameters; then $z+2$= transverse, and $z-2$ = conjugate, and by the foregoing theorems $\overline{nz+2n}\times\overline{nz-2n}$ $(=n^2z^2-4n^2)=48$ by the Question, that is, $.5z^2-2=48$, or $z=10$; therefore the two diameters are 12 and 8, and the area $12\times8\times.7854=75.3984$.

The same answered by Mr EDWARD HAUXLEY.

LET x= conjugate diameter, then $x+4$= transverse, and $\sqrt{\frac{xx}{2}}$= length, and $\sqrt{\overline{x+4}^2}$ divided by 2= breadth of the greatest inscribed parallelogram. Th. $\sqrt{\frac{x^4+8x^3+16x^2}{4}}=48$, and reduc'd, $x^4+8x^3+16x^2=9126$, hence $x=8$, the conjugate, and $x+4=12$, the transverse diameter of the ellipsis. Th. $\sqrt{\frac{xx}{2}}=\sqrt{32}=5.656$, the breadth of the parallelogram, and $\sqrt{72}=8.429$, the length thereof.

QUESTION CXIII. *answered by Mr* ROB. ROBINSON.

Pl. 8. Fig. 151. SUPPOSE the ship departs from W. and sails upon the parallel of latitude WBF, due east to B, the distance WB=400 miles, and comes back upon the great circle BW=380 miles. Here is given WC=CB=3° 10′, sine $=m$, arc of the parallel WC=200 $=a$; let x= cosine of the latitude; then $x : a :: 1 : \frac{a}{x}$ miles =

ED;

BD; let s = fine of the degrees $\frac{a}{60x} = \frac{19}{3x}$ (whofe radius is 1) then $s : m$: : 1 : x, hence $sx = m$, or $sx - .0552406 = 0$; folved by a table of natural fines, $x = .104895$, the cofine of 83° 58′ 44″ = the latitude; hence the ∠ WNB = 63° 33′ 21″, the difference of longitude required.

The fame anfwered by Mr Nic. Farrer.

LET WB the arc of the parallel circle = 400 miles = m, and WB the arc of the great circle = 380 miles = 6° 20′, the fine of its half = n, put x = cofine of the latitude, and r the earth's radius in nautical miles; then by parallel failing $x : m :: 1 : \frac{m}{x}$, whofe fine is $\frac{m}{x} - \frac{m^3}{6r^2x^3} + \frac{m^5}{120r^4x^5}$ &c. and $\frac{m}{x} - \frac{m^3}{6r^2x^3} + \frac{m^5}{120r^4x^5}$ — &c. : n :: 1 : x; Th. $\frac{m^3}{6r^2x^2} - \frac{m^5}{120r^4x^4} + \frac{m^7}{5040r^6x^6}$ &c. $= m - n$; hence $x = .104894$ = cofine of 83° 59′, the latitude required, and the difference of longitude of the ports = 63° 33′.

QUESTION CXIV. *anfwered.*

THIS Queftion of the increafe of the moon's mean motion, and her annual equation depending on the action of the fun, is founded on *Scholium* to *Prop.* 35. *B.* 3. of Sir *Ifaac Newton*'s *Principia*, and admits of three hypotheſes, *viz.* 1ſt That the orb of the moon is circular; 3d That it is elliptical; and laſtly in an hypotheſis of her orbit inclined to the ecliptic; of theſe in their order, and, firſt, on a ſuppoſition that the orb of the moon is circular.

And in order to give a compleat ſolution to this important problem, it will be requiſite to premiſe the following *Lemmata*, becauſe of their uſe in all queſtions of this nature.

LEMMA I.

SUPPOSE T the earth, P the moon, A D B C a circle which the moon deſcribes; let S T be the mean diſtance of the earth from the ſun, which we will call a, F the force of the ſun on the earth at that mean diſtance, the ſun being ſuppoſed immoveable; call P T, the diſtance of the moon from the earth, r; let C P be the diſtance of the moon from the next quadrature, which call u; and let y and z = ſine and cofine thereof; I ſay, that that part of the ſun's force Pl. 8. Fig. 152.

 which

which acteth on the moon, in a direction of the radius PT, is every where $\frac{F}{a} \times \frac{3yy}{r} - r$, by *Prop.* 66. *B.* 1. *Principia.*

DEMONSTRATION.

For, according to *Newton*'s conſtruction, *Prop.* 66. let the force of the ſun, which is called F, be repreſented by the line ST, or SK; that force of the ſun wherewith the moon is attracted in P, be repreſented by the line SL; and this force may be ſuppoſed to be compounded of theſe two, SM and LM, whereof LM is parallel to the radius PT; but when the line SM is equal to the line ST $\pm$TM, and the earth attracted by the force ST, not as the moon, the moon's and the earth's reſpective ſituation is not changed by that force ST, therefore that part alone of the force SM, which is expreſſed by TM, comes to be conſidered. Therefore, from the nature of gravity, SK is to SL as $\frac{1}{\overline{SK}^2}$ to $\frac{1}{\overline{SK \mp PK}^2}$, or as $\overline{SK}^2 \mp 2SK \times PK + PK^2$ to $\overline{SK}^2$; or, omitting the term $\overline{PK}^2$, as SK $\mp$2PK to SK; or, becauſe 2PK is ſmall in reſpect of the line SK, as SK to SK$\pm$2PK; therefore SL, or SK$\pm$KL=SK$\pm$2PK, and KL=2PK. But when the line PL is nearly parallel to the line SM, and PT parallel to LM, from the conſtruction, TM is nearly equal to the line PL, and PL=PK+KL=3PK. From the Point L draw a perpendicular to the radius PT (produced if requir'd) and the force TM, or the force PL, equal to it, may be ſuppoſed to be reſolved into the forces PE, and LE; let the force LE be perpendicular to the radius PT, and then it does not affect the central force; let the force PE act in the direction of the radius, and let the point P be attracted from the center T, and the other part, which is repreſented by LM, acting in the direction of the radius, attract the point P towards the center; therefore that part of the ſun, which acteth on the moon in the direction of the radius PT, is the difference of the forces PE and LM. Now, by reaſon of the parallels SL, SM, TP and LM, LM is =TP= r; and when PK is nearly perpendicular to TC, PK will be the ſine of the arc PC, which we call y; therefore PL=3PK=3y; and when the triangles PKT, PEL are ſimilar, PT (r) : PK (y) :: PL (3y) : PE=$\frac{3yy}{r}$, and the difference of the forces PE

and

and L M is $\frac{3yy}{r}-r$; which difference is positive when $\frac{3yy}{r}$ exceeds r, and then it attracts the moon from the center; and negative when $\frac{3yy}{r}$ is less than r, then it attracts the moon to the center; therefore when the line S T, or a, represents the whole force of the sun on the earth, and that force be called F, and the quantity $\frac{3yy}{r}-r$ represents that part of the sun's force which acteth on the moon in the direction P T, and it be made as a to $\frac{3yy}{r}-r$, so F to that part of the sun's force which causeth the central force of the earth on the moon, will therefore be $\frac{F}{a}\times\frac{3yy}{r}-r$. Q. E. D.

L E M M A II.

SUPPOSE r= rad. of the circle ADBC, ϱ= rad. of the circle adbc, or $\alpha\delta\beta\varkappa$; let p be the difference of the radii r and ϱ, V= force of a body T at the distance r, and in the same distance call the outward force Y, which let increase as the distances from the center T, and which may be supposed positive, if it attracts the body P from the center, but negative if it attracts it to the center; I say, that the radius ϱ will always be equal to $\frac{V-3Y}{V-4Y}r$, or $r\times 1+\frac{Y}{V}+\frac{4Y^2}{V^2}+\frac{16Y^3}{V^3}$, &c. and omitting the terms, they vanishing because of the smallness of Y, ϱ is $=Y\times 1+\frac{Y}{V}$. Pl. 8. Fig. 153.

For the force of the body T, at r distance, will be $\frac{rr}{\varrho\varrho}V$, the outward force will be $\frac{\varrho}{r}Y$, therefore the force whereby the circle adbc (or $\alpha\delta\beta\varkappa$) is described, is $\frac{rr}{\varrho\varrho}V-\frac{\varrho}{r}Y$. But this force ought to be to the force V, whereby the circle ABCD is described, inversely as the cubes of the radii, or as $\frac{1}{\varrho^3}$ to $\frac{1}{r^3}$; therefore $\frac{V}{\varrho^3}=\frac{V}{r\varrho^2}-\frac{Y\varrho}{r^4}$, or, reducing the terms to the same denominator, is $\varrho^4Y=r^3V\times\varrho-r=\pm r^3pV$. Instead of ϱ write $r\pm p$, and it will be $r^4Y\pm 4r^3pY+6r^2p^2Y$

$6r^2p^2Y \pm 4rp^3Y + p^4Y = \pm r^3pV$; or, blotting out the terms where p is above one dimension, becauſe this quantity is ſmall, becomes $r^4Y \pm 4r^3pY = \pm r^3pV$, or $\pm pV \mp 4pY = rY$, whence is obtained $\pm p = \frac{rY}{V-4Y}$; therefore ρ, which is $r \pm p$, becomes $\frac{V-3Y}{V-4Y}r$, which reduced into a ſeries is $r \times 1 + \frac{Y}{V} + \frac{4Y^2}{V^2}$ &c. or $r \times 1 + \frac{2Y}{V}$.

LEMMA III.

If the periodic time of a body P in the circle ADBC be called M, I ſay, that its periodic time in the circle adbc (or $\alpha\delta\beta\varkappa$) will be $M \times 1 + \frac{2Y}{V}$.

DEMONSTRATION.

The periodic time of a body P revolving in the circle adbcd, (or $\alpha\delta\beta\varkappa$) by reaſon of Y the outward force being added or diminiſhed, is to the periodic time of its body P, when it was revolved in the circle ADBC, without any outward force, as the ſquare of the radius ρ to the ſquare of the radius r. For becauſe the force Y is always directed to the center T, the areas will remain proportional to the times, which way ſoever the body P is inclined; therefore, if at laſt its way be changed into the circle adbcd (or $\alpha\delta\beta\varkappa\delta$) the time wherein the periphery adbc (or $\alpha\delta\beta\varkappa$) will be deſcribed, will be to the time wherein the periphery ADBC was deſcribed, as the whole area of the circle adbc (or $\alpha\delta\beta\varkappa$) to the whole area of the circle ADBC; therefore as the ſquares of the radii ρ and r, or (by the foregoing *Lemma*) as $\frac{V-3Y^2}{V-4Y^2}rr$ to rr, conſequently as $\frac{V-3Y^2}{V-4Y^2}$ to 1. But this fraction, reſolved into a ſeries, becomes $1 + \frac{Y}{V} + \frac{4Y^2}{V^2}$ &c. which ſeries converges ſwift, becauſe of the ſmallneſs of this fraction $\frac{Y}{V}$; and its ſquare is $1 + \frac{2Y}{V} + \frac{9Y^2}{V^2} + \frac{40Y^3}{V^3}$ &c. Therefore as 1 to $1 + \frac{2Y}{V}$ &c. ſo M to $M \times 1 + \frac{2Y}{V}$ +, which is the time wherein the periphery adbc or $\alpha\delta\beta\varkappa$ will be deſcribed.

LEMMA IV.

Things remaining as in *Lemma* I. put c = whole circumference whoſe

whole radius is r, call Y the force of the ſun acting on the moon in the direction P T, (ſee *Fig.* 152.) and at the given diſtance C P from C the quadrature, call C P, u, let M be the periodic time of the moon in the circle ADBC, without the action of the ſun, the ſmall arc taken from the point P be called $\dot{u}$; I ſay, that the time wherein a ſimilar arc will be deſcribed in an orbit, into which the moon is carried by the action of the ſun, will be $\frac{M\dot{u}}{c} \times 1 + \frac{2Y}{V} +$ &c.

For if the force Y, which is acted by the ſun on the point P, be divided into ſmall particles, and every one of which be called $\dot{Y}$, ſhould remain conſtant during one revolution of the moon, and thus by degrees ſhould carry the moon into the circle adbc, the periodic time in each circle would exceed the periodic time in the preceding circle by the quantity $\frac{2\dot{Y}}{V}$. Hence at laſt the periodic time wherein the circle adbc would be deſcribed, will be $M \times 1 + \frac{2Y}{V} +$ &c. by *Lemma* 3; and the time wherein an arc ſimilar to the arc $\dot{u}$ would be deſcribed in that circle, will be to this periodic time, as $\dot{u}$ to c, therefore will be $\frac{M\dot{u}}{c} \times 1 + \frac{2Y}{V} +$.

Before we exhibit a ſolution to the queſtion in hand, it will be requiſite *To find the whole retardation of the moon whilſt ſhe performs one revolution.*

It is manifeſt, from *Lem.* 4. that if the ſun be immoveable, and the moon in a whole revolution ſuffer that force of the ſun, which ſhe ſuffers in the point P, it will come to paſs that the time wherein the arc $\dot{u}$ is deſcribed (which ought to be $\frac{M\dot{u}}{c}$) is $\frac{M\dot{u}}{c} \times \overline{1 + \frac{2Y}{V}}$, therefore that time is augmented by $\frac{M\dot{u}}{c} \times \frac{2Y}{V}$; conſequently ſince in the time $\frac{M\dot{u}}{c}$ the arc $\dot{u}$ ſhould have been deſcribed, in this time $\frac{M\dot{u}}{c} \times \frac{2Y}{V}$ the arc $\frac{2Y}{V}\dot{u}$ would be deſcribed; therefore this is the retardation of the moon in the point P, ariſing by the action of the ſun. Pl. 8. Fig. 154.

 But

But in every point P of the lunar orbit the force Y is $\frac{F}{a} \times \overline{\frac{3yy}{r} - r}$ by *Lem.* 1. Therefore the element of the moon's retardation is $\dot{u}$ $\frac{2F}{Va} \times \overline{\frac{3yy}{r} - r}$, whoſe Fluent is $\frac{2F}{Va} \times \overline{\frac{3r^2c}{8r} - \frac{1}{4}rc}$, or $\frac{2F}{Va} \times \frac{1}{4}rc$, when P arrives at A; and when the effect of the ſun is the ſame in each quadrant, the whole retardation of the moon is $\frac{2F}{Va} \times \frac{1}{4}rc$, or $\frac{Frc}{Va}$, whilſt the moon performs one revolution in reſpect of the ſun being unmoveable.

If the ſun have a motion, and inſtead of M, a periodic month, μ a ſynodical month be underſtood, and it be ſuppoſed that the ſynodical month is nearly true, which anſwers to the periodic month, is to that periodic month as μ to M, then that ſynodical month is $\mu \times \overline{1 + \frac{2Y}{V}}$, the concluſion will come out the ſame as before, and $\frac{Frc}{Va}$ will be the retardation of the moon in its whole ſynodical time.

But here may ariſe a doubt amongſt ſome of our readers, whether in this expreſſion $\frac{Frc}{Va}$, the quantity c denotes the periphery of 360 deg. or that periphery which the ſun paſſes over in a ſynodical month; but it is evident from the method of finding the Fluent, that is, of the true quadrant of a circle, therefore c here denotes that periphery and nothing more, ſo that $\frac{Frc}{Va}$ is the abſolute retardation of the moon in a ſynodical time.

But, to uſe a better and more certain correction, it is evident from 26 *Prop. Lib.* 3. of *Newton*'s *Principia*, that the velocity of the moon is increaſed by the ſun acting in a direction perpendicular to the radius of the lunar orbit, ſo that, the moon's velocity in the quadratures be to its velocity in any other point as $109.73r$ to $109.73r + \frac{yy}{r}$, hence the time wherein the arc $\dot{u}$ is deſcribed is made ſhorter in proportion of the velocities; therefore when that time was before $\frac{\mu \dot{u}}{c} \times \overline{1 + \frac{2Y}{V}}$, it becomes $\frac{109.73r}{109.73r + \frac{yy}{r}} \times \frac{\mu \dot{u}}{c} \times \overline{1 + \frac{2Y}{V}}$, or, by reducing the fraction into a ſeries, $\overline{1 - \frac{yy}{109.73rr}} \times \frac{\mu \dot{u}}{c} \times \overline{1 + \frac{2Y}{V}}$; but

but this quantity $\frac{\mu \dot{u}}{c} \times \overline{1 + \frac{2Y}{V}}$ contains two parts. The firſt part, independent of the action of the ſun, which is in a direction of the radius, and of the acceleration belonging to this part, is elegantly treated in 26 *Prop. B.* 3. of *Newton*'s *Principia*. And hence it comes to paſs that a mean ſynodical month is ſhorter than that, which ought to be in proportion of 10973 to 11073 (when $y=r$) and the inequalities ariſing therefrom in the various parts of a ſynodical month are contained in the variation. The other part $\frac{\mu \dot{u}}{c} \times \frac{2Y}{V}$ depends on the action of the ſun in direction of the radius of the lunar orbit, and of this part alone, it comes next under our conſideration, to give the method of calculation. Therefore when from this there ariſes $\frac{2Y}{V} \dot{u}$ the retardation, and the time $\frac{\mu \dot{u}}{c}$ be made leſs in proportion as 1 to $1 - \frac{yy}{109.73r^2}$, the retardation, which will be made, whilſt the arc $\dot{u}$ ought to have been deſcribed, will be only $\frac{2Y\dot{u}}{V} - \frac{2Yyy\dot{u}}{109.73r^2V}$; for Y put $\frac{F}{a} \times \frac{3yy}{r} - r$, and it will be $\dot{u} \times \frac{2F}{Va} \times \overline{\frac{3yy}{r} - r - \frac{3y^4}{109.73r^3} + \frac{yy}{109.73r}}$, whoſe fluent for the quadrant is $\frac{2F}{Va} \times \frac{3r^2c}{8r} - \frac{1}{4}rc - \frac{3.3r^4c}{4.8.109{,}73r^3} + \frac{r^2c}{8.109{,}73r}$, or $\frac{2Frc}{Va} \times \frac{1}{8} - \frac{5}{4.8.109{,}73}$, and multiplied by 4 for the whole revolution, it becomes $\frac{Frc}{Va} \times \frac{433.92}{438.92}$.

COROLLARY.

It is evident from *Corol.* 2. *Prop.* 4. *B.* 1. Newton's *Principia*, that the central forces are one to the other directly as the radii, and inverſely as the ſquares of the periodic times; hence if A = ſydereal year, and M a ſydereal periodic month, ſuppoſe all action of the ſun taken away, F will be to V as $\frac{a}{AA}$ to $\frac{r}{MM}$, or $\frac{F}{V} = \frac{aMM}{rAA}$; wherefore ſubſtituting this value for $\frac{F}{V}$ in $\frac{Frc}{Va} \times \frac{433.92}{438.92}$, which expreſſeth

preſſeth the retardation during a ſynodical month, that retardation becomes $\frac{MM}{AA} \times \frac{433.92}{438.92} c$; and if we ſet aſide the correction, which depends on the action of the ſun, as perpendicular to the radius of the lunar orbit, that retardation will be $\frac{MM}{AA} c$.

LEMMA V.

Pl. 8. Fig. 155. Now with the focus F, greater axis NFn, which we call $2a$, deſcribe an ellipſis; let e be the eccentricity thereof, and ſuppoſe it ſmall; let the leſſer axis $= 2b$, then $b^2 = a - e^2$; from the focus as a center, with the radius a, deſcribe a circle; and from the focus draw lines cutting the circle in P, and the ellipſis in Π; call FΠ, x; let the ſine of the angle AFP$= y$, coſine $= z$; I ſay that $x = \frac{b^2 a}{a^2 \mp ez}$.

DEMONSTRATION.

From Π, draw ΠH perpendicular to the axis, and by reaſon of the ſimilitude of the triangles FPE, FΠH, FP will be to FΠ as PE to ΠH, and as FE to FH, that is, $a : x :: y : \frac{y}{a}x :: z : \frac{z}{a}x$. Suppoſe f the other focus of the ellipſis, from it draw fΠ; from the nature of the ellipſis f$\Pi = 2a - x$, but f$\Pi^2 = \Pi H^2 + fH^2$, and $\Pi H = \frac{y}{a}x$, and fH$=$FH$-$Ff, or Ff$-$FH, or Ff$+$FH, and Ff$=2e$, and FH$= \frac{z}{a}x$; hence $\Pi H^2 + fH^2 = \frac{y^2}{a^2}x^2 + \frac{z^2}{a^2}x^2 \mp \frac{4ez}{a}x + 4e^2 = f\Pi^2 = 4a^2 - 4ax + x^2$; but $\frac{y^2}{a^2} + \frac{z^2}{a^2}x^2 = x^2$, th. $\mp \frac{4ez}{a}x + 4e^2 = 4a^2 - 4ax$; by dividing by 4, and tranſpoſition, $ax \mp \frac{ez}{a}x = a^2 - e^2 = b^2$; whence $x = \frac{b^2 a}{a^2 \mp ez}$. *Q. E. D.*

COROLLARY I.

This value of x reſolved into a ſeries is $\frac{b^2}{a} \times 1 \pm \frac{ez}{a^2} + \frac{z^2 e^2}{a^4} \pm \frac{e^3 z^3}{a^6}$ &c. The ſuperior ſigns muſt be taken when E falls in the ſame part as a center, and the ſuperior ſigns, when E falls in that part in which there is no center.

COROL-

COROLLARY II.

If the fraction $\frac{a}{x} \times \frac{a^2 \mp ez}{b^2}$ be raised to superior powers, the terms in which e is of many dimensions, may be omitted, because of the supposition that the eccentricity was small; and if it be considered from the eccentricity of the sun, it does not rise to two hundredth parts of the radius, and the moon's eccentricity does not rise to seven hundredth parts.

COROLLARY III.

Hence the tardation of the moon, which depends on the action of the sun, will become (during S a synodical time) $\frac{433.92\,c}{438.92} \times \frac{M^2}{A^2} \times \frac{a^2 \mp ez^3}{b^6}$, putting a for the greater semi-axis of the sun's orbit, e for its eccentricity, and b for its lesser axis.

And now these things being premised, we come *to determine the quantity of degrees whereby the moon is retarded by the action of the sun, whilst the earth describes about the sun the least given arc possible?*

Suppose, as in the last *Lemma*, NIn be an ellipsis, which the earth describes, let the sun be in the focus F, draw, as in the figure (*see* 155.) the line FΠ, and nearly to it Fπ, cutting the arc Pp in the circle CAD; required the quantity of degrees, whereby the moon is retarded by the action of the sun, whilst the earth seemed from the sun to have described the arc Pp.

Let as before A = annual time, a the greater semi-axis of the ellipsis, k the circumference described with that radius from the focus F, e the eccentricity, $b = \sqrt{a^2 - e^2}$ the lesser semi-axis, the area of the semicircle $= \frac{ak}{4}$, which is to the area of the semi-ellipsis as a to b; hence the area of the semi-ellipsis is $= \frac{bk}{4}$.

Call the arc AP, u; the arc Pp, i; with the radius FΠ, or x, describe a small arc from x on FΠ, it will be to i as FΠ or x to a, therefore that little arc will be $\frac{xi}{a}$, consequently the area FΠx is $\frac{x^2 i}{2a} = \frac{b^2 a i}{2 \times a^2 \mp ez^2}$. But the time wherein the earth seems to have described the arc Pp is to the semi-annual time $\frac{1}{2}$A, as this area FΠx,

FΠx, or $\frac{x^2\dot{u}}{2a}$, to the ſemi-ellipſis $\frac{bk}{4}$; wherefore that time wherein the earth ſeems to have deſcribed the arc Pp, is $= \frac{4x\dot{u}}{2abk} \times \frac{1}{4}A = \frac{x^2A\dot{u}}{abk}$. But it hath been found, that in the time S, the moon was retarded, by reaſon of the action of the ſun, by this quantity $\frac{433.92c}{438.92} \times \frac{M^2a^3}{A^2x^3}$; therefore in the time $\frac{x^2A\dot{u}}{abk}$ ſhe will be retarded by this quantity $\frac{433.92c \times x^2A\dot{u}}{438.92 \times Sabk} \times \frac{M^2a^3}{A^2x^3}$, or by this $\frac{433.92c \times \dot{u}}{438.92 \times S.b.k} \times \frac{M^2a^2}{Ax}$; or, by ſubſtituting the value of the fraction $\frac{a}{x}$, it becomes $\frac{433.92.c\dot{u}}{438.92.S.b.k} \times \frac{M^2a}{A} \times \frac{aa \mp ez}{b^2}$, or $\frac{433.92c\dot{u} \times M^2a}{438.92.SAb^3k} \times a^2 \mp ez$. H.

QUEST. CXV, CXVI *and* CXVII. *anſwered by Mr* HOLLIDAY.

IN order to give the inveſtigation of theſe three problems, it is proper to obſerve that they are founded on Mr *Emerſon*'s VIII. *Rule*, *Ex.* 43. *p.* 51. of his *Fluxions*, which, for its extenſiveneſs and general uſe, in queſtions of this kind, is not to be met with in any other author, at leaſt any hitherto ſeen by me.

1. Aſſume the Fluent of $\dot{x}x^{n-1}$, or $\dot{x}x^nx^{-1} = \frac{x^{n+1}}{n+1}x^{-1} + s$; this in Fluxions gives $\dot{x}x^{n-1} = \dot{x}x^{n-1} - \frac{x^{n+1}}{n+1}x^{-2}\dot{x} + \dot{s}$, whence $\dot{s} = \frac{x^{n+1}}{n+1}x^{-2}\dot{x}$. Again, aſſume $s = \frac{x^{n+2}}{n+1.n+2}x^{-2} + t$, whence $\dot{t} = \frac{2x^{n+2}}{n+1.n+2}x^{-3}\dot{x}$; aſſume $t = \frac{2x^{n+3}}{n+1.n+2.n+3}x^{-3} + u$, &c. whence we have the Fluent of $\dot{x}x^{n-1} = \frac{x^{n+1}}{n+1.x} + \frac{x^{n+2}}{n+1.n+2.x^2} + \frac{2.x^{n+3}}{n+1...n+3.x^3} + \frac{2.3.x^{n+4}}{n+1....n+4.x^4} + \&c. = \frac{x^n}{n}$.

QUESTION CXVI. *anſwered.*

2. Aſſume Fluent $\frac{n\dot{x}}{r}x^{\frac{m-n-2r}{r}} = \frac{n}{m}x^{\frac{m-n-2r}{r}} + s = \frac{n}{m}x^{\frac{m}{r}}x^{-\frac{n+r}{r}} + s$, in Fluxions

Fluxions $\frac{n\dot{x}}{r}x^{\frac{m-n-2r}{r}} = \frac{n\dot{x}}{r}x^{\frac{m-n-2r}{r}} - \frac{n+r}{r} \times \frac{n}{m}x^{\frac{m-n-2r}{r}}\dot{x} + \dot{s}$, whence $\dot{s} = \frac{n.\overline{n+r}}{mr}\dot{x}x^{\frac{m-n-2r}{r}} = \frac{n.\overline{n+r}}{rm}x^{\frac{m}{r}}x^{-\frac{n+2r}{r}}\dot{x}$. Assume $s = \frac{n.\overline{n+r}}{m.\overline{m+r}}x^{\frac{m+r}{r}-\frac{n+2r}{r}}$ $+t$, whence $\dot{t} = \frac{n.\overline{n+r}.\overline{n+2r}}{r.m.\overline{m+r}}x^{\frac{m-n-2r}{r}}\dot{x} = \frac{n.\overline{n+r}.\overline{n+2r}}{rm.\overline{m+r}}x^{\frac{m+r}{r}-\frac{n+3r}{r}}\dot{x}$, Assume $t = \frac{n.\overline{n+r}.\overline{n+2r}}{m.\overline{m+r}.\overline{m+2r}}x^{\frac{m+2r}{r}-\frac{n+3r}{r}} + u$, whence $\dot{u} = \frac{n\ldots\overline{n+3r}}{rm\ldots\overline{m+2r}}$ $\dot{x}x^{\frac{m+2r}{r}-\frac{n+4r}{r}}$. Assume $u = \frac{n\ldots\overline{n+3r}}{m\ldots\overline{m+3r}}x^{\frac{m+3r}{r}}x^{-\frac{n+4r}{r}} + w$, &c. Hence the Fl. $\frac{n\dot{x}}{r}x^{\frac{m-n-2r}{r}} = \frac{n}{m}x^{\frac{m-n-r}{r}} + \frac{n.\overline{n+r}}{m.\overline{m+r}}x^{\frac{m-n-r}{r}} + \frac{n\ldots\overline{n+2r}}{m\ldots\overline{m+2r}}x^{\frac{m-n-r}{r}}$ $+\frac{n\ldots\overline{n+3r}}{m\ldots\overline{m+3r}}x^{\frac{m-n-r}{r}}$ &c. $= \frac{n}{m-n-r}x^{\frac{m-n-r}{r}}$.

This series may also be deduced from another fluxionary quantity, $\dot{x}x^{m-1-n-r}$, putting $x = 1$, thus:

Assume Fl. $\dot{x}x^{m-1-n-r} = \frac{x^m}{m}x^{-n-r} + s$, then will $\dot{s} = \frac{\overline{n+r}.x^m x^{-n-r-1}\dot{x}}{m}$

Assume $s = \frac{\overline{n+r}.x^{m+r}x^{-n-2r}}{m.\overline{m+r}} + t$, then $\dot{t} = \frac{\overline{n+r}.\overline{n+2r}}{m.\overline{m+r}}\dot{x}x^{m+2r-1}x^{-n-3r}$;

assume $t = \frac{\overline{n+r}.\overline{n+2r}}{m.\overline{m+r}.\overline{m+2r}}x^{m+2r}x^{-n-3r} + u$, &c. whence Fl. $\dot{x}x^{m-1-n-r}$

$= \frac{x^m}{mx^{n+r}} + \frac{\overline{n+r}.x^{m+r}}{m.\overline{m+r}.x^{n+2r}} + \frac{\overline{n+r}.\overline{n+2r}.x^{m+2r}}{m.\overline{m+r}.\overline{m+2r}.x^{n+3r}}$ &c. $= \frac{x^{m-n-r}}{m-n-r}$;

multiply all by n, and you have the series required; and by a like method infinite forms may be found.

QUESTION CXVII. *answered.*

3. Assume the Fl. $\frac{m-r}{r}a\dot{x}x^{\frac{m-n-2r}{r}} = ax^{\frac{m-n-r}{r}} + s = ax^{\frac{m-r}{r}}x^{-\frac{n}{r}} + s$,

whence $\dot{s} = \frac{na}{r}\dot{x}x^{\frac{m-r}{r}-\frac{n+r}{r}}$. Assume $s = \frac{an}{m}x^{\frac{m}{r}-\frac{n+r}{r}} + t$, whence $\dot{t} =$

$an.\overline{n+r}$

$\frac{an.\overline{n+r}}{rm}x^{\frac{m+n+2r}{r}}\dot{x}$. Assume $t=\frac{an.\overline{n+r}}{m.\overline{m+r}}x^{\frac{m+r}{r}\frac{n+2r}{r}}+u$, then $\dot{u}=$

$\frac{an..\overline{n+2r}}{rm.\overline{m+r}}\dot{x}x^{\frac{m+r}{r}\frac{n+2r}{r}}$. Assume $u=\frac{an..\overline{n+2r}}{m...m+2r}x^{\frac{m+2r}{r}\frac{n+2r}{r}}+w$,

hence $\dot{w}=\frac{an..\overline{n+3r}}{rm...m+2r}\dot{x}x^{\frac{m+2r}{r}\frac{n+4r}{r}}$. Assume $w=\frac{an...\overline{n+3r}}{m....m+3r}$

$x^{\frac{m+3r}{r}\frac{n+4r}{r}}$ &c. Hence Fl. $\frac{m-r}{r}a\dot{x}x^{\frac{m-r-2r}{r}}=ax^{\frac{m-r-r}{r}}+\frac{an}{m}x^{\frac{m-r-r}{r}}$

$+\frac{an.\overline{n+r}}{m.\overline{m+r}}x^{\frac{m-r-r}{r}}+\frac{an...\overline{n+2r}}{m....m+2r}x^{\frac{m-r-r}{r}}+\frac{an....\overline{n+3r}}{m....m+3r}x^{\frac{m-r-r}{r}}+$

&c. $=\frac{m-r}{m-n-r}ax^{\frac{m-r-r}{r}}$.

We will give only one inſtance of the elegancy of this rule, in ſolving the 70th queſtion in the *Gentleman's Diary* for 1747.

The QUESTION.

Pl. 8. Fig. 156. "GIVEN $y=rxzz=1.5xz^2$, where z is an arc Am of the ſemicircle AmB, $y=$ any variable ordinate PM of the curve AMN, "and $x=$ AP its abſciſſa; to find the area of the curve ſpace "APM (when AB=20, and AP=6) by a general *Theorem*?"

SOLUTION.

To find the Fluent of $y\dot{x}$ or $rz^2\dot{x}x$, let $a=$ diameter, then $\dot{z}=\frac{a\dot{x}}{2\sqrt{ax-xx}}=\frac{ax^{-\frac{1}{2}}\dot{x}}{2\sqrt{a-x}}$, then by our method, already preſcribed for finding the Fluent, aſſume Fl. $y\dot{x}=\frac{rz^2x^2}{2}-s$, whence $\dot{s}=rx^2z\dot{z}=\frac{rax^{\frac{1}{2}}z\dot{x}}{2\sqrt{a-x}}$; let $A=$Fl. $\frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a-x}}$, and aſſume $s=\frac{razA}{2}-t$, whence $\dot{t}=\frac{raA\dot{z}}{2}$; but (by Form 10 and 11 of *Emerſon*'s Fluxions) $A=\frac{3ax}{4}-\frac{3a+2x}{4}\times\sqrt{ax-xx}$, whence $\dot{t}=\frac{raA\dot{z}}{2}=\frac{3a^2rz\dot{z}}{8}-\frac{ra\dot{z}}{2}\times\frac{3a+2x}{4}\sqrt{ax-xx}=\frac{3a^2rz\dot{z}}{8}-\frac{ra^2\dot{x}}{4\sqrt{ax-xx}}\times\frac{3a+2x}{4}\sqrt{ax-xx}=\frac{3a^2rz\dot{z}}{8}-$

$3ra^2\dot{x}$

$\frac{3ra^3\dot{x}}{16} - \frac{2ra^2x\dot{x}}{16}$, and $t = \frac{3a^2rx\dot{z}}{16} - \frac{3ra^3\dot{x}}{16} - \frac{ra^2x\dot{x}}{16}$, whence the area $= \frac{rz^2x^2}{2} - \frac{razA}{2} + t$.

Or the area of this curve may be found from page 45 of *Emerson*'s Fluxions, by putting $\dot{z} = \frac{a}{2s}$, $ax - xx = ss$, $\dot{s} = \frac{a - 2x}{2s}$, $\dot{x} = 1$; now according to the said rule assume $Az^2x^2 + Bz^2 + Cx + D \times zs + Fx^2 + Gx =$ Fluent of $z^2x\dot{x}$; and we get the same solution as before; but room not permitting, the rest of the operation I leave to the reader's exercise.

QUESTION CXVIII. *answered.*

LET g = centripetal force in P; r = the resistance; k = density of the medium, v = the velocity of the body, y = distance PC, the radius of curvature PO = R, s = arc VP, p = perpendicular CT drawn on the tangent PT, and a, b, n, m, given quantities; then $g = \frac{a}{y^n}$, and $r = \frac{kv^m}{b}$; but $v\dot{v}$ is always $= \frac{R\dot{p}g}{q} = \frac{aR\dot{p}}{y^{n+1}} = \frac{ap\dot{y}}{y^n p}$; therefore by either of these equations, the velocity will be given. Moreover $r = \frac{-v\dot{v}}{\dot{s}} = \frac{-v\dot{v}}{y^n\dot{s}} = \frac{a\dot{y}}{y^n\dot{s}}$, or likewise $= \frac{-v\dot{v}\sqrt{yy-pp} - gy\sqrt{yy-pp}}{yy} = \frac{-v\dot{v}\sqrt{yy-pp}}{yy} - \frac{a\sqrt{yy-pp}}{y^{n+1}}$. Wherefore if, in either of these equations, instead of $v\dot{v}$ be wrote its value, which is found by taking the Fluxion of this equation $v^2 = \frac{aR\dot{p}}{y^{n+1}} = \frac{ap\dot{y}}{y^n p}$, we shall get the resistance r, or $\frac{kv^m}{b}$, and its value divided by $\frac{v^m}{b}$, which is given, because (v being now found) k the density of the medium will be given.

Pl. 8. Fig. 157.

Suppose for example it be a logarithmic spiral; because of the given angle TPC, the ratio PC to CT, or y to p, is given; therefore let $c : a :: y : p$. Th. $p = \frac{a\dot{y}}{c}$, and $\dot{p} = \frac{a\dot{y}}{c}$, and $v\dot{v} = \frac{ap\dot{y}}{y^n p} = \frac{p\dot{c}}{y^n} = \frac{a\dot{y}}{y^{n-1}}$; hence we have $\sqrt{yy-pp} = \frac{y\sqrt{cc-aa}}{c}$, and $v\dot{v} = \frac{1 - n \times a\dot{y}}{2y^n}$; whence $r = \frac{y - n \times a\sqrt{cc-aa}}{2cy^n}$, for the descent, and

$r = \frac{\overline{n-3} \times a\sqrt{cc-aa}}{2cy^n}$, for the aſcent; therefore the reſiſtance is reciprocally as y^n. But when $r = \frac{kv^m}{b} = \frac{ka^{\frac{m}{2}}}{by^{\frac{mn-m}{2}}}$, k the denſity will be as $ry^{\frac{mn-m}{2}}$, or as $\frac{y^{\frac{mn-m}{2}}}{y^n} = y^{\frac{mn-m-2n}{2}}$, and hence if $m=2$, k the denſity will be as y^{-1}, or as $\frac{1}{y}$.

Cor. I. By the above equations, from the given velocity of the body, there are found the centripetal force, the reſiſtance, and the medium's denſity. For $g = \frac{v\dot{v}y}{Rp} = \frac{v\dot{v}p}{p\dot{y}}$, whence we have g the centripetal force, and the centripetal force and celerity being given, r the reſiſtance, and k the denſity of the medium are found, as above.

Pl. 8. Fig. 158. Suppoſe it be a hyperbolic ſpiral, whoſe property is, that if thro' the center C there be erected, to the radius CP, the perpendicular CX, meeting the tangent PX, drawn thro' P in X, ſuppoſe the ſubtangent CX conſtant. Let the velocity be as the tangent PX, and the reſiſtance r as the denſity of the medium and ſq. of the velocity conjunctly, that is, $\frac{kv^2}{b} = r$, put CX $= c$, then PX $= \sqrt{yy+cc}$, and $v = \frac{e\sqrt{yy+cc}}{c}$, e being a given quantity. By reaſon of the ſimilar triangles CPT, XPC, PX $(\sqrt{yy+cc})$: CX (c) :: PC (y) : CT (p), and therefore $p = \frac{cy}{\sqrt{yy+cc}}$, $\dot{p} = \frac{c^3\dot{y}}{\overline{yy+cc}^{\frac{3}{2}}}$, and $\sqrt{yy-pp} = \frac{yy}{\sqrt{yy+cc}}$, wherefore $g = \frac{v\dot{v}p}{p\dot{y}} = \frac{e^2}{y}$, that is, the centripetal force as the diſtance PC reciprocally. But becauſe $vv = \frac{ee}{cc} \times \sqrt{yy+cc}$, $v\dot{v}$ will be $= \frac{ee y\dot{y}}{cc}$, and therefore $r = \frac{v\dot{v}\sqrt{yy-pp}}{yy} + \frac{g\sqrt{yy-pp}}{y} = \frac{eeyy}{cc\sqrt{yy+cc}} + \frac{e^2}{\sqrt{yy+cc}} = \frac{e^2 \times \overline{yy+cc}}{cc\sqrt{yy+cc}} = \frac{e^2}{c^2}\sqrt{yy+cc}$, therefore the reſiſtance as the tangent PX, or as the velocity. Therefore when $r = kv^2 = \frac{ke^2}{cc} \times \sqrt{yy+cc} = \frac{e^2}{c^2}$

$\sqrt{yy+cc}$

$\sqrt{yy+cc}$, the denſity of the medium will be $k=\frac{1}{\sqrt{yy+cc}}$, or reciprocally as the tangent P X, or reciprocally as the velocity.

Cor. 2. Given the denſity of the medium, and granting the quadratures of figures, there will be given the centripetal force, and the body's velocity. For $vv\dot{v}+\frac{vv\dot{p}}{p}=-r\dot{s}=\frac{kv^n\dot{s}}{b}$, and by dividing by v^n, and multiplying by p^{2-n}, it becomes $p^{2-n}v^{1-n}\dot{v}+v^{2-n}p^{1-n}\dot{p}=\frac{-kp^{2-n}\dot{s}}{b}$, and taking their Fluents, we have $\frac{1}{2-n}\times p^{2-n}\times v^{2-n}=\frac{-\text{Fl. of } kp^{2-n}\dot{s}}{b}$, therefore $v^{2-n}=(n-2)\frac{\text{Fl. } kp^{2-n}\dot{s}}{b\times p^{2-n}}$. Wherefore if k, the denſity of the medium, be as any power of the diſtance PC, from C the center, the Fluent of $kp^{2-n}\dot{s}$ will be found, either algebraically, or by the quadrature of figures, and $\mp\frac{y\dot{y}}{\sqrt{yy-pp}}$ may be wrote for $\dot{s}$; but v being found, g is had by this equation $g=\frac{vv\dot{p}}{p\dot{y}}=\frac{vv\dot{y}}{Rp}$.

Cor. 3. If in the above *Corollary*, $n=2$, that is, the reſiſtance as the denſity and the ſquare of the velocity conjunctly, $2-n$ will be $=0$, and the equation $p^{2-n}v^{1-n}\dot{v}+v^{2-n}p^{1-n}\dot{p}=\frac{kp^{2-n}\dot{s}}{b}$, will be changed into this $\frac{\dot{v}}{v}+\frac{\dot{p}}{p}=-\frac{k\dot{s}}{b}$, whence taking their Fluents, we have L. v + L. p = −Fl. $\frac{k\dot{s}}{b}$, and L. v = −Fl. $\frac{k\dot{s}}{b}$ − L. p; from which equation v is found, and g as above.

Cor. 4. If (in *Cor.* 3.) k the denſity of the medium be uniform, v the velocity of the body in a given place $=c$, and p the perpendicular in the ſame place $=q$, L. v will be $=-\frac{ks}{b}$ − L. p + Q, and becauſe in V, $s=0$, $v=c$, $p=q$; then will Q = L. c + L. q = L. cq. And hence L. v = L. $\frac{cq}{p}-\frac{ks}{p}$. Put L. $h=1$, that L. v may be = L. $\frac{cq}{p}-\frac{ks}{b}\times$ L. h = L. $\frac{cq}{ph^{\frac{ks}{b}}}$, whence is deduced $v=\frac{cq}{ph^{\frac{ks}{b}}}$, $vv=$

$vv=\frac{c^2q^2}{p^2b^{\frac{2ks}{b}}}$, hence $g=\frac{vvy}{Rp}=\frac{c^2q^2y}{Rp^3b^{\frac{2ks}{b}}}$, or $g=\frac{vv\dot{p}}{p\dot{y}}=\frac{c^2q^2\dot{p}}{p^3b^{\frac{2ks}{b}}\dot{y}}$.

Cor. 5. The time may be found by this equation $t=\frac{s}{v}$, or $t=$ Fluent $\frac{s}{v}$.

Cor. 6. Given the centripetal force, with the resistance, and the density of the medium, the equation to the trajectory Z P V may be found which a projectile body describes about the center of forces C. Suppose, for *example*, the medium uniform, the resistance as the square of the velocity, and the centripetal force $=\frac{a}{y^n}$, then $\frac{a}{y^n}$ will be $=\frac{c^2q^2\dot{p}}{p^2b^{\frac{2ks}{b}}\dot{y}}$, therefore $b^{\frac{2ks}{b}}=\frac{c^2q^2y^n\dot{p}}{ap^2\dot{y}}$, and L. $b^{\frac{2ks}{b}}=\frac{2ks}{b}=$ L. $\frac{c^2q^2y^n\dot{p}}{ap^2\dot{y}}$; taking their Fluxions, making $\dot{y}$ constant, it will be $\frac{2ks}{b}(=\pm\frac{2ky\dot{y}}{b\sqrt{yy-pp}})=\frac{n\dot{y}}{y}+\frac{\dot{p}}{p}-\frac{3\dot{p}}{p}$. Hence $\frac{2ks}{b}=$ L. $y+$ L. $\dot{p}-3$ L. $p-\frac{\text{L. }\dot{y}}{Q}$, when $\frac{\dot{y}}{Q}$ is a constant quantity, therefore make $\frac{2ks}{b}=$ L. $y+$ L. $\frac{Q\dot{p}}{\dot{y}}-$ L. $p^3=$ L. $\frac{Qy^n\dot{p}}{p^3\dot{y}}$, and hence $b^{\frac{2ks}{b}}=\frac{Qy^n\dot{p}}{p^3\dot{y}}$, the equation to the trajectory.

Scholium. If the curve V P Z be a conic section, whose focus is C, the greater axis c, the lesser semi-axis e, then it will be for the ellipsis $pp=\frac{eey}{c-y}$, for the hyperbola $pp=\frac{eey}{c+y}$, and for the parabola, if the *latus rectum* be called $4e$, it will be $pp=ey$; whence it is easy to apply the solutions of this prob. to those of the con. sections. Suppose V P Z be a parabola, the centrip. force $g=\frac{a}{y^n}$, the resistance $r=\frac{kv^2}{b}$, required the velocity of the body, with the density of the medium, in any place P.

For as much as $pp=ey$, in Fluxions it will be $2p\dot{p}=e\dot{y}$, therefore $\dot{p}=\frac{e\dot{y}}{2p}$, $\frac{\dot{p}}{p}=\frac{2\dot{p}p}{ey}=\frac{2\dot{y}}{y}$, whence $vv=\frac{ap\dot{y}^n}{y^n\dot{p}}=\frac{2a}{y^{n-1}}$, hence we have

$v\dot{v}=$

$v\dot{v} = \frac{\overline{1-n} \times a\dot{y}}{y^n}$, and therefore $r = \frac{v\dot{v}\sqrt{yy-pp}}{\dot{y}\dot{y}} + \frac{a\sqrt{yy-pp}}{y^{n+1}} = \frac{2a-na\sqrt{yy-ey}}{y^{n+1}}$, for the deſcent; and $r = \frac{na-2a\sqrt{yy-ey}}{y^{n+1}}$, for the aſcent; therefore the reſiſtance is always as $\frac{PT}{PC^{n+1}}$. Moreover $r = \frac{kv^2}{b} = \frac{2ak}{by^{n+1}} = \frac{\overline{\quad}\sqrt{yy-ey}}{2a-na\,y^{n+1}}$, or $\frac{\overline{\quad}\sqrt{yy-ey}}{na-2a\,y^{n+1}}$; therefore k will be as $\frac{\sqrt{yy-ey}}{y^2}$, or as $\frac{PT}{PC^2}$; and in a like way, the denſity of the medium is found in the ellipſis and hyperbola, as $\frac{PT}{PC^2}$. But in the circle $PT = 0$, therefore the denſity of the medium and reſiſtance is nothing; the reſiſtance alſo vaniſhes if $n = 2$, that is, if the centripetal be as the ſquare of the diſtance reciprocally, where the conic ſections are deſcribed in a non-reſiſting medium. If n be leſs than 2, the conic ſections may be deſcribed by the deſcent; but if n be greater than 2, by the aſcent. Laſtly, when $n = 1$, that is, the centripetal force of the diſtance PC is reciprocally proportional, the velocity in the parabola is uniform, as in the log. ſpiral. H.

This queſtion was anſwered by *Philofluentimecanalgegeomaſtrolongo*, and ſhall be inſerted in our next number.

*** I can but take notice what a great deal of pains M. *John Bernoulli* has taken to reſolve this problem, and glories exceedingly that he has given a ſolution ſhorter and more general than Sir *Iſaac Newton* in his *Principia*. But after all his triumphs, he has not been able to apply it to any further uſe, than Sir *Iſaac* has done, and at the ſame time he ſeems to be ignorant that the method of *Analyſis* is, in its own nature, ſhorter and more conciſe than the *Synthetic* method, in which the *Principia* is written. This boaſting mathematical *Hector* may as well condemn *Euclid*'s elements, and all other *Synthetic* writings, becauſe any of their propoſitions may be demonſtrated vaſtly ſhorter by common algebra. Sir *Iſaac* has left him, and all ſuch like others, to rake among the rubbiſh and remains, which he ſet aſide as uſeleſs in erecting his noble fabric the *Principia*. What was uſeleſs to him might ſerve the other well enough, in building up his great *Babel* of confuſion.

New QUESTIONS *to be anſwered in our next.*

QUESTION CXIX. *By Mr* JOHN TODD.

'TIS required to multiply 5697487 by 96488, ſo that the product may be had by the addition of three lines only?

QUESTION CXX. *By Mr* THOMAS GARVISON.

FOUR men A, B, C, and D, diſcourſing of their money, ſays A, to B, C, and D, give me half of your money, and I ſhall have eleven ſhillings; ſays B, to A, C, D, give me one third of your money, and I ſhall have eleven ſhillings; nay ſays C to A B and D, give me a fourth of your money, and I ſhall have 11 ſhillings; hold, ſays D to A, B and C, give me a fifth of your money, and I ſhall have 11 ſhillings. Quere what had each?

QUESTION CXXI. *By Mr* ROBERT THORNTON.

GIVEN two ſides of an obtuſe-angled triangle, which are 20, and 40 perches, required the third ſide, that ſhall contain juſt an acre of land.

QUESTION CXXII. *By Mr* F. HOLLIDAY.

A Gent. having a bowling green, whoſe length was 276 feet, and breadth 206 feet, has a mind to raiſe the ſaid green one foot higher, by a ditch that he will make round it. Quere the width and depth of the ditch?

QUESTION CXXIII. *By Mr* NICH. FARRER.

A Gent. has a caſtle 94.34 feet high, near to which are two towers, 50 and 78 feet high, and 224 feet diſtant from each other, which towers contain an angle of 58° 26′ 17″ at the bottom of the caſtle; and being deſirous of having a fountain in a line drawn from the caſtle, perpendicular to that joyning the towers, ſo that it may be equally diſtant from the top of each tower and caſtle, requires to know its diſtance from the caſtle?

QUESTION CXXIV. *By Mr* JOHN TODD.

THERE are two wheels, the diameter of the greater=40 inches, and the diameter of the leſſer=8 inches, placed at 50 inches diſtance from each others center; 'tis required to find the length of a ſtring, that will reach round both wheels?

QUES-

QUESTION CXXV. *By Mr* HOLLIDAY.

LET ABDE represent a cylinder, cut by a plane DC, dividing the base AB, *viz.* making AC=CB; then will $\frac{\overline{AB}^2 \times BD}{6}$ be the solidity of the slice CBD, and the area of the plane ABDE= its convex superficies. *Quere* the demonstration. Pl. 8. Fig. 157.

QUESTION CXXVI. *By* PROBLEM TOM.

AT a given hour and minute, &c. by a good pendulum clock, in the latitude 52° 40′ North, *Sirius* was observed in the plane of a certain wall, and 5^h 56′ 36″ after this (by the same clock) *Arcturus* was seen to come into the same plane. From these observations, required the plane's declination?

QUESTION. CXXVII. *By Mr* SAMUEL DUNN.

REQUIRED the area of the greatest parallelogram inscribed in the segment of a circle, whose chord is 40, and versed sine is 38?

QUESTION CXXVIII. *By* R. R. H.

SUppose the velocity of the tide to be as the time of its flowing, for the first 4 hours; and that it sets a body over the space of 16 miles in that time. Now admit a ship sets out just as the tide begins to flow, and sails perpendicular to the direction thereof uniformly at the rate of six miles an hour by the log; one hour after a second ship sets out from the same place, keeps continually bearing on the first, and comes up with her in 3 hours chace. Required the length of both ships curves, with the rate of the second ship's sailing by the log, supposing it also uniform, and that the tide begins to flow every where at the same time, so far as it affects the ships.

QUESTION CXXIX. *By* GAMSTON RETFORD.

SUPPOSE the centripetal force was changed from the square to the cube of the distance inversly, let the earth's radius be 3985 miles, and $16\frac{1}{12}$ feet the descent of heavy bodies on its surface in one second of time; if now a body, whose distance from the earth's center (which I suppose at rest) is equal to four times it radius, be projected in the direction of a right line making an angle of 60° with a line drawn from the center of the earth with a velocity sufficient to cause it to move uniformly thro' 80 miles in one minute; 'tis required to find in what time the body will perform one gyration, and what will then be its distance from the center? The

The Calculation of the Sun's Eclipse, *July* 14, 1748, for *London*, and *Newcastle*, in Lat. 54° 58′ N, Long. 1° 30′ W *per Newton*'s Theory. From Brent's Tables.

	☽ Longitude.	☽ Apoge.	☽ Node.	☉ Longitude.	☉ Anom.	
	S o ′ ″	S o ′ ″	S o ′ ″	S o ′ ″	S o ′ ″	
1748 *July* 13.	3 20 43 57	3 22 29 32	08 02 03	4 02 32 40	0 24 05 27	
Hours 23	12 37 39	6 24	3 03	56 41	56 41	
Minutes 23	12 38	(	03	57	57	
Seconds 23	13		3 (	01	01	
	4 03 34 27	3 22 36 02	10 07 58 57	4 03 30 19	25 03 06	
Equation 1st	+ 4 55	— 8 19	+ 3 51	— 48 20	4 03 30 19	
1st equated place	4 03 39 22	3 22 27 43	10 08 02 53	4 02 41 59	3 08 27 13	
Equation 2d	— 1 14	+ 3 33 17		☉'s Place.	☉ Apoge	
2d Equated place	4 03 38 08	3 26 01 00				
Equation 3d	+ 09					
☽ 3d equated	4 03 38 17					
Equation 4th	— 37					
☽ 4th equated			10 14 16			= ☽ Apo. à ☉
Elliptick equation	4 03 37 40		5 24 39 06			= Node à ☉
	— 55 41		14 56 48			= ☉à☽ 3d eq. + ☉ap. à ☽ ap.
☽'s Orbit place	4 02 41 59		7 36 40			= ☽ Mean anomaly
☉ Place	4 02 41 59					= ☉ à ☽ 5th equated, hence the variation and 7th equation vanish
Hence the above is the true time of the orbit ♂						
Reduction	+ 1 19		— 16 15			= 2d equat. ☽ node
☽ Ecliptick place	4 02 43 18		10 07 46 38			= true place of the node
☽'s True latitude	N.D. 28 01		5 24 55 21			= node à ☽ in orbit
Hor. parallax	53 58		17 39			= inc. of the limit.
Semidiameter	14 47					
Eccentricity	66 163					

	Newcastle. D.	H.	M.	S.	London. D.	H.	M.	S.
Equal time true ♂ *July*					13	23	23	23
Time of reduction							2	54
Equal time true ecl. ♂						23	20	29
Equation of time						—	5	56
Ap. time true ecl. ♂						23	14	33
		S	o	′ ″		S	o	′ ″
Sun's place then			23	08 33		4	♌2	41 50
—— Right ascension							124	59 17
Right asc. med. celi.			112	07 32			113	37 32
Med. celi. in eclip.		♋	20	27 03		♋	21	51 36
Meridian angle			81	22 06			80	48 39
Declin. culm. point		N.	21	55 24		N.	21	42 18
Alt. equator			35	02			38	28
Alt. med. celi.			56	57 24			60	10 18
Alt. nonagesima			57	22 41			60	35 33
Dist. med. celi. & nona.			5	34 33			5	13 46
Place nonagesima		♋	14	52 30		♋	16	37 50
Dist. ☉ à nonag.			17	49 22			16	04 02
Hor. parallax. ☽ à ☉				13 51				53 47
Par. long. ☽ à ☉				28 19				12 58
☽ true lat.	N.D.			28 59	N.D.		28	10
Par. lat. ☽ à ☉				49			26	24
☽ Vif. lat.					N.		1	46 ♋
☽ Ecliptick place					4		02	41 32

And since this eclipse happens in the oriental quadrant, the vif. ♂ is before the true; therefore compute to one hour before the true ♂, as follows:

At

	London.				Newcastle.			
	D.	H.	M.	S.	D.	H.	M.	S.
At 1 hour before true ♂	13	22	14	33		22	08	33
	S.	o	′	″	S.	o	′	″
☉ place then	4	02	39	28				
— right ascension		124	56	49				
☽ ecliptic place	4	02	12	25				
Hourly mot. ☽ à ☉			26	45				
Hor. parallax ☽ à ☉			53	47				
Right asc. med. celi		98	35	04		97	05	04
Med. celi in ecliptic	♋	7	52	58	♋	6	30	10
Meridian angle		86	35	25		87	10	59
Declin. culm. point	N.	23	14	54	N.	23	19	23
Alt. equator		38	28			35	02	
Alt. med. celi		61	42	54		58	21	23
Alt. nonagesima		61	46	13		58	23	57
Dist. med. celi & nona.		1	49	59		1	44	05
Place nonagesima	♋	6	02	59	♋	4	46	05
Dist. ☉ à nonag.		26	36	29		27	53	23
Par. long. ☽ à ☉		—	24	19			21	25
☽ true lat.	N.	D.	30	52			30	52
Par. lat. ☽ à ☉			25	26			28	11
☽ vis. lat.	N.	D.	5	26	N.	D.	2	41
Vis. hor. mat. ☽ à ☉			18	36			19	11
Interv. bet. true and vis. ♂			42	03			48	19
	D.	H.	M.	S.	D.	H.	M.	S.
Ap. time eclip. ♂ *July*	13	23	14	33	13	23	08	33
Interval sub.			42	03			43	19
Ap. time vis. ♂	13	22	32	30	13	22	25	14
	S.	o	′	″	S.	o	′	″
Sun's place then	4	02	40	12	4	02	40	08
—— right ascension		124	57	34		124	57	29
Moon's ecl. place	4	02	21	13	4	02	20	36
Right asc. med. celi		103	05	04		101	15	59
Place med. celi	♋	12	01	52	♋	10	21	15
Meridian angle		84	49	28		85	32	06
Decl culm. point	N.	22	56	14	N.	23	04	42
Alt. equator		38	28			35	02	
Alt. med. celi		61	24	14		58	06	42
Alt. nonagesima		61	31	52		58	13	12
Dist. med. celi à non.		2	48	54		2	46	22
Place nonagesima	♋	9	12	58	♋	7	34	53
Dist. ☉ à nonag.		23	27	14		25	05	15
Par. long. ☽ à ☉			18	49			19	23
True lat. ☽	N.	D.	30	00	N.	D.	30	05
Par. lat. ☽ à ☉			25	38			28	19
Vis. lat. ☽	N.	D.	4	22	N.	D.	1	46
Semid. of the ☉			15	52				
Semid. of the ☽			14	47				
Semid. of the penumbra			30	39			30	39
☽ vis. lat. at vis. ♂			4	22			1	46
Parts deficient			26	17			28	53
Digits eclips'd		9	56	21		10	55	20
Scruples of incid.			30	20			30	36

	London.				Newcastle.			
	D.	H.	M.	S.	D.	H.	M.	S.
To 1 hour bef. vis. ♂	13	21				21	25	14
Sun's place then	4	02	37	48	4	02	37	44
—— right ascension		124	55	04		124	55	00
Right asc. med. celi		88	02	34		86	13	30
Place of med. celi	♊	28	12	18	♊	26	32	13
Meridian angle		89	13	13		88	29	47
Declin. culm. point	N.	23	28	16	N.	23	26	16
Alt. equator		38	28			35	02	
Alt. med. celi		61	56	16		58	28	16
Alt. nonagesima		61	55	55		58	28	28
Dist. med. celi à nonag.		00	25	10		00	56	43
Place nonagesima	♊	28	37	28	♊	27	28	56
Dist. ☉ à nonagesima		34	00	20		35	08	48
Moon's ecl. place	4	01	51	49	4	01	51	09
—— true lat.	N.	D.	32	46				
True hor. mot. ☽ à ☉			27	00			27	03
Par. long. ☽ à ☉			26	32			26	24
Vis. hor. mot. ☽ à ☉			19	12			19	54
	D.	H.	M.	S.	D.	H.	M.	S.
At 1 hour past vis. ♂	13	23	32	30		23	25	14
	S.	o	′	″	S.	o	′	″
Sun's place then	4	02	42	35	4	02	42	31
—— right ascension		125	00	01		124	59	56
Right asc. med. celi		118	07	31		116	18	26
Place med. celi	♋	26	06	58	♋	24	23	29
Meridian angle		79	10	23		79	49	42
Declin. culm. point	N.	20	57	54	N.	21	16	45
Alt. equator		38	28			35	02	
Alt. med. celi		59	25	54		56	18	45
Alt. nonagesima		60	01	56		56	54	39
Dist. med. celi à nona.		6	20	15		6	33	52
Place nonagesima	♋	19	46	43	♋	17	49	37
Dist. ☉ à nonagesima		12	55	52		14	52	54
Moon's ecl. place	4	02	50	36	4	02	50	01
True hor. mot. ☽ à ☉			27	00			27	02
Par. long. ☽ à ☉			10	25			11	34
Vis. hor. mot. ☽ à ☉			18	36			19	13
Time of incidence			1	34	23		1	32	19
Time of repletion			1	37	51		1	35	35
Ap. time of begin.		13	20	59	18	13	20	53	17
middle			22	33	41		22	25	36
end		14	00	11	32	14	00	01	11
Vis. lat. ☽ at Beg.	N.	D.		8.	43	N.	D.	5	54
Vis. lat. ☽ at End	S.	A.		2	23	S.	A.	1	36

London.	H.	M.	S.		Newcastle.	H.	M.	S.	
1748, *July* 14.					1748, *July* 14.				
Beginning	8	59	18	A.M.	Beginning	8	53	17	A.M.
Middle	10	33	41	A.M.	Middle	10	25	36	A.M.
End	00	11	32	P.M.	End	00	01	11	P.M.
Duration	3	12	14		Duration	3	07	54	
Digits	9	56	21		Digits	10	55	20	

The GENERAL ECLIPSE.

	Time. H.	M.	S.	Latitude. o	′		Longit. o	′		
Penumb. touches the disk, and the eclipse first begins	8	22	10	38	36	N.	48	02	W.	
Central eclipse begins	9	37	02	49	12		69	06		
Central eclipsed in the meridian	10	56	11	53	17		16	11	E.	
Central eclipse in the nonages.	11	11	18	50	22		22	15		
At the nearest approach to the center of the disk	11	18	02	49	01		25	31		From *London*
Center of the penum. passes off the disk, and cent. ecl. ends	12	59	14	12	03		78	02		
The penumbra leaves the disk, and the eclipse ends	2	14	08	00	00		56	13		
Central duration	3	22	12							
Total duration	5	51	58							

This eclipse first touches the earth in the *Atlantic Ocean*, between the *Azores* and island *Bermudas*; the annular center enters *North America*, and, passing over *Newfoundland*, enters *Europe* at the isle of *Skie*, leaving *Scotland* at *Aberdeen*, bending its course over the north sea, enters *Germany* between *Hamburgh* and *Copenhagen*, running thro' part of *Poland*, *Turkey*, the *Levant*, and *Arabia*, to the *Indian* sea, and leaves the earth on the south part of the Mogul's empire; and soon after the eclipse ends.

A Calculation of the SOLAR ECLIPSE, *July* 14, 1748; by Mr MACHIN's Numbers in BRENT's Tables.

	D.	H.	′	″
Equal time of true ☌ *July*	13	23	19	46
	S.	o	′	″
Mean anomaly of ☉	o	25	2	57
☽	o	7	36	55
Place of ☉	♋	2	41	50
☽ in her orbit	♋	2	41	49
☊	♒	7	46	25

	S.	o	′	″
Argument of latitude	5	24	55	24
Reduction add			1	18
Time of reduction ſub.			2	52
Equal time of ecliptic ☌	13	23	16	54
Equation of time ſub.			5	55
Apparent time of ecliptic ☌ at *London*	13	23	10	59

	Ecliptic ☌				30′ bef. Ecl. ☌				Viſible ☌.				1 ho. bef. viſ. ☌				1 h. aft. viſ. ☌				Begin. of eclip.				End of eclipſe.			
	S.	o	′	″	S.	o	′	″	S.	o	′	″	S.	o	′	″	S	o	′	″	S.	o	′	″	S.	o	′	″
Place of ☉	♋	2	41	45	♋	2	40	32	♋	2	40	1	♋	2	37	38	♋	2	42	24	♋	2	36	12	♌	2	43	54
Right aſc. of ☉		124	59	7		124	57	54		124	57	22		124	54	54		124	59	50		124	53	25		125	1	23
Time from noon		347	44	45		340	14	45		337	1	15		322	1	15		352	1	15		313	59			1	29	30
Right aſc. of med. celi		112	43	52		105	12	39		101	58	37		86	56	9		117	1	5		78	52	25		126	30	53
Place of med. celi	♑	21	1	9	♑	14	0	9	♑	11	0	37	♊	27	11	22	♑	25	3	54	♊	19	46	30	♌	4	10	41
Meridian angle		81	9			84	0			85	15			88	47			79	34			85	35			76	17	
Decl. of med. celi N.		21	50			22	45			23	2			23	27			21	10			23	5			19	15	
Alt. of the equat. *Lond.*		38	28																									
Alt. of med. celi		60	18			61	43			61	30			61	55			59	38			61	33			57	43	
Alt. of nonag. deg.		60	41			61	23			61	36			61	55			60	11			61	38			58	44	
Diſt. 90° from med. celi		5	1			3	17			2	35			0	39			6	3			2	23			8	31	
Place of nonageſima	♑	16	0	9	♑	10	43	9	♑	8	25	37	♊	27	50	22	♑	19	0	54	♊	22	9	30	♑	25	39	41
Diſt. of ☾ from nonag. E.		16	41	34		21	57	23		24	14	24		34	47	16		13	41	30		40	26	42		7	4	13
Hor. paral. ☽ 53′57″ ☉ 12″ diff.			53	45																								
Par. of ☽ from ☉ in long.			13	28			17	38			19	25			27	0			21	2								
——— in lat.			26	19			25	45			25	34			25	19			26	43			25	32			27	54
True lat. ☽ N. D.			28	1	Int. of true ☌						29	57					Time of incid.		1	33	14		33	5			26	49
Viſible lat. ☽ N. D.			1	42	Viſ. ☌ S.	42	54				4	23					Time of replet.		1	36	43		7	33	S. A.	1	5	
Viſ. hour. mot. of ☽ fr. ☉							18	50	Digits eclips'd		0	57			19	30			18	48	Interv. of viſ. ☌ and great obſ.							
									Scrup. of incid.		30	18					Total duration.		3	9	57	Add	1	10						

Hence from the above calculation are deduced for the meridian of *London*,

	H.	M.
Beginning *July* 14	8	56
Viſible ☌	10	28
Middle of the eclipſe	10	29
End	12	6

A. M.

	H.	M.
Duration	3	10
Dipits eclips'd	9	57

A TABLE exhibiting the Latitudes and Longitudes of those Places (to every 4 Minutes of Time) where the central Annulus is conspicuous, in the SOLAR ECLIPSE, the 14th of *July*, 1748.

By Mr T. COWPER.

Time at *London*.	N. Lat. ° ′	Longit. ° ′		Time at *London*.	N. Lat. ° ′	Longit. ° ′		Time at *London*.	N. Lat. ° ′	Longit. ° ′	
Cen. Ecl. begins.				H. M.				H. M.			
H. M.	46 30	76 5	W.	10 45	54 56	10 3	E.	11 53	39 42	38 3	E
9 37	50 0	66 26		10 49	54 15	12 19		11 57	38 36	39 20	
9 41	53 23	55 20		10 53	53 32	14 32		12 1	37 28	40 37	
9 45	55 11	47 48		Mer. ⊙ cen. ecl.	53 2	15 58		12 5	36 19	41 55	
9 49	56 23	41 32		10 57	52 48	16 35		12 9	35 8	43 15	
9 53	57 13	36 0		11 1	52 1	18 32		12 13	33 55	44 36	
9 57	57 47	30 57		11 5	51 13	20 22		12 17	32 40	45 59	
10 1	58 10	26 19		11 9	50 22	22 11		12 21	31 22	47 26	
10 5	58 21	21 52		11 13	49 31	23 53		12 25	30 2	48 58	
10 9	58 23	17 44		Middle.	48 38	25 30		12 29	28 39	50 35	
10 13	58 21	13 54		11 21	47 43	27 4		12 33	27 14	52 23	
10 17	58 11	10 16		11 25	46 47	28 34		12 37	25 39	54 17	
10 21	57 57	6 50		11 29	45 50	30 2		12 41	24 0	56 26	
10 25	57 34	3 35		11 33	44 52	31 26		12 45	22 14	58 56	
10 29	57 10	0 32		11 37	43 52	32 48		12 49	20 14	61 57	
10 33	56 41	2 21	E.	11 41	42 52	34 9		12 53	17 52	65 57	
10 37	56 9	5 4		11 45	41 50	35 28		12 57	14 28	72 42	
10 41	55 34	7 38		11 49	40 47	36 46		Cent. Ecl. ends.	11 43	79 36	

N. B. If there be any typographical errors, through the author's great distance from the press, they shall be corrected in our next number.

The End of NUMBER VIII.

Miſcellanea Curioſa, &c.

NUMBER IX.

QUST. CXVIII. *anſwered by* Philofluentimecanalgegeomaſtrolongo.

LET SP=y, radius of curvature CP=r, CD perpend. to SP, and PD=s, po perpendicular to SP, and pd perpendicular to the tangent PT; po=$\dot{x}$, Po=$\dot{y}$, pP=$\dot{z}$; and let v= velocity at P, R the reſiſtance, D the denſity, F the force, t the time of deſcribing ZP; let a heavy body, with a given force, deſcend thro' the ſpace a in the time I, and let F be repreſented by y^n, and R by Dv^m, m being the index of velocity. Pl. 8. Fig. 159.

By compoſition of forces $\frac{F\dot{x}}{\dot{z}}$= force towards C by gravity, and $\frac{F\dot{y}}{\dot{z}}$= the force in direction PT by the ſame; therefore $\frac{F\dot{y}}{\dot{z}}\pm R$= force in direction PT by gravity and reſiſtance both, according as the body aſcends or deſcends. In uniform motions, the ſpaces are as the times multiplied by the velocities; Th. $v\dot{t}=\dot{z}$, whence $\dot{t}=\frac{\dot{z}}{v}$. By ſimilar triangles $r\dot{x}=s\dot{z}$, and by the nature of the circle $\frac{\dot{z}^2}{2r}=\frac{v^2\dot{t}^2}{2r}=$dp. In accelerated motion, the velocities generated are as the forces multiplied by the times; Th. $2a\times\frac{f\dot{x}}{\dot{z}}\times\dot{t}$, or $2a\times\frac{f\dot{x}}{z}\times\frac{\dot{z}}{v}$, that is, $\frac{2aF\dot{x}}{v}$= velocity generated in direction PC in the time $\dot{t}$.

Alſo the forces are as the velocities generated, when the times are given; therefore $\frac{F\dot{x}}{\dot{z}}:\frac{2aF\dot{x}}{v}::\frac{\pm F\dot{y}}{\dot{z}}+R:-\dot{v}$, and $v\dot{v}=\mp 2aF\dot{y}-2aR\dot{z}$, and by reduction $R=\frac{2aF\dot{y}\pm v\dot{v}}{\mp 2a\dot{z}}=\frac{\pm 2aF\dot{y}+v\dot{v}}{-2a\dot{z}}$.

But in accelerated motion, the ſpaces deſcended are as the forces

multiplied by the ſquares of the times; therefore $1 : a :: \frac{F\dot{x}}{\dot{z}} \times \dot{t}\dot{t}$: dp, or $\frac{v^2\dot{t}^2}{2r}$; whence $v^2 = \frac{2aFr\dot{x}}{\dot{z}} = 2aFs$, and $v\dot{v} = a \times$ *Fluxion* Fs; therefore $R = \frac{F\dot{y} + \frac{1}{2}\,Flux.\,Fs}{-\dot{z}}$ for the aſcent. Alſo $R = Dv^n$, and $D = \frac{R}{\overline{2aFs}^{\frac{n}{2}}}$, or $D = \frac{F\dot{y} + \frac{1}{2}\,Flux.\,Fs}{-\dot{z} \times \overline{2aFs}^{\frac{n}{2}}}$; likewiſe $\dot{t} = \frac{\dot{z}}{\sqrt{2aFs}}$, and $t =$ *Fluent* of $\frac{\dot{z}}{\sqrt{2aFs}}$.

Therefore to find R, D or t, ſeek s by the nature of the curve, in terms of y, and put y^n for F, and find the *Fluxion* Fs, and expunge $\dot{z}$ by the nature of the curve, which done you'll have R, D and t.

Note, If R or D comes out negative, the aſcent is impoſſible, and the curve will then be deſcribed by deſcent. I ſhall add but two Examples.

EXAMPLE I.

Pl. 8. Fig. 160. LET ZP be a parabola, S the focus, r = *latus rectum*, ST perpendicular to the tangent PT; then $PT = \sqrt{yy - \frac{1}{4}ry}$, $\dot{z} = \frac{y\dot{y}}{\sqrt{yy - \frac{1}{4}ry}}$, $s = 2y$, and $\frac{1}{2}\dot{s} = \dot{y}$; whence $v = \sqrt{4ay^{n+1}}$, $Fs = 2y^{n+1}$, Fs in *Fluxions* $= \overline{2n+2}.y^n\dot{y}$; therefore $R = \frac{+\overline{n+2}.y^{n-1}}{-1}PT$, for the aſcent, and $D = \frac{\overline{n+2}.PT}{-\sqrt{4a^m \times y^{\frac{nm+m-2n+2}{2}}}}$. Let $m = 2$, then $D = \frac{n+2}{-4ay^2}PT$, whence it follows,

1. If $n > -2$, as -1, 0, 1, 2, 3, &c. the parabola is deſcribed by deſcent only; but if $n < -2$, as -3, -4, -5, &c. 'tis deſcribed only by aſcent.

2. If $n = -2$, the reſiſtance and denſity vaniſh, and the parabola may be deſcribed both by aſcent and deſcent, and $v = 2\sqrt{\frac{a}{y}}$, as is known, and $t = \frac{2y + r}{12a}\sqrt{4ay - ar}$.

3. In the vertex z, the reſiſtance and denſity are 0.

4. If

4. If $n=-1$, then the velocity $=2\sqrt{a}$, and is every where the ſame; alſo $R=\frac{PT}{yy}$, and $D=\frac{PT}{4ayy}$.

5. If $n=-3$, $v=\sqrt{\frac{4a}{yy}}=\frac{2}{y}\sqrt{a}$, $R=\frac{PT}{y^4}$, $D=\frac{PT}{4ayy}$, and $t=$ Fl. $\frac{y^{\frac{1}{2}}\dot{y}}{\sqrt{4ay-ar}}$.

EXAMPLE II.

LET ZP be the hyperbolic ſpiral, S, the center of it, the center of force; ſubtangent $SC=b$; then $s=\frac{bby+y^3}{bb}$, and $\dot{s}=\frac{bb\dot{y}+3yy\dot{y}}{bb}$. Pl. 8. Fig. 161.

Th. $v=\sqrt{2aFs}=\frac{CP}{CS}\sqrt{2ay^{n+1}}$, and $R=\frac{n+3.y^{n+1}}{-2bb}\sqrt{bb+yy}$, and $D=$

$$\frac{\frac{n+3}{2}b^{n-2}\sqrt{bb+yy}}{\overline{-2abb+2ayy}^{\frac{n}{2}}\times y^{\frac{nn+2n-2}{2}}}$$ for the aſcent.

Note, This curious ſolution was omitted in our laſt for want of room.

In Prop. 20. B. 3. *of* Newton's *Principia he gives a Theorem for finding and comparing together the weights of bodies. We hope an inveſtigation thereof may be acceptable to our readers.*

THEOREM.

THAT the increaſe of weight in paſſing from the equator to the poles, is nearly as the verſed ſine of double the latitude; or, which comes to the ſame thing, as the ſquare of the right ſine of the latitude. And the arcs of the degrees of latitude, in the meridian, increaſe nearly in the ſame proportion.

INVESTIGATION.

LET APBA be an ellipſis, which let reſemble the meridian of the earth; and ARBLA a circle deſcribed with the radius CA, nearly to which is the ellipſis APBA; and let the radius CA be the ſemidiameter of the terreſtrial equator. Then (by the nature of the ellipſis) RP : MG :: CR : ME; Therefore $MG=\frac{RP\times EM}{CR}$. Pl. 8. Fig. 163.

But, by reaſon of the ſimilar triangles DMG, EMC, when the ellipſis

ellipſis accedes nearly to the circle (for then DG may be taken for a right line touching the ellipſis in D, and that tangent is nearly perpendicular to the radius DC) it is MG : MD :: MC : ME; hence $MG=\frac{MD\times MC}{ME}$. Therefore $\frac{RP\times ME}{CR}=\frac{MD\times MC}{ME}$, whence $MD=\frac{RP\times\overline{ME}^2}{\overline{CR}^2}$. Now from M draw the perpendicular M*m* to the right line F*e*, *em* will be the verſed ſine of double the arc AM, that is, of the arc M*e*; or, becauſe AM exhibits the latitude, *em* will be the verſed ſine of double the latitude; but $em\times eF=MM$ (from the property of the circle) wherefore, becauſe of the given line *e*F, *em* is as eM^2, or likewiſe as $\overline{ME}^2$; therefore MD is as $\frac{RP\times em}{\overline{CR}^2}$; or, becauſe of the given lines $\frac{RP}{\overline{CR}^2}$, MD is as *em*, or as $\overline{ME}^2$. But becauſe the weights in the places A and D are as the diſtances of the places from the center reciprocally, the increment of the weight in D will be as $\frac{1}{CD}-\frac{1}{CA}$, that is, as CA—CD, or as CM—CD, therefore as MD. Wherefore *the increaſe of weight in paſſing from the equator to the poles, is nearly as the verſed ſine of double the latitude, or, which comes to the ſame thing, as the ſquare of the right ſine of the latitude.*

Moreover Sir *Iſaac Newton* further ſays, *that the arcs of the degrees of latitude in the meridian, increaſe nearly in the ſame proportion.*

DEMONSTRATION.

THE leaſt arc of a circle, touching any curve in a given point, may be uſed inſtead of an infiniteſimal arc of a curve in that point. But the whole degrees are as the leaſt ſimilar arcs, and thoſe arcs are as radii of circles touching the curve; whence the whole degrees will be as the ſame radii; wherefore in D a degree will be as the radius of a circle touching the ellipſis in the ſame place; and in A a degree is the ſame as the radius of a circle touching the ellipſis in the ſame point A.

Now drawing the perpendicular CN to the tangent DN, and taking D*r* for the radius of curvature in D, D*r* will be as DK^3; or, ſince $CP^2=CN\times DK$, becauſe CP^2 is given, DK will be as $\frac{1}{CN^3}$; therefore the radius of a circle which is as DK^3, will be as

$\frac{1}{CN}$; therefore the radius of a circle which is as DK^3, will be as $\frac{1}{CN^3}$, that is, the radius of a circle touching the ellipsis is reciprocally as the cube of a perpendicular let fall from the center C on the tangent DN. Wherefore the increases of the degrees in D, by going forwards from the equator to the poles, will be as $\frac{1}{\overline{CN}^3} - \frac{1}{\overline{AC}^3}$, that is, as $CA^3 - \overline{CN}^3$, or as $CM^3 - CN^3$, or likewise as $CM^3 - CD^3$, because the difference of the right lines CN, CD, is very small. But $CM^3 = \overline{CD+DM}^3 = CD^3 + 3CD^2 \times DM + 3DM^2 \times CD + DM^3$, therefore $CM^3 - CD^3 = 3CD^2 \times DM + 3DM^2 \times CD + DM^3 = 3CD^2 \times DM$, because the quantities DM^2, DM^3, almost vanishing in respect of $3CD^2 \times DM$, therefore the increments of the degrees are as $3CD^2 \times DM$, or as DM, because the right line CD is nearly constant. Wherefore the increments of the degrees are as the increments of the weights.

SIR,

THE following proposition having been often taken for granted, but no where demonstrated, that I have seen, I here send you a demonstration of it.

Yours, &c. W. CHAPPLE.

PROPOSITION.

IN any acute-angled triangle, if perpendiculars be let fall from each angle on the opposite side, they will all three intersect each other in one and the same point within the triangle.

And in any obtuse-angled triangle, if the sides including the obtuse angle be continued, and perpendiculars let fall on them from the acute angles, and from the obtuse angle on its subtending side, those perpendiculars will, if continued, intersect each other in the same point without the triangle.

DEMONSTRATION.

IN any obtuse-angled triangle, A ⊙ B, produce the sides A ⊙ and B ⊙ to E and G, and let fall the perpendiculars BE, AG, and D ⊙, and continue AG and D ⊙ till they intersect in F; then will BE, if continued, also intersect them in F, *if the proposition be true*; however let us, for the present, suppose it otherwise, and that it inter- Pl. 8. Fig. 164.

ſects ⊙F in ſome other point, as f, ſo that Ef (not EF) be perpendicular to ⊙E. Now 'tis plain that the triangles ABE, AD⊙, and ⊙Ef, are ſimilar, as are alſo the triangles ABG, DB⊙, and ⊙GF, and the triangles B⊙E, and A⊙G. Put $a=90$, $u=\angle DA\odot$, and x its complement $=a-u$; $y=\angle\odot AG$, and z its complement $=a-y$; $\angle G\odot F=u+y$; $\angle GF\odot=a-u-y$, &c. and the ſum of the angles at A, B, F, and f, will be found $=2a$. But this will be the ſum of the angles, ſuppoſing F and f coincident, *per* 37 *E.* 1. And as the angle ⊙fE will be either greater or leſs than the $\angle\odot FE$, if f be removed any the leaſt diſtance from F, 'tis therefore evident that they will be coincident, and conſequently the perpendiculars BE, D⊙, and AG, will, if continued, interſect each other in F. And as this will always raiſe an acute-angled triangle on the baſe of the obtuſe one, 'tis plain that the ſame demonſtration ſerves for both.

The Method for finding Fluents, according to Mr Emerſon's *Table, continued from Page* 252.

EXAMPLE XXI.

To find the *Fluent* of $\frac{r\dot{y}}{y\sqrt{yy-rr}}$?

Divide by yy, then $\frac{r\dot{y}}{y\sqrt{yy-rr}}$ is reduced to $\frac{ry^{-2}\dot{y}}{\sqrt{1-rry^{-2}}}$, which belongs to *Form* 10. where $z=y$, $n=-2$, $\alpha=1$, $\beta=rr$; and the *Fluent* of $\frac{y^{-2}\dot{y}}{\sqrt{1-rry^{-2}}}$ or of $\frac{\dot{y}}{y\sqrt{yy-rr}}$, is $=\frac{2N}{-2\sqrt{rr}}\times$ degrees in the arc whoſe ſine is $\sqrt{rry^{-2}}$, or $\sqrt{\frac{rr}{yy}}$; and Fluent of $\frac{r\dot{y}}{y\sqrt{yy-rr}}$ is $=-.017453\times$ degrees in the arc, whoſe ſine is $\frac{r}{y}$, and radius $=1$.

I preſume it will be quite needleſs to give the operation in particular numbers to any of theſe examples, the proceſs being ſo plain and eaſy.

Having now gone through the firſt ten Forms which the author, at page 53. of his *Doctrine of Fluxions*, calls original forms, becauſe from them (as it ſeems to me) the reſt are deduced, before I proceed any further, I think it not amiſs to inſert ſome more examples to ſeveral of theſe his Forms, which will ſerve ſtill further to illuſtrate their uſe to the learner. And theſe I ſhall diſpoſe into a ſort of

a table, ſetting down only the Fluxions, and their correſpondent Fluents over againſt them, leaving the reſt of the operation for the learner's exerciſe.

Form III.

Fluxions.	Fluents.
$\overline{a+\beta z^n}^{-\frac{1}{2}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{1}{2}}}{\frac{1}{2}n\beta}$
$\overline{a+\beta z^n}^{\frac{1}{2}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{3}{2}}}{\frac{3}{2}n\beta}$
$\overline{a+\beta z^n}^{\frac{3}{2}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{5}{2}}}{\frac{5}{2}n\beta}$
$\overline{a+\beta z^n}^{-\frac{1}{3}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{2}{3}}}{\frac{2}{3}n\beta}$
$\overline{a+\beta z^n}^{\frac{1}{3}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{4}{3}}}{\frac{4}{3}n\beta}$
$\overline{a+\beta z^n}^{\frac{2}{3}} z^{n-1}\dot{z}$	$\frac{\overline{a+\beta z^n}^{\frac{5}{3}}}{\frac{5}{3}n\beta}$
$\overline{a+\beta z^{-2}}^{\mu} z^{-3}\dot{z}$	$\frac{\overline{a+\beta z^{-2}}^{\mu+1}}{\mu+1\times-2\beta}$
$\overline{a+\beta z^{-1}}^{\mu} z^{-2}\dot{z}$	$\frac{\overline{a+\beta z^{-1}}^{\mu+1}}{\mu+1\times-\beta}$
$\overline{a+\beta z}^{\mu}\dot{z}$	$\frac{\overline{a+\beta z}^{\mu+1}}{\mu+1.\beta}$
$\overline{a+\beta z^2}^{\mu} z\dot{z}$	$\frac{\overline{a+\beta z^2}^{\mu+1}}{\mu+1.2\beta}$
$\overline{a+\beta z^3}^{\mu} z^2\dot{z}$	$\frac{\overline{a+\beta z^3}^{\mu+1}}{\mu+1.3\beta}$
&c.	

Form

Form IV.

Fluxions.	Fluents.
$\frac{z^{-3}\dot{z}}{a+\beta z^{-2}}$	$\frac{2.30258}{-2\beta} \times \log. \frac{a+\beta z^{-2}}{}$
$\frac{z^{-2}\dot{z}}{a+\beta z^{-1}}$	$\frac{2.30258}{-\beta} \times \log. \frac{a+\beta z^{-1}}{}$
$\frac{\dot{z}}{a+\beta z}$	$\frac{2.30258}{\beta} \times \log. \frac{a+\beta z}{}$
$\frac{z\dot{z}}{a+\beta z^2}$	$\frac{2.30258}{2\beta} \times \log. \frac{a+\beta z^2}{}$
$\frac{z^2\dot{z}}{a+\beta z^3}$	$\frac{2.30258}{3\beta} \times \log. \frac{a+\beta z^3}{}$
&c.	&c.

Form V.

Fluxions	Fluents.
$\frac{z^{-2}\dot{z}}{a+\beta z^{-2}}$	$\frac{.017453}{-\sqrt{a\beta}} \times$ deg. in the arc, whose tang. $= \sqrt{\frac{\beta}{azz}}$, rad. $= 1$
$\frac{z^{-\frac{1}{2}}\dot{z}}{a+\beta z^{-1}}$	$\frac{.017453}{-\frac{1}{2}\sqrt{a\beta}} \times$ deg. tang. $= \sqrt{\frac{\beta}{az}}$, rad. $= 1$
$\frac{z^{-\frac{1}{2}}\dot{z}}{a+\beta z}$	$\frac{.017453}{\frac{1}{2}\sqrt{a\beta}} \times$ deg. tang. $= \sqrt{\frac{\beta z}{a}}$, rad. $= 1$
$\frac{\dot{z}}{a+\beta z^2}$	$\frac{.017453}{\sqrt{a\beta}} \times$ deg. tang. $= z\sqrt{\frac{\beta}{a}}$, rad. $= 1$
$\frac{z^{\frac{1}{2}}\dot{z}}{a+\beta z^3}$	$\frac{.017453}{\frac{3}{2}\sqrt{a\beta}} \times$ deg. tang. $= z\sqrt{\frac{\beta z}{a}}$, rad. $= 1$
&c.	&c.

Form VI.

Fluxions	Fluents
$\frac{z^{-2}z}{a-\beta z^{-2}}$	$\frac{2.30258}{-2\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta z^{-2}}}{\sqrt{a}-\sqrt{\beta z^{-2}}}$
$\frac{z^{-\frac{1}{2}}\dot{z}}{a-\beta z^{-1}}$	$\frac{2.30258}{-\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta z^{-1}}}{\sqrt{a}-\sqrt{\beta z^{-1}}}$
$\frac{z^{-\frac{1}{2}}\dot{z}}{a-\beta z}$	$\frac{2.30258}{\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta z}}{\sqrt{a}-\sqrt{\beta z}}$

$\frac{\dot{x}}{a-\beta x^2}$	$\frac{2.30258}{2\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta x^2}}{\sqrt{a}-\sqrt{\beta x^2}}$
$\frac{x^{\frac{1}{2}}\dot{x}}{a-\beta x^3}$	$\frac{2.30258}{3\sqrt{a\beta}} \times \log. \frac{\sqrt{a}+\sqrt{\beta x^3}}{\sqrt{a}-\sqrt{\beta x^3}}$
&c.	

Form IX.

Fluxions	Fluents.
$\frac{x^{-2}\dot{x}}{\sqrt{a+\beta x^{-2}}}$	$\frac{2.30258}{-\sqrt{\beta}} \times \log. \sqrt{\beta x^{-2}}+\sqrt{a+\beta x^{-2}}$
$\frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{a+\beta x^{-1}}}$	$\frac{2.30258}{-\frac{1}{2}\sqrt{\beta}} \times \log. \sqrt{\beta x^{-1}}+\sqrt{a+\beta x^{-1}}$
$\frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{a+\beta x}}$	$\frac{2.30258}{\frac{1}{2}\sqrt{\beta}} \times \log. \sqrt{\beta x}+\sqrt{a+\beta x}$
$\frac{\dot{x}}{\sqrt{a+\beta x^2}}$	$\frac{2.30258}{\sqrt{\beta}} \times \log. \sqrt{\beta x^2}+\sqrt{a+\beta x^2}$
$\frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a+\beta x^3}}$	$\frac{2.30258}{\frac{3}{2}\sqrt{\beta}} \times \log. \sqrt{\beta x^3}+\sqrt{a+\beta x^3}$
$\frac{x\dot{x}}{\sqrt{a+\beta x^4}}$	$\frac{2.30258}{2\sqrt{\beta}} \times \log. \sqrt{\beta x^4}+\sqrt{a+\beta x^4}$
&c.	&c.

Form X.

Fluxions	Fluents.
$\frac{x^{-2}\dot{x}}{\sqrt{a-\beta x^{-2}}}$	$\frac{.017453}{-\sqrt{\beta}} \times$ deg. in the arc, whose fine $= \frac{1}{x}\sqrt{\frac{\beta}{a}}$, rad. $= 1$.
$\frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{a-\beta x^{-1}}}$	$\frac{.01745}{-\sqrt{\beta}} \times$ deg. fine $= \frac{2}{a}\sqrt{\frac{a\beta x-\beta^2}{xx}}$, rad. $= 1$.
$\frac{x^{-\frac{1}{2}}\dot{x}}{\sqrt{a-\beta x}}$	$\frac{.01745}{\sqrt{\beta}} \times$ deg. fine $= \frac{2}{a}\sqrt{a\beta x-\beta^2 x^2}$, rad. $= 1$.
$\frac{\dot{x}}{\sqrt{a-\beta x^2}}$	$\frac{.01745}{\sqrt{\beta}} \times$ deg. fine $= x\sqrt{\frac{\beta}{a}}$, rad. $= 1$.
$\frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{a-\beta x^3}}$	$\frac{.01745}{3\sqrt{\beta}} \times$ deg. fine $= \frac{2}{a}\sqrt{a\beta x^3-\beta^2 x^6}$, rad. $= 1$.
$\frac{x\dot{x}}{\sqrt{a-\beta x^4}}$	$\frac{.01745}{2\sqrt{\beta}} \times$ deg. fine $= xx\sqrt{\frac{\beta}{a}}$, rad. $= 1$.
&c.	&c.

An useful and general Propositon for finding Fluents.

The PROPOSITION.

THE *Fluent* of the *Fluxion* $\overset{A}{\overline{e+fz^n}}^{m-1}.a\dot{z}z^{v-1}+\overset{B}{\overline{e+fz^n}}^{m-1}\times b\dot{z}z^{v+n-1}+\overset{C}{\overline{e+fz^n}}^{m-1}.c\dot{z}z^{v+2n-1}+\overset{D}{\overline{e+fz^n}}^{m-1}.d\dot{z}z^{v+2n-1}$ &c. (puting $r=v+nm$) will be $A+\frac{abz^v.\overline{e+fz^n}^m-v.beA}{af.r}+\frac{bcz^{v+n}.\overline{e+fz^n}^m-\overline{v+n}.ceB}{bf.\overline{r+n}}+\frac{cdz^{v+2n}.\overline{e+fz^n}^m-\overline{v+2n}.deC}{cf.\overline{r+2n}}$ &c. *ad infinitum*; where the law of continuation is visible.

DERIVATION.

(1) $\overline{e+fz^n}.a\dot{z}z^{v-1}=ae\dot{z}z^{v-1}+af\dot{z}z^{v+n-1}$, multiplying both sides by $\overline{e+fz^n}^{m-1}$, $\overline{e+fz^n}^m.a\dot{z}z^{v-1}=\overline{e+fz^n}^{m-1}.ae\dot{z}z^{v-1}+\overline{e+fz^n}^{m-1}.af\dot{z}z^{v+n-1}$, (*i. e.*) puting Q= the *Fluent* of $\overline{e+fz^n}^m\times\dot{z}z^{v-1}$, $a\dot{Q}=e\dot{A}+\frac{af\dot{B}}{b}$ $\therefore aQ=eA+\frac{afB}{b}$, and $Q=\frac{eA}{a}+\frac{fB}{b}$.

(2) $z^v.\overline{e+fz^n}^m$ in *Fluxions* is $v\dot{z}z^{v-1}.\overline{e+fz^n}^m+\overline{e+fz^n}^{m-1}.mfn\dot{z}z^{v+n-1}$, $=vQ+\frac{mfn\dot{B}}{b}$. But the contemporary *Fluents* of the same or equal *Fluxions* are equal; consequently $z^v.\overline{e+fz^n}^m=vQ+\frac{mnfB}{b}$. By writing for Q here its equal, found above, and proceeding according to the rules of common algebra, we get $B=\frac{abz^v.\overline{e+fz^n}^m-v.beA}{af.v+nm}$.

After the same manner, taking $\overline{e+fz^n}.b\dot{z}z^{v+n-1}=be\dot{z}z^{v+n-1}+bf\dot{z}z^{v+2n-1}$ (and proceeding as before ($C=\frac{bcz^{v+n}.\overline{e+fz^n}^m-\overline{v+n}.ceB}{bf.v+n+nm}$, $D=\frac{cdz^{v+2n}\overline{e+fz^n}^m-v+2n.deC}{cf.v+2n+nm}$ &c. by writing r for $v+nm$ the proposition is evident.

COROLLARY I.

If A, the *Fluent* of $\overline{e+fz^n}^{m-1}.a\dot{z}z^{v-1}$ were given, to find B the *Fluent* of $\overline{e+fz^n}^{m-1}.b\dot{z}z^{v+n-1}$, it will be $B=\frac{abz^v.\overline{e+fz^n}^m-v.beA}{afr}$.

CO-

COROLLARY II.

If A = the *Fluent* of $\overline{e+fz^n}|^{m-1} . a\dot{z}z^{v-1}$, C the *Fluent* of $\overline{e+fz^n}|^{m-1} . c\dot{z}z^{v+2n-1}$ will be

$$\frac{acfr . z^{v+n} . \overline{e+fz^n}|^m - \overline{v+n} . ceaz^v . \overline{e+fz^n}|^m + \overline{v+n} . vce^2A}{affr . \overline{r+n}}.$$

For, by Propoſition, $C = \frac{cbz^{v+n} . \overline{e+fz^n}|^m - \overline{v+n} . ceB}{bf . \overline{r+n}}$, and $B = \frac{abz^v . \overline{e+fz^n}|^m - v . beA}{afr}$; Th. by ſubſtitution, C as above.

COROLLARY III.

If A, or the *Fluent* of $\overline{e+fz^n}|^{m-1} . a\dot{z}z^{v-1}$, be given, to find D, or the *Fluent* of $\overline{e+fz^n}|^{m-1} . d\dot{z}z^{v+3n-1}$, it will be

$$D = \frac{\overline{r+n} . a^2fdrz^{v+2n} . \overline{e+fz^n}|^m - \overline{v+2n} . daefrz^{v+n} . \overline{e+fz^n}|^m + \overline{v+n} . \overline{v+2n} . dae^2z^v . \overline{e+fz^n}|^m - \overline{v+2n} . \overline{v+n} . vde^3A}{\overline{r+n} . a^3fr^2 + 2a^3frn}$$

Thus we may proceed to get Corollaries at pleaſure, which may ſerve as ſo many *Formulas* for the gaining of *Fluents* leſs general, as will appear by the following examples.

EXAMPLE I.

If it were required to find the *Fluent* of $\overline{q+cz^n}|^{-1} . d\dot{z}z^{n-1}$, having the *Fluent* of $\overline{q+cz^n}|^{-1} . d\dot{z}z^{2n-1}$ given?

Here A is given to find B, $m=0$, $a=b=d$, $q=e$, $c=f$, $n=v$, $r\ (=\overline{v+nm})=n$, &c. therefore by ſubſtitution at Corollary 1. $B = \frac{d^2z^n - dqnA}{dc . n} = \frac{z^n}{c} - \frac{qA}{c}$.

EXAMPLE II.

Given the *Fluent* of $\overline{p+qz^n}|^{-1} . d\dot{z}z^{n-1}$, to find that of $\overline{p+qz^n}|^{-1} . d\dot{z}z^{3n-1}$?

Here A is given to find C, $m=0$, $a=c=d$, $p=e$, $q=f$, $n=v=r$, &c. therefore by ſubſtituting at Corollary 2. $C = \frac{d^2qn . z^{2n} - 2nd^2pz^n + 2n . ndp^2A}{dq^2n . 2n} = \frac{dz^{2n}}{2qn} - \frac{dpz^n}{q^2n} + \frac{p^2A}{q^2}$, the *Fluent* ſought.

EX-

EXAMPLE III.

If the *Fluent* of $\overline{p+qx^n}|^{-\frac{1}{2}}.gx^{\frac{n}{2}-1}$ were given, to find that of $\overline{p+qx^n}|^{-\frac{1}{2}}.bx^{\frac{5n}{2}-1}$?

Here again A is given, to find C, $p=e$, $q=f$, $x=z$, $m=\frac{3}{2}$, $v=\frac{n}{2}$, $r=2n$, $g=a$, $b=c$, &c. Th. by ſubſtituting as before at Cor. 2. C, or the *Fluent* ſought, is

$$\frac{2gbqnx^{\frac{3n}{2}}.\overline{p+qx^n}|^{\frac{3}{2}}-\frac{3n}{2}bpgx^{\frac{n}{2}}.\overline{p+qx^n}|^{\frac{3}{2}}+\frac{1}{4}bn^2p^2A}{2gq^2n.3n}=\frac{bx^{\frac{3n}{2}}}{3qn}.\overline{p+qx^n}|^{\frac{3}{2}}$$

$$-\frac{bpx^{\frac{n}{2}}}{4q^2n}.\overline{p+qx^n}|^{\frac{1}{2}}+\frac{bp^2A}{8gq^2}.$$

EXAMPLE IV.

Suppose the *Fluent* of $\overline{p+qz^n}|^{\frac{1}{3}}.bx^{\frac{n}{3}-1}$ were given, to find the *Fluent* of $\overline{p+qz^n}|^{\frac{1}{3}}.dxz^{\frac{4n}{3}-1}$?

Here A is given to find D, $m=\frac{4}{3}$, $p=e$, $q=f$, $v=\frac{n}{3}$, $b=a$, $d=d$, $r=\frac{5n}{3}$, &c. Th. by ſubſtituting at Corrollary 3.

$$D=\frac{40n^2}{9}bq^2dz^{\frac{7n}{3}}.\overline{p+qz^n}|^{\frac{4}{3}}-\frac{35n^2}{9}dbpqz^{\frac{4n}{3}}.\overline{p+zq^n}|^{\frac{4}{3}}+$$

$$\frac{\frac{28n^2}{9}dq^2bz^{\frac{n}{3}}.\overline{p+qz^n}|^{\frac{4}{3}}-\frac{28n^2}{27}.dp^2A}{\frac{8n}{3}.\frac{25bq^2n^2}{9}+\frac{10bq^2n^2}{3}}$$

$$=\frac{3dz^{\frac{7n}{3}}}{11qn}.\overline{p+qz^n}|^{\frac{4}{3}}-\frac{21pd^2pz^{\frac{4n}{3}}}{88q^2n}.\overline{p+qz^n}|^{\frac{4}{3}}+\frac{21dz^{\frac{n}{3}}}{110qn}.\overline{p+qz^n}|^{\frac{4}{3}}-\frac{7dp^2A}{110bq^2}$$

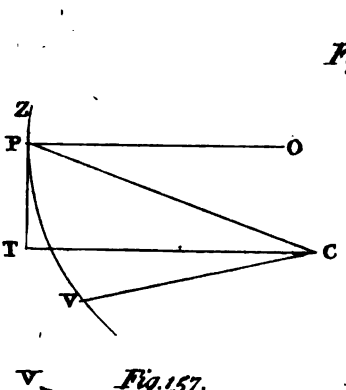
the *Fluent* required.

Other *Fluents* leſs general than theſe may eaſily be found from what has paſt.

Yours, &c. J. Powle.

A

Fig. 160.



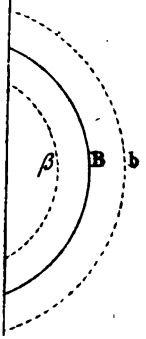


Fig. 157.

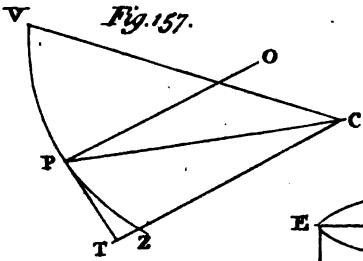


Fig. 161.

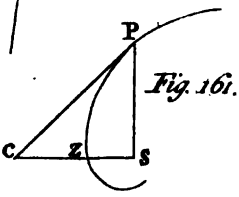


Fig. 162.

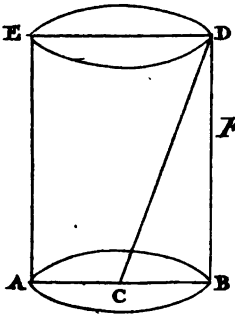


Fig. 158.

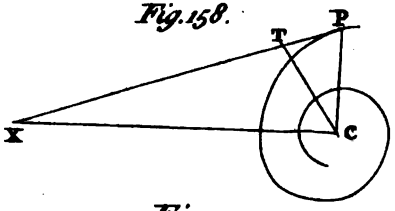


Fig. 159.

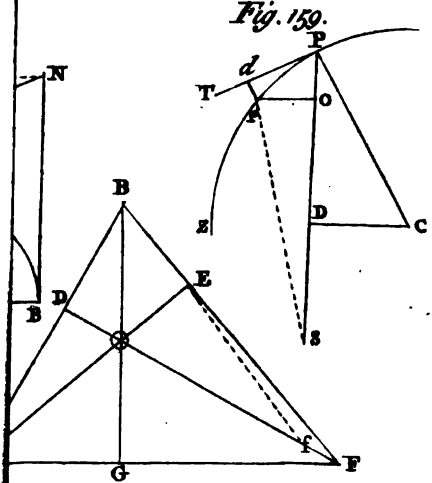


Fig. 163.

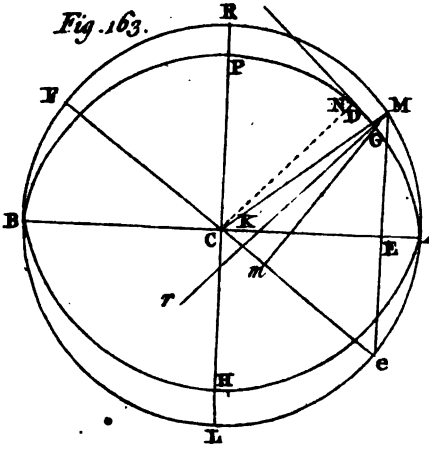


PLATE IX.

Fig. 366

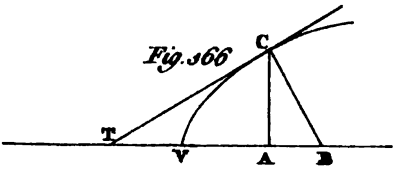


Fig. 367

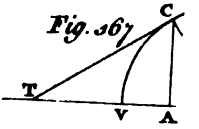
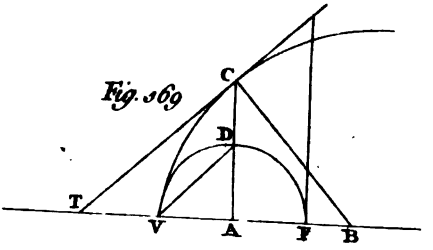
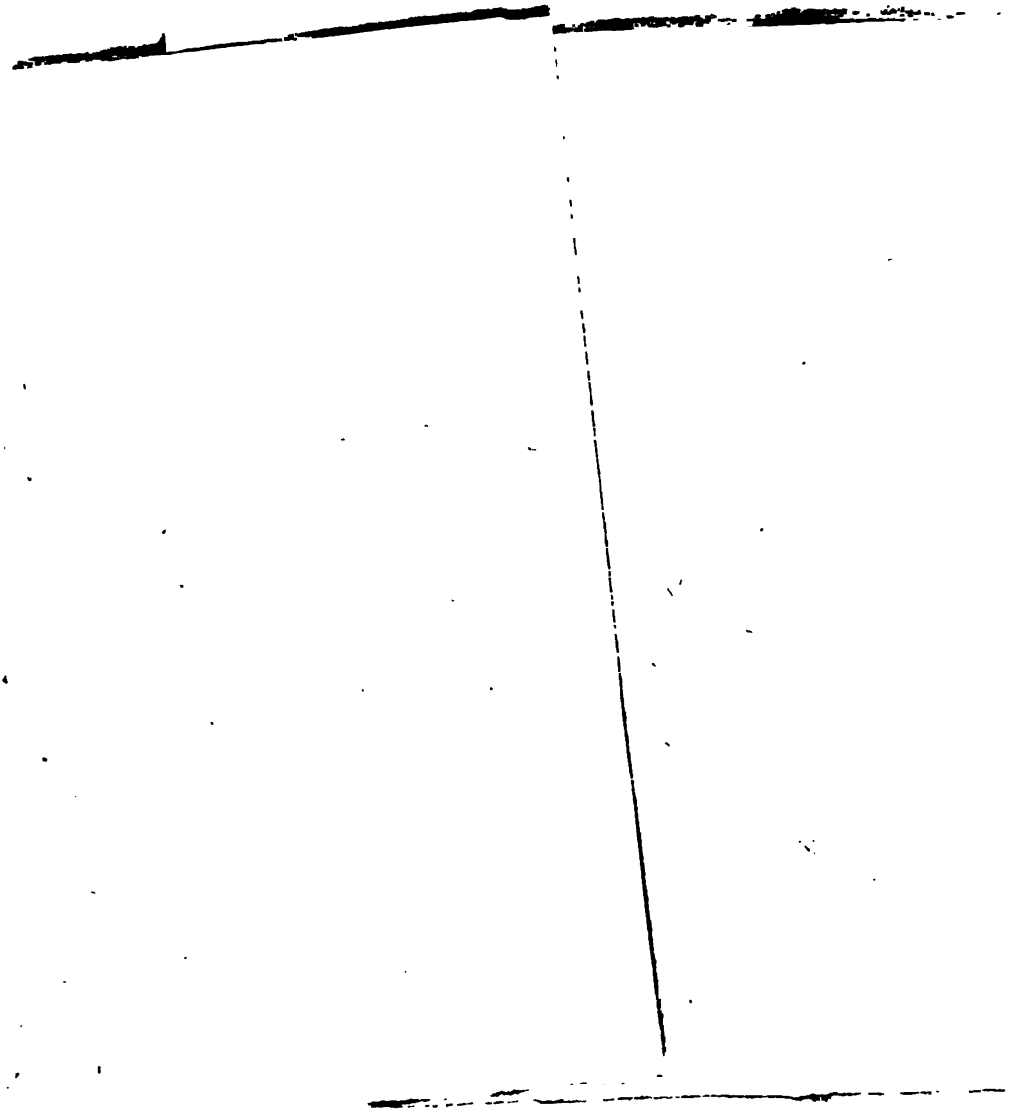


Fig. 369



$\frac{2}{9\times 2}\times\frac{s}{rr}A-\frac{5\times 8}{9\times 3\times 4}\times\frac{s}{rr}B-\frac{11\times 14}{9\times 5\times 6}\times\frac{s}{rr}C-\frac{17\times 20}{9\times 7\times 8}\times\frac{s}{rr}D$, &c. *ad infinitum*, putting A, B, C, D, &c. equal to the first, second, third, fourth, &c. terms respectively; from which method of continuing the series is apparent.

Art. II. But when s is $\sqsubset rr$, then $a=\sqrt[3]{}:\sqrt{-s}+r:+\sqrt[3]{}:-\sqrt{-s}+r:$ is $=\frac{2r}{3\sqrt[3]{-s}}+\frac{10r^3}{81s\sqrt[3]{-s}}$, &c. From the like with the above method of proceeding, *viz.* by Sir *Isaac Newton*'s theorem $=-\frac{2r}{3\times s}-\frac{5}{9\times 3}\times\frac{rr}{s}A-\frac{8\times 11}{9\times 4\times 5}\times\frac{rr}{s}B-\frac{14\times 17}{9\times 6\times 7}\times\frac{rr}{s}C$, &c. *ad infinitum*,



A SCHOLIUM *intirely new, shewing how* Cardan's *Theorems for Cubic Equations are deduced, illustrated with Examples.*

INstead of what is in inserted between the 6th line of page 186 and the 25th line of page 187, 2d edition of *Ronayne*'s algebra, I think the following scholium will be found useful.

SCHOLIUM.

FROM the 3d and 19th steps *Cardan*'s theorems may be easily deduced, and that case wherein the value of $\frac{1}{4}cc+\frac{1}{27}b^3$ is $\sqsubset 0$, may be solved, nearly, by series deduced from the particular theorem for that purpose, which series shall be inserted in the four following articles; thus,

If $a^3-pa=q$; then the particular theorem for finding the affirmative value of a in this equation, deducible from the 3d and 19th steps, is $a=\sqrt[3]{:\frac{q}{2}+\sqrt{\frac{qq}{4}-\frac{p^3}{27}}:}+\sqrt[3]{:\frac{q}{2}-\sqrt{\frac{qq}{4}-\frac{p^3}{27}}:}$; that is making $r=\frac{q}{2}$, and $\sqrt{-s}=\sqrt{:\frac{qq}{4}-\frac{p^3}{27}:}$ a is $=\sqrt[3]{:r+\sqrt{-s}:}+\sqrt[3]{:r-\sqrt{-s}}$. Now since in any infinite series, whether it arises from division or evolution, the first terms are always supposed to be the greatest, so as the last may vanish, or become $=0$: Therefore

Art. I. When rr is $\sqsubset s$, then $\sqrt[3]{r}\pm\sqrt{-s}$ will be found by Sir *Isaac Newton*'s theorem (*See part* XV. *chap.* I.) $=r^{\frac{1}{3}}+\frac{1}{3}r^{\frac{1}{3}-1}\times\pm\sqrt{-s}+\frac{\frac{1}{3}}{1}\times\frac{\frac{1}{3}-1}{2}\times r^{\frac{1}{3}-2}\times-s+\frac{\frac{1}{3}}{1}\times\frac{\frac{1}{3}-1}{2}\times\frac{\frac{1}{3}-2}{3}\times r^{\frac{1}{3}-3}\times\pm\sqrt{-s^3}+\frac{\frac{1}{3}}{1}\times\frac{\frac{1}{3}-1}{2}\times\frac{\frac{1}{3}-2}{3}\times\frac{\frac{1}{3}-3}{4}\times r^{\frac{1}{3}-4}+ss$, &c. Consequently a $(=\sqrt[3]{:r+\sqrt{-s}:}+\sqrt[3]{:r-\sqrt{-s}:})$ is $=2r^{\frac{1}{3}}*+\frac{2s}{9r^3}*-\frac{20ss}{243r^{1\frac{2}{3}}}*$ &c. $=2\times r^{\frac{1}{3}}+\frac{2}{9\times2}\times\frac{s}{rr}A-\frac{5\times8}{9\times3\times4}\times\frac{s}{rr}B-\frac{11\times14}{9\times5\times6}\times\frac{s}{rr}C-\frac{17\times20}{9\times7\times8}\times\frac{s}{rr}D$, &c. *ad infinitum*, putting A, B, C, D, &c. equal to the first, second, third, fourth, &c. terms respectively; from which method of continuing the series is apparent.

Art. II. But when s is $\sqsubset rr$, then $a=\sqrt[3]{:\sqrt{-s}+r:}+\sqrt[3]{:-\sqrt{-s}+r:}$ is $=\frac{2r}{3\sqrt[3]{-s}}+\frac{10r^3}{81s\sqrt[3]{-s}}$, &c. From the like with the above method of proceeding, *viz.* by Sir *Isaac Newton*'s theorem $=-\frac{2r}{3\times s^{\frac{1}{3}}}-\frac{5}{9\times3}\times\frac{rr}{s}A-\frac{8\times11}{9\times4\times5}\times\frac{rr}{s}B-\frac{14\times17}{9\times6\times7}\times\frac{rr}{s}C$, &c. *ad infinitum*,

nitum, putting A, B, C, &c. equal to the firſt, ſecond, third, &c. terms reſpectively.

This ſeries expreſſes one of the two negative values of a in the equation $a^3 - pa = q$.

Art. III. When rr is $\sqsubset s$, and $r^{\frac{1}{3}}$ is = a ſurd number; then from the cube of the value of a, in the former ſeries, ſubtract pa, and the remainder, by the propoſed equation, will be $= q$; and then the equation, thus had, when reduc'd, will give $a \left(= \frac{a^3 - q}{p}\right) =$

$8r + \frac{s}{3rr}A - \frac{7s}{27rr}B - \frac{37s}{189rr}C - \frac{3899s}{61111rr}D$ &c. $- q \div p$, putting A, B, C, D, &c. equal to the firſt, ſecond, third, fourth, &c. terms reſpectively; in which value of a, conſiſting of known terms, ſurds are intirely excluded.

Art. IV. When s is $\sqsubset rr$, and $s^{-\frac{1}{3}}$ is = a ſurd number; then from the cube of the value of a, expreſs'd in *Art.* 2. ſubtract pa, and the remainder, by the propoſed equation, will $= q$; and then the equation, thus had, when reduc'd, will give $a \left(= \frac{a^3 - q}{p}\right)$

$= -\frac{8r}{27s} - \frac{5rr}{9s}A - \frac{91rr}{135s}B - \frac{5471rr}{7371s}C$, &c. $- q \div p$, putting A, B, C, &c. equal to the firſt, ſecond, third, &c. terms reſpectively. In which equation you have one of the negative values of a in known terms, free from ſurdity.

I will here inſert two Examples of this remarkable caſe.

EXAMPLE I.

If $a^3 - 1203a = 16000$, *Quere a?*

Here $r = \frac{1}{2}q = \frac{16000}{2} = 8000$, and $-s = \frac{1}{4}qq - \frac{1}{27}p^3 = 64000000 - 64481281 = -481201$ is $\sqsupset 0$: Likewiſe rr is $\sqsubset s$; wherefore this example relates to the aforementioned caſe, *Art.* 1 or 3; and $r^{\frac{1}{3}} = \sqrt[3]{8000} = 20$, is a rational number; therefore the ſeries in *Art.* 1. is preferable to that in the 3d for our preſent purpoſe.

Operation

$2r^{\frac{1}{3}} = A = +40.$

$9rr)sA = B = +\ 0.03341673611$

$-27rr)105sB = C = -\ .0000930565\overline{2}$

$-135rr)775sC = D = +\ .0000003990\overline{7}$

$-126rr)855sD = E = -\ .0000000020\overline{2}$

Anſwer $a = 40.0333240766\overline{4}$

EX-

EXAMPLE II.

If $a^3-15a=4$, Quere a?

Here $r=\frac{1}{2}q=\frac{4}{2}=2$, and $-s=\frac{1}{4}qq-\frac{1}{27}p^3=4-125=-121$ is $\sqsupset 0$; likewiſe s is $\sqsubset rr$, wherefore this example relates to the abovementioned caſe *Art.* 2[d] or 4[th]; but $r^{\frac{1}{3}}=\sqrt[3]{121}$ is a ſurd number, therefore the ſeries in *Art.* 4. is preferable here to that in the ſecond.

OPERATION.

$$\begin{aligned}
-27s)8r^3 \quad &=A=-0.01958983\\
-9s)5rrA \quad &=B=+0.00035977\\
-135s)91rrB \quad &=C=-0.00000801\\
-7371s)5471rrC &=D=+0.00000019\\
\&c. \quad &= \quad \ldots\ldots\ldots\\
-q= \quad &-4.0\\
&-4.01923788
\end{aligned}$$

And $p=15)-4.01923788(-0.2679491=a$, *Anſwer.*

Note $\sqsubset$ ſtands for greater, and $\sqsupset$ leſs.

H.

An ESSAY *on the Golden Number, Cycle of the Sun,* Roman *Indiction, the* Julian *and* Dionyſian *Periods, &c. By Mr* W. BROWN.

Of the GOLDEN NUMBER.

THE golden number, prime, lunar cycle, or enneadecaterides, is a revolution of 19 years, in which time the new moons, *&c.* return (within $1\frac{1}{2}$ hour nearly) to the ſame days they were on before; and indeed *Meton,* an old *Athenian,* the inventor of this number, who liv'd long before our Saviour's nativity, and the fathers of the primitive church after it, by reaſon of their unſkilfulneſs in calculations of this ſort, thought that the moon's change, and full, fell preciſely again on the ſame day of the month that they did 19 years before: hence at the time of the *Nicene* council, when the way of the ſettling the time for obſerving the feaſt of *Eaſter* was eſtabliſh'd, the numbers of this cycle were inſerted in the firſt column of kalendar in golden letters, from whence came the names of prime, and golden number; but, as before was taken notice of, the lunations in every 304 years anticipating one whole day by reaſon of the $1\frac{1}{2}$ hour's return ſooner in 19 years, occaſions their happening almoſt 5 days ſooner now, than at the council of *Nice,* which was in the year of our lord 325.

Of

Of the Cycle of the SUN.

THE cycle of the ſun is a revolution of 28 years, in which time the ſame dominical letter comes about again, in the ſame order, and leap years expire: This cycle ſerves to find the dominical letter for any year.

Of the ROMAN *Indiction.*

THE cycle of indiction is a revolution of three luſtrums, or 15 years, after which, thoſe who uſed it began it again; this is more antient than the two preceding cycles, and hath nothing to do with the heavenly motions, being eſtabliſh'd by *Conſtantine*, A. D. 312, who ſubſtituted them in the room of the *Olympiads*. They were ſo called, becauſe they denoted the year that tribute was to be paid to the republic.

Having juſt defin'd the three foregoing cycles, it may be proper to ſhew how to find either of them for any year of Chriſt. And here it is to be obſerv'd that the year before the Chriſtian Æra the golden number was 1, the cycle of the ſun 9, and the *Roman* indiction 3; therefore as theſe three cycles conſiſt, reſpectively, of 19, 28, and 15 years, we have this eaſy rule: Add 1 to the year, and divide by 19, for the golden number; add 9 to the year, and divide by 28, for the cycle of the ſun; and add 3 to the year, and divide by 15 for the indiction, the quotients will ſhew the number of cycles of each ſince Chriſt, and the remainder, after each diviſion, the required reſpective cycle.

Of the DIONYSIAN *Period.*

From the cycles of the ſun and moon (before explain'd) multiplied into one another, ariſes another period of 532 years (for $28 \times 19 = 532$) called the *Dionyſian* period; after the completion of which, not only the new and full moons return to the ſame days of the month, but the days of the month return to the ſame days of the week; and therefore the dominical letter, and moveable feaſts return again in the ſame order, whence this is called the great paſchal cycle.

To find the year of the *Dionyſian* period for any year of Chriſt, to the current year, add the number 457 (the year of that cycle when the Chriſtian Æra commenc'd) and divide the ſum by 532, the remainder after the diviſion is the year of the period required.

Of the JULIAN *Period.*

From the multiplication of the lunar cycle of 19 years, the ſolar of 28 years, and the indiction of 15 years, ariſes the great *Julian* period

period of 7980 years (for $19\times28\times15=7980$) which is ſuppos'd to have its beginning 764 years before the creation; it comprehends all other period, cycles, and epochas; for there is but one year in this whole period, which has the ſame numbers for the three cycles of which it is made up: the year before Chriſt happen'd in the 4713th year of this period; therefore to find the year of the *Julian* period for any year of Chriſt, to the current year, add 4713, the ſum will be the year of the *Julian* period.

Here let the foregoing rules be illuſtrated by finding what is the golden number, cycle of the ſun, *Roman* indiction, *Dionyſian* and *Julian* periods, for the preſent year 1747.

I.

1747
1
19)1748(92
171
38
38
0 Since the remainder is 0, the golden number is the ſame with the diviſor = 19.

II.

1747
9
28)1756(62
168
76
56
20 = the cycle of the Sun.

III.

1747
3
15)1750(116
15
25
15
100
90
10 = the *Roman* indiction.

IV.

1747
457
532)2204(4
2128
76 = the *Dionyſian* period.

V.

1747
4713
6460 = the *Julian* period.

Now if the cycles of the moon, and ſun, are given, to find the year of *Chriſt*; or if the cycles of the moon, ſun, and indiction are given, to find the year of *Chriſt*; then they become problems of a different nature, but may be eaſily ſolv'd by the two following.

PROBLEM I.

Requir'd the year, when the prime is 5, and cycle of the ſun 6?

SOLUTION.

Put x for the requir'd year of Chriſt; then (by the nature of the queſtion) we have $\frac{x+1-5}{19}=b$, and $\frac{x+9-6}{28}=a$; which will

be whole numbers; Th. $x=28a-3$, and $x=19b+4$, conſequently $19b+4=28a-3$, and $b=\frac{28a-7}{19}=a+\frac{9a}{19}-\frac{7}{19}$, Th. $\frac{9a-7}{19}=m$, ſome integer, and $a=\frac{19m+7}{9}=2m+\frac{m}{9}+\frac{7}{9}$, and Th. $\frac{m+7}{9}=k$, an integer; Th. $m=9k-7$, and this, put for m above, gives $a=19k-14$, and Th. this put for a in $x=28a-3$, gives $x=532k-395$, the required year.

Here ſuppoſe $k=$to any integer, as 1, 2, 3, or 4, &c. then will $x=532\times1-395=137$, the year; or $x=532\times2-395=669$, the year; or $x=532\times3-395=1201$, the year; or laſtly $x=532\times4-395=1733$, the requir'd year, &c.

Or UNIVERSALLY.

Put x for the year of our lord; $m=$the golden number; and $s=$the cycle of the ſun; then we have $\frac{x+1-m}{19}=b$, and $\frac{x+9-s}{28}=a$; and, by a like proceſs with that above, we ſhall get $x=532p+57s-56m-457$, the required year; where $p=1$, 2, or 3, &c. Hence this general

CANON.

To 532 times p, add 57 times the cycle of the ſun; from that ſum ſubſtract 56 times the golden number added to 457, and the remainder will be the year of Chriſt.

EXAMPLE.

If the cycle of the moon be 19, that of the ſun 20, quere the year of our lord? Here by the general canon we have (making $p=4$) $532\times4+57\times20-56\times19-457=1747$, the year requir'd.

COROLLARY.

From the univerſal equation, $x=532p+57s-56m-457$, by tranſpoſition, we have $x+457=532p+57s-56m$; but $x+457=$the *Dionyſian* period: hence we obtain this general rule for finding the *Dionyſian* period, from the cycles of the ſun and moon given.

The RULE.

Multiply the cycle of the ſun by 57, and the cycle of the moon by 56; divide the difference of their products by 532, the remainder, after

after the quotient, will give the year of the *Dionyſian* period. N.B. when the latter product exceeds the former, then ſubſtract the remainder, after the quotient, from 532, for the *Dionyſian* period.

EXAMPLE I.

What is the *Dionyſian* period when the cycle of the ſun is 20, and the cycle of the moon is 19? Here $57\times20-56\times19=76$ (which cannot be divided by 532) the *Dionyſian* period.

EXAMPLE II.

What is the *Dionyſian* period, when the cycle of the ſun is 1, and the cycle of the moon 19? Here in this caſe $1\times57-19\times56=-1007$; Th. $\frac{1007}{532}$ quotes 1, and leaves 475, which ſubſtracted from 532 there will remain 57 for the *Dionyſian* period.

PROBLEM II.

When the prime is 8, the cycle of the ſun 9, and the *Roman* indiction 14, what year of our Lord is it?

SOLUTION.

Putting x (as before) we have by the nature of the queſtion, $\frac{x+1-8}{19}=b$; $\frac{x+9-9}{28}=a$; and $\frac{x+3-14}{15}=c$; to find x? Here (1) $x=19b+7$, (2) $x=28a$, and (3) $x=15c+11$. Th. $19b+7=28a$, and $b=\frac{28a-7}{19}=a+\frac{9a}{19}-\frac{7}{19}$; Th. $\frac{9a-7}{19}$(make)$=m$ ſome integer; and $a=\frac{19m+7}{9}=am+\frac{m}{9}+\frac{7}{9}$; Th. $\frac{m+7}{9}=k$, ſome whole number, and $m=9k-7$; conſequently by ſubſtitution $a=19k-14$, and Th. $x=532k-392$ (by the 2d); but $x=15c+11$ (by the 3d); Therefore $15c+11=532k-392$, and $c=\frac{532k-403}{15}=35k+\frac{7k}{15}-26-\frac{13}{15}$; Th. $\frac{7k-13}{15}$ muſt be ſome whole number: Hence k muſt be ſome integer, that multiplied by 7, and 13 taken from the product, may be diviſible by 15; Th. k by a few tryals is found to be 4 (for $\frac{7\times4-13}{15}=1$) which value of k, ſubſtituted for it, in $x=532k-392$, will give $x=532\times4-392=1736$, the required year; and ſo for any other.

Or

Or UNIVERSALLY.

Put x=year of Chriſt; m=cycle of the moon; s=the cycle of the ſun; and r=*Roman* indiction: Then by the nature of the queſtion $\frac{x+1-m}{19}=b$; $\frac{x+9-s}{28}=a$; and $\frac{x+3-r}{15}=c$, will be whole numbers; now by purſuing the above method, we ſhall get $x=4845s-3780m-1064r-4713-7980k$=the required year. Whence this general

CANON.

Multiply the golden number by 3780, the *Roman* indiction by 1064, the ſum of the two products, and 4713, taken from 4845 times the cycle of the ſun, will leave a remainder, which divided by 7980, the quotient will be the value of k, and the remainder the requir'd year. N. B. when the ſum of the negative products exceeds the poſitive one, divide the difference by 7980, and take the remain from 7980 for the year.

EXAMPLE I.

If the cycle of the moon be 19, that of the ſun 20, and that of the indiction 10, quere the year of the Lord? Here by the general canon we have $4845\times20-3780\times19-1064\times10-4713=9727$. Th. $\frac{9727}{7980}=1=k$; and the remainder is 1747, the year of our Lord requir'd.

EXAMPLE II.

If $m=19$, $s=1$, and $r=6$, quere x? In this caſe $1\times4845-19\times3780-6\times1064-4713=-78072$. Th. $\frac{78072}{7980}$ quotes 9, and leaves 6252, which ſubſtracted from 7980, there will remain 1728 for the year requir'd.

COROLLARY.

From the univerſal equation $x=4845s-3780m-1064r-4713-7980k$, by tranſpoſition, we have $x+4713=4845s-3780m-1064r-7980k$; but $x+4713$= the *Julian* period: hence we obtain this general rule for finding the *Julian* period from the cycle of the ſun, moon, and indiction given.

The RULE.

Multiply the cycle of the ſun by 4845, the cycle of the moon by

by 3780, and the *Roman* indiction by 1064; divide the difference betwixt the former, and sum of the two latter products, by 7980, and the remainder, after the quotient, will give the year of the *Julian* period.

N. B. When the sum of the two latter products exceeds the former, then subtract the remainder, after the quotient, from 7980, for the *Julian* period.

EXAMPLE I.

What is the *Julian* period when the cycle of the sun is 20, cycle of the moon 19, and the *Roman* indiction 10? Here $20\times4845-19\times3780-10\times1064=14440$. Th. $\frac{14440}{7980}$ quotes 1, and leaves 6460 the remainder for the *Julian* period.

EXAMPLE II.

What is the *Julian* period when the cycle of the sun is 1, that of the moon 19, and that of the indiction 6? Here in this case $1\times4845-19\times3780-6\times1064=-73359$. Th. $\frac{73359}{7980}$ quotes 9, and leaves 1539, which subtracted from 7980, there will remain 6441 for the *Julian* period.

I am, SIR, Yours, &c.

Cleobury, June 12, 1747. W. BROWN.

A Specimen of a general Method of drawing Tangents to Curves, deduced from the Doctrine of Maxima *and* Minima. *By Mr* WILLIAM JEPSON.

TO the tangent TC, at the point of contact, draw the perpendicular BC, which will be a *minimum*; and let AV$=x$, AC$=y$, BV$=n$, and AB$=n-x$; then by 47 *Euc.* I. $y^2+n^2-2nx+x^2=\overline{BC}^2$: Th. $2y\dot{y}-2n\dot{x}+2x\dot{x}=0$, out of which, by help of the equation to the curve, exterminate $\dot{y}$, or $2y\dot{y}$; when it can be done conveniently, and the resulting equation being reduc'd, will give the value of n, and thence AB; whence by similar triangles $\frac{\overline{AC}^2}{AB}=AT$. Plate 9. Fig. 166.

EXAMPLE I.

In the common parabola, where $ax=yy$; $2y\dot{y}=a\dot{x}$: Therefore $a\dot{x}-2n\dot{x}+2x\dot{x}=0$: Th. $n=\frac{1}{2}a+x$, and $AB=\frac{a}{2}$; hence $AT=\frac{2yy}{a}=2x$.

EXAMPLE II.

In the ellipſe, where $2a$=tranſverſe, and p=parameter; $2y\dot{y}=p\dot{x}-\frac{px\dot{x}}{a}$: Th. $p\dot{x}-\frac{px\dot{x}}{a}-2n\dot{x}+2x\dot{x}=0$: Th. $n=\frac{p}{2}+x-\frac{px}{2a}$, and $AB=\frac{p}{2}-\frac{px}{2a}$: Hence $AT=\frac{px-\frac{px^2}{2a}}{\frac{p}{2}-\frac{px}{2a}}=\frac{2ax-xx}{a-x}$; and in the hyperbola, by a like proceſs, we get $AB=\frac{p}{2}+\frac{px}{2a}$, and thence $AT=\frac{2ax+xx}{a+x}$.

EXAMPLE III.

Plate 9. Fig. 167. In the circle, where a=radius; $2y\dot{y}=2a\dot{x}-2x\dot{x}$: Th. $2a\dot{x}-2n\dot{x}=0$. Th. $n=a$, and $AB=a-x$; hence $AT=\frac{2ax-xx}{a-x}$.

EXAMPLE IV.

Plate 9. Fig. 168. In the conical hyperbola, where $ay=x^2$; $2y\dot{y}=\frac{4x^3\dot{x}}{aa}$; and Th. $\frac{4x^3\dot{x}}{aa}-2n\dot{x}+2x\dot{x}=0$. Th. $n=\frac{2x^3}{aa}+x$, and $AB=\frac{2x^3}{aa}$; hence $AT=\frac{x^4}{2x^3}=\frac{x}{2}$.

EXAMPLE V.

In the ciſſoid, where $VD=a$, DE being the aſymptote, and $yy=\frac{x^3}{a-x}$; $2y\dot{y}=\frac{3ax^2\dot{x}-2x^3\dot{x}}{\overline{a-x}^2}$: Th. $\frac{3ax^2\dot{x}-2x^3\dot{x}}{\overline{a-x}^2}-2n\dot{x}+2x\dot{x}=0$, which reduced gives $n=\frac{a^2x-\frac{1}{2}ax^2}{\overline{a-x}^2}$, and thence $AB=\frac{3ax^2-2x^3}{2x\overline{a-x}^2}$; hence $AT=\frac{2ax-2xx}{3a-2x}$.

EX-

EXAMPLE VI.

In the cycloid, let VA=x, AC=y, AD=v, arch VD=z; and radius of the generating circle=a; then by the nature of the cycloid $y=v+z$; and therefore $\dot{v}+\dot{z}=\dot{y}$; by the property of the circle $\dot{v}=\frac{a\dot{x}-x\dot{x}}{v}$, and $\dot{z}=\frac{a\dot{x}}{v}$. Th. $\dot{y}=\frac{2a\dot{x}-x\dot{x}}{v}$. Theref. $\frac{4ya\dot{x}-2yx\dot{x}}{v}-2n\dot{x}+2x\dot{x}=0$, whence $n=\frac{2ya-yx}{v}+x$, and AB=$\frac{2ya-yx}{v}$; hence AT=$\frac{vy}{2a-x}=\frac{xy}{v}$. Plate 9. Fig. 169.

But the ſubtangent may be more eaſily found, for the property of the cycloid is ſuch, that the chord VD of the circle is always parallel to the tangent TC of the cycloid. Therefore by ſimilar triangles DA : VA : : AC : AT=$\frac{VA\times AC}{DA}$.

Suppoſe VDE to be (the ſemicubical parabola) a given curve, and VCE another curve of ſuch a nature that DC ſhall always be a mean proportional between VA and AF, 'tis required to draw the tangent TC. Plate 9. Fig. 170.

EXAMPLE VII.

Let the parameter of the parabola=VF=a, VA=x, AC=y, AD=v, DC=z; then $y=v+\dot{z}$, and $\dot{y}=\dot{v}+\dot{z}$. By the property of the parabola $\dot{v}=\frac{2a^{\frac{1}{3}}\dot{x}}{3x^{\frac{1}{3}}}$, and by the nature of the figure $\dot{z}=\frac{a\dot{x}-2x\dot{x}}{2\sqrt{ax-xx}}$. Th. $\dot{y}=\frac{2a^{\frac{1}{3}}\dot{x}}{3x^{\frac{1}{3}}}+\frac{a\dot{x}-2x\dot{x}}{2\sqrt{ax-xx}}$; and therefore $\frac{4ya^{\frac{1}{3}}x}{3x^{\frac{1}{3}}}+\frac{ya\dot{x}-2yx\dot{x}}{\sqrt{ax-xx}}-2n\dot{x}+2x\dot{x}=0$; whence $n=\frac{2ya^{\frac{1}{3}}}{3x^{\frac{1}{3}}}+\frac{ya-2yx}{2\sqrt{ax-xx}}+x$, and $\frac{2ya^{\frac{1}{3}}}{3x^{\frac{1}{3}}}+\frac{ya-2yx}{2\sqrt{ax-xx}}$=AB. Hence AT=

$$\frac{6yx^{\frac{1}{3}}\sqrt{ax-xx}}{4a^{\frac{1}{3}}\sqrt{ax-xx}+3ax^{\frac{1}{3}}-6x^{\frac{4}{3}}}=\frac{6a^{\frac{1}{3}}x\sqrt{ax-xx}+6ax^{\frac{4}{3}}-6x^{\frac{7}{3}}}{4a^{\frac{1}{3}}\sqrt{ax-xx}+3ax^{\frac{1}{3}}-6x^{\frac{4}{3}}}=\frac{a^{\frac{1}{3}}x^{\frac{2}{3}}\sqrt{ax-xx}+ax-xx}{\frac{2}{3}a^{\frac{1}{3}}x^{\frac{1}{6}}\sqrt{a-x}+\frac{1}{2}a-x}.$$

EX-

EXAMPLE VIII.

Plate 9. Fig. 174. LET DC be an exponential curve, VA=x, AC=y, and $a^x=y$; and let A, Y be the hyp. logarithms of a, y; then xA=Y, and $A\dot{x}=\dot{Y}=\frac{\dot{y}}{y}$. Th. $\dot{y}=Ay\dot{x}$, whence $2Ay^2\dot{x}-2n\dot{x}+2x\ddot{x}=0$. Th. $n=Ay^2+x$, and AB=Ay^2, hence AT=$\frac{1}{A}$.

EXAMPLE IX.

LET the nature of the curve be expreſs'd by this equation $x^x=y$, and let X, Y be the hyp. logarithms of x, y; then $x\dot{X}+X\dot{x}=\dot{Y}$, and $\dot{X}=\frac{\dot{x}}{x}$, $\dot{Y}=\frac{\dot{y}}{y}$. Th. $y\dot{x}+yX\dot{x}=\dot{y}$, whence $2y^2\dot{x}+2y^2X\dot{x}-2n\dot{x}+2x\ddot{x}=0$. Th. $n=y^2+y^2X+x$, and AB=y^2+y^2X; hence AT=$\frac{1}{1+X}$.

Lincoln, July 20, 1747.

Yours, &c.

WILLIAM JEPSON.

EMENDATIONS.

The Solution to Queſtion XV. *p.* 63, *corrected by Mr* Wm Chapple.

PUT $Pm=20$, $AP=30=c$, $DE=x$, and $EP=z$; alſo let a=the parameter $=\frac{b^4}{c}=5333.33$, and $f=.7854$. Plate 2. Fig. 36.

Then $c-z=AE$, and $a\times\overline{c-z}=x^4$, by the nature of the curve; hence $z=c-\frac{x^4}{a}$, which ſubſtituted in fx^2z (the ſolidity of the cylinder) gives $\frac{f}{a}\times\overline{acx^2-x^6}$; which being a *maximum*, its fluxion $\frac{f}{a}\times\overline{2acx\dot{x}-6x^5\dot{x}}=0$; conſequently $2ac=6x^4$, and $\overline{\frac{ac}{3}}\Big|^{\frac{1}{4}}=x=15.1967$, the ſemidiameter of the cylinder's baſe. Hence z the altitude (being $=c-\frac{x^4}{a}$ as above) $=20$, and the ſolidity $=14510.3813$.

The Anſwer to Queſtion 47, *by* Logarithmicus, *corrected.*

PUTTING d=the depth, b=the breadth, and l=the length in inches.

$$\left.\begin{array}{l} d+\frac{l}{5}=b.\ \text{Th. } l=5b-5d \\ d+\frac{2l}{5}=l.\ \text{Th. } l=\frac{5d}{3} \end{array}\right\}\text{whence } b=\frac{4d}{3};\text{but}(\sqrt{ll+bb+dd}=)$$

$\sqrt{\frac{25dd}{9}+\frac{16dd}{9}+dd}$=length of the longeſt line, that can be inſerted in the ſolid, and $(lxbxd=)\ \frac{5d}{3}\times\frac{4d}{3}\times d=\frac{20d^3}{9}$=the ſolidity. Th. $\sqrt{\frac{25dd}{9}+\frac{16dd}{9}+dd}=\frac{20d^3}{9\times1728}$, and $\frac{50d^2}{9}=\frac{400d^6}{\overline{9}|^2\times\overline{1728}|^2}$, whence $\left(\frac{\overline{9\times1728}|^2}{8}=\right)\ 3^8\times2^9=d^4$; whence the malt buſhels may be found by the following logarithmical proceſs.

To 8 times the log. of 3	3.8169704
Add 9 times the log. of 2	2.7092700
Divide the ſum by	4)6.5262404
The quotient is the log. of *d*	1.6315601
To 3 times the log. of *d*	4.8946803
Add the log. of 20	1.3010300
And the arith. comp. of 9	$\bar{1}$.0457575
The ſum is the log. of the ſolidity in inches	5.2414678
To which add the log. $\bar{4}$.6674767 (ſee *Dodſon*'s Anti-Logarithmic Canon, Page 72)	$\bar{4}$.6674767
The ſum is the log. of 81.0849, the malt ~~buſhels~~ requir'd	1.9089445

The following Corrections of the Eſſay of the Julian *and* Gregorian *Accounts, in the Celebration of Eaſter, came from an unknown Hand, to whom we are much oblig'd for his candid Obſervations, viz.*

I Beg leave to obſerve to you, in your emendation of print errors, that, I think, page 228, line 5, 6, and 7, had better be thus: In the *Julian* years 1700 to 1800, in 1800 to 1900, in 1900 to 2100, in 2100 to 2200, in 2200 to 2300, in 2300 to 2500, in 2500 to 2600, in 2600 to 2700, *&c.* And again, in lines 24, 25, and 26, in p. 228, thus: Years of *Chriſt* 1700, and onwards to 1800, 1800 to 1900, 1900 to 2100, 2100 to 2200, 2200 to 2300, 2300 to 2500, *&c.* *Page* 231, line 8, read 39—y days. ~~And in the 3d~~ paragraph of the ſame page, laſt four lines, thus: $6+7z$ greater than $(54-y=)$ 33, we ſhall have $z=4$, and, conſequently, $6+7z=34$, *viz.* the 3d ~~day of~~ *April* old ſtyle, or $3+11=14$th day of *April* new ſtyle, ~~will be the *Gregorian*~~ Eaſter day in the year 1748. I ~~think your laſt paragraph~~, in page 232, had better have propoſed a method of ſubtracting 3 days ~~out of 391 years~~, which might eaſily be done, by ordering by fives and ſevens reſpectively each other 132d and 128th years to be common years, both the fives and ſevens beginning and ending with a 132d year. I conceive that the *Gregorian* epacts muſt each be leſſen'd by 1, every 100th year, that is, not made a leap year; becauſe, the addition of 1 for every 312 years are *Julian* years.

New

New QUESTIONS *to be answered in our next.*

QUESTION CXXX. *By Mr* HOLLIDAY.

REQUIRED the fewest number of weights, and the weight of each, that shall weigh from one pound to twenty nine hundred weight?

QUESTION CXXXI. *By Mr* WILLIAM DYKE.

ON the top of a mountain with a common quadrant, turn'd upside down, I took an angle at the setting sun of 0° 4′; at the same time, I saw two ships at sea in a right line with one another, and I took an angle at the neareſt ship of 1° 50′, and at the fartheſt ship of 0° 10′; required the mountain's height, the distance of each ship from the place of observation, and the distance of the ships from one another?

QUESTION CXXXII. *By Mr* THOMAS BOULTON.

SUPPOSE the diameter of a grinding stone be 40 inches, 4 inches thick at the edge, and to rise conically on each side to the center, so that at the center it shall be 10 inches thick; three men A, B and C were each to grind down $\frac{1}{7}$ of this stone; required a theorem, to shew each part; A grinding first?

QUESTION CXXXIII. *By Mr* JOHN ASH.

IN a semicircular space, two semicircles are inscribed, whose radii, together, are equal to the radius of the said semicircular space; required to find the radius of the greatest circle that can possibly be inscribed in the curvilineal triangle, made by the three semicircular curves?

N. B. The radius of the semicircular space is =50, that of the greatest inscribed circle =30, consequently that of the less =20.

QUESTION CXXXIV. *By Mr* PAUL SHARP.

SUPPOSE the periphery of a circle be divided into 10000 equal parts, and two indices, *viz.* A and B to move from the same place, at the same time; but contrary in motion; A to move in arithmetical progression, 10 of those equal parts the first day, 20 the 2d, 30 the 3d, *&c.* and B to move 9 the first day, 90 the 2d, *&c.* in a geometrical proportion whose common ratio is 10; how many days, and how many parts must each move before they coincide?

QUES-